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Input Impedance of a Probe Antenna Exciting a TEM Cell

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CONTENTS

FOREWORD

This report describes theoretical and experimental analyses developed by staff of the University of Colorado at Boulder in collaboration with the Electromagnetic Fields Division of the National Bureau of Standards (NBS), under a contract sponsored by NBS. Professor David C. Chang heads the University team. Dr. Mark T. Ma of NBS serves as the technical contract monitor. The period covered by this report extends from July 1980 to July 1981.

The work presented in this report represents a further aspect of establishing a theoretical basis for the technical analyses of transverse electromagnetic (TEM) transmission line cells. The general purpose of pursuing theoretical studies is to evaluate the use of TEM cells for (1) measuring the total rf radiated power by a device inserted into the cell for test, or (2) performing necessary susceptibility tests on a small electronic device.

The particular topic addressed herein is to determine the input impedance of a probe antenna which may be inserted into a TEM cell to measure or excite fields. The formulation of the problem is based on a variational principle, which insures that the input impedance so obtained is stationary for small arbitrary variations in the probe current distribution about its true value. The evaluation of the variational integral requires a knowledge of the field distribution inside the cell excited by a vertical short electric dipole. This latter problem was the subject of a previous report by the same authors [4], the results of which are readily adapted to the present problem. Thus, the input impedance treated in this report represents an extension to the previous studies.

The analysis of the present problem is significantly simplified by assuming that the gap between the inner septum and the outer walls of the cell is electrically small, and that the operating frequency is such that no higher order modes are excited inside the cell. These assumptions are consistent with the practical cell utilization and design principle.

The final impedance expression is shown to consist of two distinct terms, which can be clearly identified as the respective contributions by the ordinary rectangular waveguide and the gap perturbation.

Previous publications under the same effort include:

Tippet, J. C. and Chang, D. C., Radiation characteristics of dipole sources located inside a rectangular coaxial transmission line, NBSIR 75-829 (Jan. 1976).

Tippet, J. C., Chang, D. C., and Crawford, M. L., An analytical and experimental determination of the cut-off frequencies of higher-order TE modes in a TEM cell, NBSIR 76-841 (June 1976).

Tippet, J. C. and Chang, D. C., Higher-order modes in rectangular coaxial line with infinitely thin inner conductor, NBSIR 78-873 (March 1978).

Sreenivasiah, I. and Chang, D. C., A variational expression for the scattering matrix of a coaxial line step discontinuity and its application to an over moded coaxial TEM cell, NBSIR 79-1606 (May 1979).

Tippet, J. C. and Chang, D. C., Dispersion and attenuation characteristics of modes in a TEM cell with a lossy dielectric slab, NBSIR 79-1615 (Aug. 1979).

Sreenivasiah, I., Chang, D. C., and Ma, M. T., Characterization of electrically small radiating sources by tests inside a transmission line cell, NBS Tech Note 1017 (Feb. 1980).

Wilson, P. F., Chang, D. C., and Ma, M. T., Excitation of a TEM cell by a vertical electric Hertzian dipole, NBS Tech Note 1037 (March 1981).

Sreenivasiah, I., Chang, D. C., and Ma, M. T., A method of determining the emission and susceptibility levels of electrically small objects using a TEM cell, NBS Tech Note 1040 (April 1981).

INPUT IMPEDANCE OF A PROBE ANTENNA EXCITING A TEM CELL

P. F. Wilson, D. C. Chang and M. T. Ma

The input impedance of a probe antenna exciting a transverse electromagnetic (TEM) transmission line cell is formulated by a variational approach. The formulation also utilizes the results from a previous work on the field distribution inside a TEM cell excited by a vertical electrical Hertzian dipole. The final result of impedance is shown to consist of two distinct terms, which are respectively contributed by the ordinary rectangular waveguide and the gap perturbation. Numerical results for both the real and imaginary parts of the impedance are given. The resistive part is found to be proportional to the square of the probe length, and the reactive part largely capacitive.

Key words: Green's function; input impedance; probe antenna; radiation resistance; rectangular coaxial transmission line; TEM cell; variational method.

1. INTRODUCTION

The National Bureau of Standards (NBS) has been interested in developing a method for testing the emission and susceptibility properties of electronic equipment. One approach to this problem is the transverse electromagnetic (TEM) transmission line cell. The TEM cell, shown in figure 1, consists of a section of rectangular coaxial tramission line (RCTL) coupled at each end to standard 50Ω cylindrical coaxial line via a tapered section. The RCTL section supports a dominant TEM mode and provides an isolated, standard test field environment. The TEM cell results may be related to the free space environment via results given by Tippet [1] and Sreenivasiah [2].

This report concerns the manner in which an idealized coaxial probe excites an infinite section of RCTL. The results may be used to study the effect of coaxial probes inserted into the TEM cell either to excite, or to measure fields. In addition, the configuration studied may be useful in modeling feed lines to electronic equipment. The method of solution is to formulate a variational expression for the input impedance of the probe at the aperture plane, as was done by Collin [3] in analyzing the coaxial probe excitation of a rectangular waveguide. The evaluation of the variational integral requires that the Green's function corresponding to the fields excited in an RCTL by a vertical electric dipole (VED) be known. The VED excitation of an RCTL was the subject of a previous report by the authors [4], the results of which will be adapted to the present problem. The analysis of the Green's function is significantly simplified by assuming that the gap between the inner septum and the outer walls, is electrically small, and that the operating frequency is not too large. Both these premises are consistent with the TEM cell usage.

Evaluating the variational impedance integral results in two physically distinct terms, which may be referred to as the ordinary rectangular waveguide contribution, and the gap perturbation contribution. The ordinary term gives the input reactance due to a coaxial probe in a rectangular waveguide with the same dimensions as that of the RCTL's upper chamber. Since the dimensions of the cell are such that the rectangular waveguide modes in the upper chamber would be cutoff, the result is a highly capacitive term for which the analysis yields an approximate analytic expression. The gap perturbation term represents the change from the ordinary result due to the presence of the gap fields, and contains the radiation resistance resulting from the dominant TEM mode. The perturbation term, as derived, remains in an integral form which must be treated numerically. However, if the TEM mode radiation resistance is extracted, the remaining reactive integral is negligible when compared to the highly capacitive ordinary contribution. Thus neglecting the perturbation reactance results in a computationally simple algebraic form for the probe input impedance.

Section 2 of this report will define the variational impedance integral to be evaluated and discuss the necessary probe idealizations. The Green's function associated with the VED excitation of an RCTL is examined in Section 3. Section 4 gives the result of the evaluation of the impedance integral and examines the distinct physical nature of the resulting terms. Certain special cases are used to verify the solution. These checks are examined in Section 5 along with numerical results. Many details of the analysis and a listing of the FORTRAN program written to evaluate the impedance terms are left to the Appendices.

2. VARIATIONAL FORMULA FOR THE PROBE INPUT IMPEDANCE

The RCTL cross section and the coaxial-line probe antenna are depicted in figure 2. The septum, of width 2w, is allowed to be vertically offset with the heights of the upper and lower chambers given by b_1, and b_2, respectively. The width of the upper chamber is $2a_1$, and the width of the lower chamber is $2a_2$. The gap between the septum and the outer wall is g. The probe, of length d and radius t, is located in the upper chamber. A TEM cell would have an RCTL cross section with equal chamber widths and previous discussions have assumed this geometry. However, in order to avoid any degeneracies caused by letting $a_1 = a_2 = a$, the analysis will initially assume general chamber widths. The time convention will be $\exp(-i\omega t)$.

If the coaxial-line aperture is small, it is reasonable to neglect the higher order modes excited at the aperture discontinuity and assume only incident and reflected TEM modes exist in the coaxial-line. As shown by Collin [3] in his analysis of the coaxial-line probe antenna excitation of an ordinary rectangular waveguide, the above approximation allows the antenna input impedance Z to be expressed as follows

$$Z = \frac{-1}{I_t^2} \int_S \int_{S'} \bar{J}(\bar{x}) \cdot \bar{\bar{K}}(\bar{x},\bar{x}') \cdot \bar{J}(\bar{x}') ds' ds \qquad (1)$$

where $\bar{x}$ and $\bar{x}'$ denote source and observation points, $\bar{J}(\bar{x})$ is the unknown current density on the probe, $\bar{\bar{K}}(\bar{x},\bar{x}')$ is a dyadic Green's function corresponding to the electric field excited by a unit strength dipole source, and I_t is the total current at the aperture plane. S denotes the probe surface plus the aperture plane which carries an equivalent electric current $\bar{J}(\bar{x}) = \bar{n} \times \bar{H}(\bar{x})$ where $\bar{n}$ is a unit vector in the outward normal direction. Experimental studies have shown that Z is effectively independent of the aperture size for electrically small apertures [3]. Therefore it is reasonable to idealize the probe as driven by a y-directed E-field concentrated at the base. Thus we need consider a y-directed current only, which reduces $\bar{J}(\bar{x})$ to $\bar{a}_y J(\bar{x})$, and $\bar{\bar{K}}(\bar{x},\bar{x}')$ to $\bar{a}_y K(\bar{x},\bar{x}')\bar{a}_y$ with $\bar{a}_y$ being the y-directed unit vector.

The above impedance expression may be shown to be variational in $J(\bar{x})$. That is, a first order approximation for $J(\bar{x})$ leads to a second order approximation for Z. At normal TEM cell operating frequencies, the length of the probe will be small compared to a wavelength. Therefore, it is reasonable to assume that the current distribution is approximately sinusoidal,

$$J(\bar{x}) = \frac{I_0}{2\pi t} \sin k(b_1-d-y)\delta(\rho-t) \qquad (2)$$

where the delta function is interpreted in the sense of a generalized function. The form of the Green's function $K(\bar{x},\bar{x}')$ will be considered next.

3. THE GREEN'S FUNCTION $K(\bar{x},\bar{x}')$

The appropriate Green's function for the special case of equal chamber widths has been derived previously [4]. However, the modifications due to generalizing the chamber widths are not difficult. Since a VED in an RCTL will excite both transverse electric (TE or h-type), and transverse magnetic (TM or e-type) modes, both cases need to be treated. For each modal type we begin by formulating an integral equation for the unknown field in the gap. These integral equations could be solved by Chebyshev polynomial expansion methods [5]. However, a less cumbersome solution may be achieved by making the following pair of assumptions. First, the gap is assumed to be small, and second, the frequency is not allowed to be too high. These assumptions lead to an approximate integral equation which may be solved directly. The small gap assumption amounts to discarding terms which are of the order $G^2 \ell n G$, where G is a normalized gap parameter defined by $G=\pi g/2a$, $a=\min(a_1,a_2)$. Up to the order of this approximation only the TE type modes contribute significantly to the gap field.

The derivation of the Green's function begins with the removal of the z-dependence via a Fourier-transform defined by

$$K^{(j)}(\bar{x},\bar{x}') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{K}^{(j)}(\bar{x}_t,\bar{x}'_t)e^{i\alpha z}d\alpha \qquad (3)$$

where j=1,2 refer to the upper, and lower chambers respectively, the subscript t denotes the transverse plane, α is the transform variable representing a continuum of propagation factors, and the superscript tilde signifies a function in the transform domain. The analysis outlined above yields the following result

$$\tilde{K}^{(j)}(\bar{x}_t,\bar{x}_t') = \tilde{K}_1^{(j)}(\bar{x}_t,\bar{x}_t') + \tilde{K}_2^{(j)}(\bar{x}_t,\bar{x}_t') \qquad (4)$$

where $\tilde{K}_1^{(j)}$ represents the Green's function expected if there were no gap, i.e., the ordinary rectangular waveguide result, and $\tilde{K}_2^{(j)}$ gives the gap field perturbation contribution. $\tilde{K}_1^{(j)}$ and $\tilde{K}_2^{(j)}$ derived in Appendix A are found to be

$$\tilde{K}_1^{(j)}(\bar{x}_t,\bar{x}_t') = -\delta_{1j}\frac{i\omega\mu}{k^2}\left\{\frac{\alpha^2}{\zeta^2}\delta(\bar{x}_t-\bar{x}_t') + \sum_{m,n=0}^{\infty}\frac{\Delta_n[M_j^2+\alpha^2]}{M_j^2+N_j^2-\zeta^2}g_{mn}^{(j)}(\bar{x}_t,\bar{x}_t')\right\} \qquad (5)$$

$$\tilde{K}_2^{(j)}(\bar{x}_t,\bar{x}_t') = \frac{i\omega\mu\pi}{2a_1a_j\zeta^2}L(\alpha)f_j(\bar{x}_t)f_1(\bar{x}_t') \qquad (6)$$

where $\omega = 2\pi f$ is the angular frequency, μ is the permeability, $k = \omega(\mu\varepsilon)^{1/2}$ is the TEM mode propagation constant, δ_{ij} is the Kronecker delta function, $\zeta^2 = k^2-\alpha^2$, $M_j = m\pi/2a_j$, and $N_j = n\pi/b_j$. The remaining functions are defined by

$$g_{mn}^{(j)}(\bar{x}_t,\bar{x}_t') = \left(\frac{2}{a_jb_j}\right)\sin M_j(x+a_j)\sin M_j(x'+a_j)\cos N_jy\cos N_jy' \qquad (7)$$

$$L(\alpha) = \left\{\ln\frac{8\sqrt{a_1a_2}}{\pi g} + \frac{\pi}{2}\sum_{j=1}^{2}\frac{1}{a_j}\sum_{m_o}\left(\frac{\cot K_m^{(j)}b_j}{K_m^{(j)}} + \frac{1}{M_j}\right)\right\}^{-1} \qquad (8)$$

$$f_j(\bar{x}_t) = \sum_{m_o} \frac{M_j \cos K_m^{(j)}(b_j - y)}{K_m^{(j)} \sin K_m^{(j)} b_j} \sin\frac{m\pi}{2} \cos M_j x \, J_o\,(M_j g) \tag{9}$$

$$K_m^{(j)^2} = \zeta^2 - M_j^2 \tag{10}$$

$$\Delta_n = \begin{cases} 1/2 & n=0 \\ 1 & n=1 \end{cases} \tag{11}$$

with m_0 denoting the summation over odd m only (m=1,3,5...), and J_0 is the zeroth order Bessel function. Again the delta function should be understood in the sense of a generalized function.

4. THE INPUT IMPEDANCE Z

4.1. The Green's function terms $\tilde{K}_1^{(j)}$ and $\tilde{K}_2^{(j)}$ will lead to two distinct impedance contributions Z_1 and Z_2 which represent the ordinary and perturbation contributions to the probe input impedance. The total impedance is the sum of these two terms. Combining the form of the Green's function given by (4), with the impedance integral (1), yields $Z = Z_1 + Z_2$, where

$$Z_1 = \frac{-1}{2\pi I_t^2} \int_S \int_{S'} J(\bar{x}) \int_{-\infty}^{\infty} \tilde{K}_1^{(1)}(\bar{x}_t, \bar{x}_t')e^{i\alpha z}\, J(\bar{x}')\, d\alpha\, ds'\, ds \tag{12}$$

$$Z_2 = \frac{-1}{2\pi I_t^2} \int_S \int_{S'} J(\bar{x}) \int_{-\infty}^{\infty} \tilde{K}_2^{(1)}(\bar{x}_t, \bar{x}_t')e^{i\alpha z} J(\bar{x}')\, d\alpha\, ds'\, ds \tag{13}$$

The integrals defined for Z_1 and Z_2 are evaluated in Appendices B and C for a centrally located probe. The approximate form found for Z_1 depends upon two assumptions: (1) the probe is taken to be thin, i.e., $\pi t/2a_1 << 1$, and (2) the frequency is restricted so that $(ka_1)^2$ is small. More precisely, terms in t of the order $(\pi t/2a_1)^2$ are discarded and terms in ka_1 of the order $(k/2M_1)^4$

are discarded. The evaluation of Z_2 involves no approximations. However, Z_2 remains in an integral form. The results of Appendices B and C are as follows

$$Z_1 \approx \frac{i\omega\mu}{2\pi b_1 k^2} \tan^2 \frac{kd}{2} \left\{ \ln \left(\frac{4a_1}{\pi t}\right) + \frac{a_1^2 k^2}{\pi^2} (4.207175) \right.$$

$$\left. -2k^2 \sum_{n=1}^{\infty} \left(1 - \frac{\sin^2 N_1 d/2}{\sin^2 kd/2}\right)^2 \frac{K_o[t(N_1^2 - K^2)^{1/2}]}{N_1^2 - K^2} \right\} \qquad (14)$$

$$Z_2 = \frac{-i\omega\mu k^2}{4a_1^2} \csc^2 kd \int_{-\infty}^{\infty} \frac{L(\alpha)}{\zeta^2} \sum_{m_o} h_{m'}(\alpha,d) \sum_{m_o} h_m(\alpha,d) J_o[t(\alpha^2 + M_1^2)^{1/2}] \, d\alpha \qquad (15)$$

$$h_m(\alpha,d) = \frac{M_1 \sin \frac{m\pi}{2} J_o(M_1 g)}{K_m^{(1)} \sin K_m^{(1)} b_1} \; \frac{(\cos kd - \cos K_m^{(1)} d)}{M_1^2 + \alpha^2} \qquad (16)$$

where $K_o(x)$ is a modified Bessel function.

The perturbation term Z_2 contains the radiation resistance due to the propagating TEM mode, which may be extracted by evaluating the half residues at $\alpha = \pm k$ due to the forward and backward propagating TEM modes. The TEM mode radiation resistance is found to be:

$$R = \frac{\pi\eta_o k^2}{4a_1^2} \csc^2 kd \;\; L(k) \sum_{m_o} h_{m'}(k,d) \sum_{m_o} h_m(k,d) J_o[t(k^2 + M^2)^{1/2}] \qquad (17)$$

where $\eta_o = 120\pi$.

If we now let $a_1 = a_2 = a$ so that we are considering a normal TEM cell geometry, and recall that the characteristic impedance Z_c of an RCTL is given by [4]

$$Z_c = \frac{\eta_o \pi}{8} L(k) \qquad (18)$$

The radiation resistance may be written

$$R = \frac{2k^2}{a^2} Z_c \csc^2 kd \sum_{m_o} h_{m'}(k,d) \sum_{m_o} h_m(k,d) J_o[t(k^2+M^2)^{1/2}] \qquad (19)$$

Because only the TEM mode is allowed to propagate, R represents the total resistance. We may also express Z as the sum of the radiation resistance R and a reactance $i(X+\delta X)$, with X and δX defined by

$$X = \frac{\eta_o}{2\pi b_1 k} \tan^2 \frac{kd}{2} \Bigg\{ \ln\left(\frac{4a}{\pi t}\right) + \frac{a^2 k^2}{\pi^2} (4.207175)$$

$$-2k^2 \sum_{n=1}^{\infty} \left(1 - \frac{\sin^2 N_1 d/2}{\sin^2 kd/2}\right)^2 \frac{K_o[t(N_1^2-k^2)^{1/2}]}{N_1^2 - k^2} \Bigg\} \qquad (20)$$

$$\delta X = \frac{-\omega\mu k^2}{4a^2} \csc^2 kd \; P \int_{-\infty}^{\infty} \frac{L(\alpha)}{\zeta^2} \sum_{m_o} h_{m'}(\alpha,d) \sum_{m_o} h_m(\alpha,d) J_o[t(\alpha^2+M^2)^{1/2}] \; d\alpha \qquad (21)$$

where the integral is now understood in the principal value sense. The dominant reactive contribution comes from the sum over n and therefore the ractance is highly capacitive. The capacitive behavior is expected from comparisons to either a monopole in free space, or a similar probe in a rectangular guide. For typical TEM cell geometries, the principal value integral does not significantly add to the reactance. Therefore, a very good algebraic approximation for the reactance is simply X which allows us to write Z as follows:

$$Z \simeq R + iX, \qquad (22)$$

where R and X are given respectively in (19) and (20).

5. NUMERICAL RESULTS AND DISCUSSIONS

In order to check our result we would like to compare our expression to certain special cases for which solutions are known. The impedance Z consists of three terms: (i) the radiation resistance R due to the propagating TEM mode, (ii) the reactive ordinary rectangular waveguide contribution Z_1, and (iii) the gap field perturbation reactance principal value integral. Ideally, since each of these terms has a distinct physical interpretation, each term should be checked individually. Because the probe is electrically short, the probe radiation resistance may be compared to that of a Hertzian dipole via a result given earlier by Tippet [1]. His expression for the radiation resistance of a Hertzian dipole involves the square of $IE_0 d\ell$ where I is the current magnitude, E_0 is the TEM mode electric field distribution, and $d\ell$ is the effective dipole length. In the monopole case, the dipole quantity $(IE_0 d\ell)$, may be replaced by an effective value which results from integrating the product $I(y)\ E_0(y)$ over the monople length. The two results agree up to the order of our approximations. Next, by letting the gap width go to zero $(g \to 0)$ thereby reducing the upper chamber to an ordinary rectangular waveguide, Z_1 may be compared to a result given by Collin [3]. If we allow the frequency to be large enough that the ordinary rectangular waveguide TE_{10} mode will propagate, then our result reduces exactly to that found by Collin. Both these checks are detailed in Appendix D for the special case when $a_1 = a_2 = a$.

The perturbation reactance term is more difficult to verify and is not comparable to any simple special case result. However, a good check of the self consistency of our expression exists. If we had allowed k to be sufficiently large, such that $k^2 > M_1^2$, the expression (12) for Z_1 would contain poles at $\alpha = \pm\beta_{10}^{(1)}$, where $\beta_{10}^{(1)} = (k^2 - M_1^2)^{1/2}$. This would imply an apparent mode propagating the same as an upper chamber TE_{10} mode. Equation (15) for Z_2 would also imply the existence of this mode. However, the guide as a whole supports no such mode, therefore, we expect that the contributions from Z_1 and Z_2, to this evident $TE_{10}^{(1)}$ mode, should cancel out to the order of our previous approximations. It is found in Appendix D that the radiation resistance due to a propagating $TE_{10}^{(1)}$ mode in a rectangular waveguide of the dimensions of the upper chamber, is given by

$$R = \frac{\eta_o}{2a_1 b_1 k\beta_{10}^{(1)}} \tan^2 \frac{kd}{2} \tag{23}$$

Now consider the perturbation contribution given by Z_2 in (15). In order to avoid the degeneracy due to an additional $TE_{10}^{(1)}$ mode also propagating in the lower chamber, we refrain from letting the chamber widths be equal until after we have evaluated the pole contribution at $\alpha = \pm\beta_{10}^{(1)}$. As α approaches $\pm\beta_{10}^{(1)}$, we see that $k_1^{(1)} = (\beta_{10}^{(1)2} - \alpha^2)^{1/2} \to 0$. Evaluating the half residues at $\alpha = \pm\beta_{10}^{(1)}$, we find

$$L(\alpha) \to \frac{2a_1 b_1(\beta_{10}^{(1)2} - \alpha^2)}{\pi} \tag{24}$$

$$\zeta^2 \to (\pi/2a_1)^2 \tag{25}$$

$$h_m(\alpha,d) \to \frac{\pi(1 - \cos kd)}{2a_1 b_1 k^2(\beta_{10}^{(1)2} - \alpha^2)} J_o\left(\frac{\pi g}{2a_1}\right) \tag{26}$$

$$J_o[t(M_1^2+\alpha^2)^{1/2}] \to J_o(kt) \tag{27}$$

Because both $\pi g/2a_1$ and kt are small, the Bessel functions may be replaced by unity. Collecting results gives

$$Z_2\Big|_{\alpha=\pm\beta_{10}^{(1)}} = \frac{-i\omega\mu k^2}{4a_1^2} \csc^2 kd(-2\pi i) \lim_{\alpha\to\pm\beta_{10}^{(1)}} \frac{L(\alpha)}{(\pi/2a_1)^2} h^2_1(\beta_{10}^{(1)},d)$$

$$= \frac{-\eta_o}{2a_1 b_1 k\, \beta_{10}^{(1)}} \tan^2 \frac{kd}{2} \tag{28}$$

which indeed will cancel the unperturbed result given by expression (23). Thus the poles at $\alpha = \pm\beta_{10}^{(1)}$, that apparently imply a physically meaningless mode, do not contribute up to the order of our approximations. The impedance expressions will next be used to generate numerical data for typical TEM cell geometries.

The radiation resistance R given by (19) is plotted in figure 3 for g/a=0.1 and 0.2. The dimensions of the RCTL cross section are $a_1=a_2=1.0$ m, and $b_1=b_2=1.0$ m. The probe varies in length from d/a=0.0 to d/a=0.95, and is of radius t=1 mm. The frequency is set at 1 MHz. The curves show a basically parabolic dependence on d with the main difference between the two curves resulting from the logarithmic dependence of Z_c on the gap size. This d^2 variation is significantly larger than the free-space probe radiation resistance which is proportional to $(kd)^2$, a frequency dependent quantity. This difference between the radiated power by a short monopole in free space and that delivered to a TEM mode has also been encountered in other types of problems such as the coupling between a dipole antenna and an infinite cable [8]. Figure 4 gives the dependence on d of the ordinary rectangular waveguide reactance X. The same set of parameters is used to plot X, given by (20), as that used to plot R. The curve shows a highly capacitive contribution which is independent of the gap size. Again, this result may be compared to the free-space result for an electrically short monopole which is known as proportional to $-i(kd)^{-1}$ [9]. From (20), the dominant reactance behavior for small kd is more like $-i(kd)^{-2}$. Thus, we find that the reactance of a monopole in a TEM cell is more highly capacitive than that in free space. Finally, the perturbation reactance δX is plotted in figure 5 using (21), again for the same parameter set. We find that $|\delta X|<1\Omega$ for the entire range of probe lengths. Thus δX is at least three orders of magnitude less than X. Therefore, neglecting δX will yield excellent results. The FORTRAN program used to compute Z is listed in Appendix E.

6. CONCLUDING REMARKS

This report has detailed the manner in which a coaxial probe excites a TEM cell. The analysis utilizes three simplifying assumptions, which are (i) the gap is small, that is $G^2 \ell n G << 1$ where $G = \pi g/2a$, (ii) the probe is thin, or $\pi t/2a << 1$ where t is the probe radius, and (iii) the frequency is not too high so that only the TEM mode propagates and $(ka)^2$ is small when compared to unity. None of the requirements is restrictive in terms of common TEM cell usage. The resulting expression for the probe input impedance consists of three distinct terms R, X, and δX, these being the radiation resistance due to the propagating TEM mode, the ordinary rectangular waveguide reactance, and the reactance perturbation due to the gap fields. The first two terms, R and X, are computationally simple, with R showing a basically d^2 dependence on probe length and X giving a highly capacitive reactance. Although δX consists of a nontrivial principal value integral, numerical results show that $|\delta X| << X$, and therefore δX may be neglected. Thus a reasonably straightforward form for Z is achieved which may be used to analyze probes inserted into a TEM cell either to measure, or excite fields as well as other types of feed lines.

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APPENDIX A

DERIVATION OF $\tilde{K}_1^{(j)}$ and $\tilde{K}_2^{(j)}$

Generalizing the chamber widths involves some modifications in the expressions derived in NBS Technical Note 103 [4]. For the most part, this involves nothing more than replacing a by a_j. The details will be briefly outlined beginning with the TE type mode kernel which becomes

$$Q^{(h)}(t,t') = Q_s^{(h)}(t,t') - Q_f^{(h)}(t,t') \tag{a-1}$$

$$Q_s^{(h)}(t,t') \approx -\frac{1}{\pi}\ell n \left[\left(\frac{\pi}{4a_1}\right)\left(\frac{\pi}{4a_2}\right) |t^2 - t'^2|\right] \tag{a-2}$$

$$Q_f^{(h)}(t,t') \approx \sum_{j=1}^{2} \frac{1}{a_j} \sum_{mo} \left[\frac{\cot K_m^{(j)} b_j}{K_m^{(j)}} + \frac{1}{M_j}\right] \equiv Q_f \tag{a-3}$$

where $t = a - x$, $t' = a - x'$, $a = \min(a_1, a_2)$, $K_m^{(j)2} = \zeta^2 - M_j^2$, $M_j = m\pi/2a_j$, and mo refers to the summation over odd m. This kernel will result in the following solution for the unknown gap field

$$f^{(h)}(t') = \frac{-Id\ell}{2a_1\sqrt{g^2 - t'^2}} L(\alpha) f_1(\overline{x}_t') \tag{a-4}$$

$$L(\alpha) = \left[\ell n\left(\frac{8\sqrt{a_1 a_2}}{\pi g}\right) + \frac{\pi}{2} Q_f\right]^{-1} \tag{a-5}$$

$$f_j(\overline{x}_t) = \sum_{mo} \frac{M_j \cos K_m^{(j)}(b_j - y)}{K_m^{(j)} \sin K_m^{(j)} b_j} \sin\frac{m\pi}{2} \cos M_j x \, J_o(M_j g) \tag{a-6}$$

where we recall that $\overline{x}_t'$ is the dipole location. The above solution assumes that the dipole is centrally located.

The y-component of the electric field in the upper, and lower chambers may now be found. Both TE and TM type unperturbed (ordinary) modes, as well as TE type perturbed modes will contribute significantly to $\tilde{E}_y^{(j)}$. The unperturbed modes exist only in the upper chamber. Denoting the unperturbed contribution by $\tilde{K}_1^{(j)}$, and the gap perturbation contribution by $\tilde{K}_2^{(j)}$, and referring to Appendix C of NBS Technical Note 1037 [4], we find

$$\tilde{K}_1^{(j)}(\overline{x}_t,\overline{x}_t') = \delta_{ij} \left\{ \frac{-i\omega\mu}{\zeta^2} \partial_x \tilde{H}_z^{(j)}(\overline{x}_t, \overline{x}_t') + \frac{i\alpha}{\zeta^2} \partial_y \tilde{E}_z^{(j)}(\overline{x}_t,\overline{x}_t') \right\} \qquad \text{(a-7)}$$

Suitably modifying equation (49) of NBS Technical Note 1037 [4] yields

$$\tilde{K}_2^{(j)}(\overline{x}_t, \overline{x}_t') = \frac{i\omega\mu\pi}{2a_1 a_j \zeta^2} \; Id\ell \; L(\alpha) \; f_j \; (\overline{x}_t) \; f_1 \; (\overline{x}_t') \qquad \text{(a-8)}$$

Taking the proper derivatives required in (a-7) and substituting the proper forms for $\tilde{E}_z^{(j)}$, and $\tilde{H}_z^{(j)}$ allows $\tilde{K}_1^{(j)}$ to be written as follows.

$$\tilde{K}_1^{(j)}(\overline{x}_t,\overline{x}_t') = \delta_{1j} \frac{i\omega\mu}{\zeta^2} Id\ell \sum_{m,n=0}^{\infty} \frac{\Delta_n M_j^2 + \frac{\alpha^2}{k^2} N_j^2}{M_j^2 + N_j^2 - \zeta^2} g_{mn}^{(j)}(\overline{x}_t,\overline{x}_t') \qquad \text{(a-9)}$$

$$g_{mn}^{(j)}(\overline{x}_t,\overline{x}_t') = \left(\frac{2}{a_j b_j}\right) \sin M_j(x + a_j) \; \sin(x'+a_j) \; \cos N_j y \; \cos N_j y' \qquad \text{(a-10)}$$

where

$$\Delta_n = \begin{cases} \tfrac{1}{2} & n = 0 \\ 1 & n \geq 1 \end{cases} \qquad \text{(a-11)}$$

and $N_j = n\pi/b_j$. It will prove convenient to extract the delta function nature in $\tilde{K}_1^{(j)}$ which occurs when $\overline{x}_t \to \overline{x}_t'$. Notice that

$$\Delta_n M_j^2 + \frac{\alpha^2}{k^2} N_j^2 = \Delta_n \frac{\alpha^2}{k^2} [M_j^2 + N_j^2 - \zeta^2 + \frac{\zeta^2}{\alpha^2} M_j^2 + \zeta^2] \qquad \text{(a-12)}$$

Therefore $\tilde{K}_1^{(j)}$ becomes

$$\tilde{K}_1^{(j)}(\overline{x}_t,\overline{x}_t') = \delta_{1j} \frac{i\omega\mu}{k^2} Id\ell \left\{ \frac{\alpha^2}{\zeta^2} \sum_{m,n=0}^{\infty} \Delta_n g_{mn}^{(j)}(\overline{x}_t,\overline{x}_t') + \sum_{m,n=0}^{\infty} \frac{\Delta_n [M_j^2 + \alpha^2]}{M_j^2 + N_j^2 - \zeta^2} g_{mn}^{(j)}(\overline{x}_t,\overline{x}_t') \right\} \qquad \text{(a-13)}$$

If we expand a delta function in a double Fourier series over a chamber cross section we will find

$$\delta(\overline{x}_t - \overline{x}_t') = \sum_{m,n=0}^{\infty} \Delta_n \; g_{mn}^{(j)}(\overline{x}_t,\overline{x}_t') \qquad \text{(a-14)}$$

Therefore $\tilde{K}_1^{(j)}$ becomes

$$\tilde{K}_1^{(j)}(\bar{x}_t, \bar{x}_t') = \delta_{1j} \frac{i\omega\mu}{k^2} Id\ell \left\{ \frac{\alpha^2}{\zeta^2} \delta(\bar{x}_t - \bar{x}_t') + \sum_{m,n=0}^{\infty} \frac{\Delta_n [M_j^2 + \alpha^2]}{M_j^2 + N_j^2 - \zeta^2} g_{mn}^{(j)}(\bar{x}_t, \bar{x}_t') \right\} \tag{a-15}$$

$\tilde{K}_1^{(j)}$ and $\tilde{K}_2^{(j)}$ represent the y-component of the electric field excited by a symmetrically located VED at $\bar{x}_t'$. There remain only two small changes to adapt this result to the probe excitation problem. First, the probe carries a negative y-directed current. Thus $Id\ell \rightarrow - Id\ell$. Second, the Green's function represents results from a unit strength dipole. Therefore $-Id\ell \rightarrow -1$. Therefore $\tilde{K}_1^{(j)}$ and $\tilde{K}_2^{(j)}$ finally become

$$\tilde{K}_1^{(j)}(\bar{x}_t, \bar{x}_t) = -\delta_{1j} \frac{i\omega\mu}{k^2} \left\{ \frac{\alpha^2}{\zeta^2} \delta(\bar{x}_t - \bar{x}_t') + \sum_{m,n=0}^{\infty} \frac{\Delta_n [M_j^2 + \alpha^2]}{M_j^2 + N_j^2 - \zeta^2} g_{mn}^{(j)} (\bar{x}_t, \bar{x}_t') \right\} \tag{a-16}$$

$$\tilde{K}_2^{(j)}(\bar{x}_t, \bar{x}_t') = \frac{i\omega\mu\pi}{2a_1 a_j \zeta^2} L(\alpha) f_j(\bar{x}_t) f_1(\bar{x}_t') \tag{a-17}$$

APPENDIX B

EVALUATION OF Z_1

The integral defined for Z_1 is to be evaluated.

$$Z_1 = \frac{-1}{2\pi I_t^2} \int_S \int_{S'} J(\overline{x}) \int_{-\infty}^{\infty} \tilde{K}_1^{(1)}(\overline{x}_t,\overline{x}_t')e^{i\alpha z}J(\overline{x}')d\alpha\, ds'\, ds \tag{b-1}$$

$$\tilde{K}_1^{(1)}(\overline{x}_t,\overline{x}_t') = -\frac{i\omega\mu}{k^2}\left\{\frac{\alpha^2}{\zeta^2}\delta(\overline{x}_t-\overline{x}_t') + \sum_{m,n=0}^{\infty} \Delta_n \frac{M_1^2+\alpha^2}{M_1^2+N_1^2-\zeta^2} g_{mn}^{(1)}(\overline{x}_t,\overline{x}_t')\right\} \tag{b-2}$$

The α-integration may be performed first, which requires evaluating the following pair of integrals.

$$I_1 = \int_{-\infty}^{\infty} \frac{\alpha^2}{\zeta^2}\delta(\overline{x}_t - \overline{x}_t')e^{i\alpha z}d\alpha \tag{b-3}$$

$$I_2 = \int_{-\infty}^{\infty} \sum_{m,n=0}^{\infty} \Delta_n \frac{M_1^2+\alpha^2}{M_1^2+N_1^2-\zeta^2}\, g_{mn}^{(1)}(\overline{x}_t,\overline{x}_t')e^{i\alpha z}\, d\alpha \tag{b-4}$$

In each integral we will deform into the upper α half plane for $z > 0$ (forward waves), and into the lower α half plane for $z < 0$ (backward waves). For I_1, we find

$$I_1 = -i\pi k\delta(\overline{x}_t - \overline{x}_t')e^{ik|z|} \tag{b-5}$$

The denominator of I_2 may be factored as follows.

$$M_1^2 + N_1^2 - \zeta^2 = \alpha^2 + {\Gamma_{mn}^{(1)}}^2 \tag{b-6}$$

$$\Gamma_{mn}^{(1)} = [M_1^2 + N_1^2 - k^2]^{1/2} \tag{b-7}$$

Thus we see that a simple pole arises in the I_2 integrand when $\alpha = \pm i\Gamma_{mn}^{(1)}$. Again deforming as for I_1 gives the following result.

$$I_2 = \pi \sum_{m,n=0}^{\infty} \Delta_n \frac{k^2 - N_1^2}{\Gamma_{mn}^{(1)}} g_{mn}^{(1)}(\bar{x}_t, \bar{x}_t') e^{-\Gamma_{mn}^{(1)}|z|} \tag{b-8}$$

We next proceed to the integration over the current distributions $J(\bar{x})$ and $J(\bar{x}')$. The source current $J(\bar{x}')$ will be concentrated on an idealized linear antenna located at the origin, i.e. $\bar{x}_t' = (0,y')$. Therefore $\bar{x}_t \neq \bar{x}_t'$ on S and the delta function term will not contribute to Z_1. Thus Z_1 reduces to

$$Z_1 = \frac{i\omega\mu}{2k^2 I_t^2} \sum_{m,n=0}^{\infty} \Delta_n \frac{k^2 - N_1^2}{\Gamma_{mn}^{(1)}} \int_S \int_{S'} J(\bar{x}) g_{mn}^{(1)}(\bar{x}_t, \bar{x}_t') J(\bar{x}') e^{-\Gamma_{mn}^{(1)}|z|} ds'ds \tag{b-9}$$

The simpler integration over the source current will be considered first. We recall that the assumed current density on the probe will be

$$J(\bar{x}) = \frac{I_o}{2\pi t} \sin k\,(b_1 - d - y)\delta(\rho - t) \tag{b-10}$$

For the δ' integration we let $t' \to 0$, and therefore

$$\int_{S'} J(\bar{x}') g_{mn}^{(1)}(\bar{x}_t, \bar{x}_t') ds' = I_3 \left(\frac{2}{a_1 b_1}\right) I_o \sin M_1(x + a_1) \sin \frac{m\pi}{2} \cos N_1 y \; e^{-\Gamma_{mn}^{(1)}|z|} \tag{b-11}$$

where the integral I_3 is given by

$$I_3 = \int_{b_1 - d}^{b_1} \sin k\,(b_1 - d - y')\cos N_1 d'\,dy' = \frac{k \cos n\pi(\cos kd - \cos N_1 d)}{k^2 - N_1^2} \tag{b-12}$$

Because of the factor $\sin m\pi/2$, only the summation over odd m will contribute. Therefore Z_1 takes the form

$$Z_1 = \frac{i\omega\mu I_o}{a_1 b_1 k I_t^2} \sum_{mo,n=0}^{\infty} \frac{\Delta_n \cos n\pi}{\Gamma_{mn}^{(1)}} (\cos kd - \cos N_1 d) \int_S J(\bar{x}) \cos M_1 x \cos N_1 y \times e^{-\Gamma_{mn}^{(1)}|z|} ds \tag{b-13}$$

The integration over S may be split into separate integrations over y and θ. The y-integration has already been encountered and we find

$$\int_S J(\bar{x})\cos M_1 x \cos N_1 y \, e^{-\Gamma_{mn}^{(1)}|z|} ds = \frac{I_o I_3}{2\pi t}\int_o^{2\pi} \cos(M_1 t \cos\theta) e^{-\Gamma_{mn}^{(1)} t|\sin\theta|} t\, d\theta \tag{b-14}$$

If we now interchange the remaining theta integration with the double summation, Z_1 may be written

$$Z_1 = \frac{i\omega\mu I_o^2}{\pi a_1 b_1 I_t^2}\int_o^{\pi}\left\{\sum_{n=0}^{\infty}\Delta_n \frac{(\cos kd - \cos N_1 d)^2}{k^2 - N_1^2}\right.$$

$$\left. \times \sum_{mo}\frac{1}{\Gamma_{mn}^{(1)}}\cos(M_1 t\cos\theta) e^{-\Gamma_{mn}^{(1)} t\sin\theta}\right\} d\theta \tag{b-15}$$

where we have noted that the theta integration is symmetric about π. Let us now consider the sum over m

$$S_n(\theta) = \sum_{mo}\frac{1}{\Gamma_{mn}^{(1)}}\cos(M_1 t\cos\theta) e^{-\Gamma_{mn}^{(1)} t\sin\theta} \tag{b-16}$$

The dominant series will be for $n = 0$. $S_o(\theta)$ may be expressed as follows.

$$S_o(\theta) = \mathrm{Re}\sum_{mo}\left.\frac{e^{iM_1 t\cos\theta}\, e^{-\Gamma_{mn}^{(1)} t\sin\theta}}{\Gamma_{mn}^{(1)}}\right|_{n=0} \tag{b-17}$$

For large m we see that $\Gamma_{mo}^{(1)} \sim M_1$. Adding and subtracting the asymptotic form of the summand will thus give

$$S_o(\theta) = \mathrm{Re}\sum_{mo}\frac{e^{M_1 t(i\cos\theta - \sin\theta)}}{M_1}$$

$$+ \mathrm{Re}\sum_{mo} e^{iM_1 t\cos\theta}\left[\frac{e^{-\Gamma_{mn}^{(1)} t\sin\theta}}{\Gamma_{mn}^{(1)}} - \frac{e^{-M_1 t\sin\theta}}{M_1}\right]_{n=0} \tag{b-18}$$

where the first series represents the major contribution. The first series may be summed

$$\sum_{mo} \frac{e^{iM_1 t(e^{i\theta})}}{M_1} = \frac{-a_1}{\pi} \ln \left[\tan\left(\frac{\pi t e^{i\theta}}{4a_1}\right)\right] + \frac{ia_1}{2} \tag{b-19}$$

We now assume that the probe is thin, $\pi t/4a_1 << 1$, which allows us to replace the tangent by its argument

$$\ln\left[\tan\left(\frac{\pi t e^{i\theta}}{4a_1}\right)\right] \approx \ln\left(\frac{\pi t}{4a_1} e^{i\theta}\right) = \ln\left(\frac{\pi t}{4a_1}\right) + i\theta \tag{b-20}$$

If we now take the real part of the first sum in (b-18) we see that

$$S_0(\theta) = \frac{a_1}{\pi} \ln\left(\frac{4a_1}{\pi t}\right) + \mathrm{Re} \sum_{mo} e^{iM_1 t \cos\theta} \left[\frac{e^{-\Gamma_{mn}^{(1)} t \sin\theta}}{\Gamma_{mn}^{(1)}} - \frac{e^{-M_1 t \sin}}{M_1}\right]_{n=0} \tag{b-21}$$

The correction series will be considered next. Again consider $\Gamma_{mo}^{(1)}$

$$\Gamma_{mo}^{(1)} = M_1 (1 - k^2/M_1^2)^{\frac{1}{2}} \approx M_1 (1 - k^2/2M_1^2) = M_1 (1 - \varepsilon) \tag{b-22}$$

where $\varepsilon = k^2/2M_1^2$ is assumed to be small. The correction series may be written

$$\sum_{mo} \frac{e^{M_1 t(i \cos\theta - \sin\theta)}}{M_1} \left[\frac{e^{M_1 t \varepsilon \sin\theta}}{1 - \varepsilon} - 1\right] \tag{b-23}$$

For ε small, we have $(1 - \varepsilon)^{-1} \approx 1 + \varepsilon$. This series converges very rapidly, thus only a few significant terms may be considered. Because $\pi t/2a_1 << 1$ for these few terms, the exponents may be replaced by unity with little error giving

$$\sum_{mo} \frac{\varepsilon}{M_1} = \frac{k^2}{2} \sum_{mo} \frac{1}{M_1^3} = \frac{4a_1^3 k^2}{\pi^3} \sum_{mo} \frac{1}{m^3} \tag{b-24}$$

The sum of the inverse cube over odd m may be done in terms of the Riemann Zeta function. Recalling that

$$\zeta(s) = \sum_{k=1}^{\infty} k^{-s} \tag{b-25}$$

For $\mathrm{Re}(s) > 1$, we see that our expression may be written

$$\sum_{mo} \frac{1}{m^3} = \sum_{m=1}^{\infty} \frac{1}{m^3} - \sum_{m=1}^{\infty} \frac{1}{(2m)^3} = \frac{7}{8} \zeta(3) \tag{b-26}$$

The Riemann-Zeta function is well tabulated, and from [6] we find

$$\zeta(3) \approx 1.20205 \tag{b-27}$$

Collecting these results we have the following for the dominant series $S_o(\theta)$

$$S_o(\theta) \approx \frac{a_1}{\pi} \ln \left(\frac{4a_1}{\pi t}\right) + \frac{a_1^3 k^2}{\pi^3} (4.207175) \tag{b-28}$$

The remaining series $S_n(\theta)$ for $n \geq 1$ will be summed in a manner similar to that used by Collin [3] in treating the probe excitation of a rectangular waveguide. We will make use of the following Poisson summation formula: if $f(x)$ is a square integrable function, then

$$\sum_{n=-\infty}^{\infty} f(n\alpha) = \frac{1}{\alpha} \sum_{n=-\infty}^{\infty} F\left(\frac{2\pi n}{\alpha}\right) \tag{b-29}$$

where $F(\omega)$ is the Fourier-transform of $f(x)$ defined by

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx \tag{b-30}$$

Our particular problem involves the function $f(x)$ defined by

$$f(x) = e^{ixt\cos\theta - \Gamma_{mn}^{(1)}(x)t\sin\theta} / \Gamma_{mn}^{(1)}(x) \tag{b-31}$$

$$\Gamma_{mn}^{(1)}(x) = [x^2 + N_1^2 - k^2]^{1/2} \tag{b-32}$$

This function is clearly L_2 so Poisson's summation formula applies. The Fourier transform is given by

$$F(\omega) = \int_{-\infty}^{\infty} \frac{e^{-\Gamma_{mn}^{(1)}(x)t\sin\theta}}{\Gamma_{mn}^{(1)}(x)} e^{ix(t\cos\theta + \omega)} dx \tag{b-33}$$

If we now rewrite the complex exponential, and notice that $\Gamma_{mn}^{(1)}(x)$ is even in x, then $F(\omega)$ takes the form

$$F(\omega) = 2\int_{0}^{\infty} \frac{e^{-\Gamma_{mn}^{(1)}(x)t\sin\theta}}{\Gamma_{mn}^{(1)}(x)} \cos((t\cos\theta + \omega)x) dx \tag{b-34}$$

This integral may be evaluated in terms of the modified Bessel function K_o [7] with the result

$$F(\omega) = 2K_o[(N_1^2 - k^2)^{\frac{1}{2}} (t^2 \sin^2\theta + (t\cos\theta + \omega)^2)^{\frac{1}{2}}] \tag{b-35}$$

If we now return to the definition of $S_n(\theta)$ we see that

$$S_n(\theta) = \frac{1}{2} \sum_{mo=-\infty}^{+\infty} f(\frac{m\pi}{2a_1}) = \frac{1}{2} \sum_{m=-\infty}^{\infty} \left\{ f(\frac{m\pi}{2a_1}) - f(\frac{m\pi}{a_1}) \right\} \tag{b-36}$$

Thus applying Poisson's summation formula we have

$$S_n(\theta) = \frac{1}{2} \sum_{m=-\infty}^{\infty} \left\{ \frac{2a_1}{\pi} F(4a_1 m) - \frac{a_1}{\pi} F(2a_1 m) \right\} \tag{b-37}$$

which in terms of modified Bessel functions K_o becomes

$$S_n(\theta) = \frac{a_1}{\pi} \sum_{m=-\infty}^{\infty} \left\{ 2K_o[(N_1^2 - K^2)^{\frac{1}{2}}((4a_1 m)^2 + 8a_1 mt\cos\theta + t^2)^{\frac{1}{2}}] \right.$$

$$\left. - K_o[(N_1^2 - k^2)^{\frac{1}{2}} ((2a_1 m)^2 + 4a_1 mt\cos\theta + t^2)^{\frac{1}{2}}] \right\} \tag{b-38}$$

The modified Bessel function decays very rapidly so the only significant contribution comes from $m = 0$. Thus

$$S_n(\theta) \approx \frac{a_1}{\pi} K_o[(N_1^2 - k^2)^{\frac{1}{2}} t] \qquad (n \geq 1) \tag{b-39}$$

Returning to the integral for Z_1 in (b-15) we have the following

$$Z_1 \approx \frac{i\omega\mu I_o^2}{\pi a_1 b_1 I_t^2} \int_o^{\pi} \left\{ \frac{(1 - \cos kd)^2}{2k^2} S_o(\theta) + \sum_{n=1}^{\infty} \frac{(\cos kd - \cos N_1 d)^2}{k^2 - N_1^2} S_n(\theta) \right\} d\theta \tag{b-40}$$

Note that the approximate form for $S_n(\theta)$ in (b-39) is now independent of θ. Thus combining all these results, we have

$$Z_1 \approx \frac{i\omega\mu I_o^2}{2\pi b_1 k^2 I_t^2} (1 - \cos kd)^2 \left\{ \ln (\frac{4a_1}{\pi t}) + \frac{a_1^2 k^2}{\pi^2} (4.207175) \right.$$

$$\left. - 2k^2 \sum_{n=1}^{\infty} \left(1 - \frac{\sin^2 N_1 d/2}{\sin^2 kd/2} \right)^2 \frac{K_o[t(N_1^2 - k^2)^{\frac{1}{2}}]}{N_1^2 - k^2} \right\} \tag{b-41}$$

Note that Z_1 given in (b-41) is approximate, based on two assumptions that the probe is thin ($\pi t/2a_1 << 1$) and that the frequency is not too high ($k^2 a_1^2 << 1$).

Finally, we note that the total current at the aperture ($y = b_1$) is given by $I_t = - I_o \sin k d$. Thus,

$$Z_1 \approx \frac{i\omega\mu}{2\pi b_1 k^2} \tan^2 \frac{kd}{2} \left\{ \ell n \left(\frac{4a_1}{\pi t}\right) + \frac{a_1^2 k^2}{\pi^2}(4.207175) - 2k^2 \sum_{n=1}^{\infty} \left(1 - \frac{\sin^2 N_1 d/2}{\sin^2 kd/2}\right)^2 \frac{K_o[t(N_1^2 - k^2)^{\frac{1}{2}}]}{N_1^2 - k^2} \right\}. \tag{b-42}$$

APPENDIX C

EVALUATION OF Z_2

The integral defined for Z_2 in (13) is to be evaluated.

$$Z_2 = \frac{-1}{2\pi I_t^2} \int_S \int_{S'} J(\bar{x}) \int_{-\infty}^{\infty} \tilde{K}_2^{(1)} (\bar{x}_t, \bar{x}_t') e^{i\alpha z} J(\bar{x}') d\alpha \, ds' \, ds \qquad \text{(c-1)}$$

where

$$\tilde{K}_2^{(1)}(\bar{x}_t, \bar{x}_t') = \frac{i\omega\mu\pi}{2a_1^2\zeta^2} L(\alpha) f_1(\bar{x}_t) f_1(\bar{x}_t') \qquad \text{(c-2)}$$

$$L(\alpha) = \left\{ \ln\left(\frac{8\sqrt{a_1 a_2}}{\pi g}\right) + \frac{\pi}{2} \sum_{j=1}^{2} \frac{1}{a_j} \sum_{mo} \left[\frac{\cot K_m^{(j)} b_j}{K_m^{(j)}} + \frac{1}{M_j} \right] \right\}^{-1} \qquad \text{(c-3)}$$

and

$$f_1(\bar{x}_t) = \sum_{mo} \frac{M_1 \cos K_m^{(1)} (b_1 - y)}{K_m^{(1)} \sin K_m^{(1)} b_1} \sin \frac{m\pi}{2} \cos M_1 x \; J_o(M_1 g) \qquad \text{(c-4)}$$

If we define two integrals I_4 and I_5 via

$$I_4 = \int_{S'} J(\bar{x}') \; f_1(\bar{x}_t') ds' \qquad \text{(c-5)}$$

$$I_5 = \int_S J(\bar{x}) \; f_1(\bar{x}_t) e^{i\alpha z} ds \qquad \text{(c-6)}$$

and interchange the α- and surface integrations, we may write Z_2 in the following form

$$Z_2 = \frac{-i\omega\mu}{4a_1^2 I_t^2} \int_{-\infty}^{\infty} \frac{L(\alpha)}{\zeta^2} I_4 \, I_5 \, d\alpha \qquad \text{(c-7)}$$

The integrals I_4 and I_5 now need to be evaluated.

The assumed current distribution is given by (2). Therefore I_4 becomes

$$I_4 = I_o \sum_{mo} \frac{M_1 \sin \frac{m\pi}{2} J_o(M_1 g)}{K_m^{(1)} \sin K_m^{(1)} b_1} \int_{b_1-d}^{b_1} \sin k(b_1 - d - y') \cos K_m^{(1)}(b_1 - y')dy' \tag{c-8}$$

Performing the y-integration gives

$$I_4 = kI_o \sum_{mo} \frac{M_1 \sin \frac{m\pi}{2} J_o(M_1 g)}{K_m^{(1)} \sin K_m^{(1)} b_1} \cdot \frac{(\cos kd - \cos K_m^{(1)} d)}{M_1^2 + \alpha^2} \tag{c-9}$$

The integral I_5 is similar to I_4 except that an additional azimuthal integration around a probe must be performed. The y-integration is the same as in I_4 so I_5 becomes

$$I_5 = \frac{kI_o}{2\pi t} \sum_{mo} \frac{M_1 \sin \frac{m\pi}{2} J_o(M_1 g)}{K_m^{(1)} \sin K_m^{(1)} b_1} \quad \frac{(\cos kd - \cos K_m^{(1)} d)}{M_1^2 + \alpha^2}$$

$$\times \int_o^{2\pi} \cos(M_1 t \cos\theta) e^{i\alpha t \sin\theta}\, t\, d\theta \tag{c-10}$$

If we designate the theta integration by $H_m(t)$, then letting $\theta \to 2\pi - \theta$ in the interval $[\pi, 2\pi]$ allows $H_m(t)$ to be written

$$H_m(t) = 2t \int_o^{\pi} \cos(M_1 t \cos\theta)\cos(\alpha t \sin\theta)d\theta \tag{c-11}$$

The second cosine term may be expanded in terms of Bessel functions [7]

$$\cos(\alpha t \sin\theta) = J_o(\alpha t) + 2 \sum_{k=1}^{\infty} J_{2k}(\alpha t)\cos 2k\theta \tag{c-12}$$

Interchanging the sum and integral gives

$$H_m(t) = 2t \Big\{ J_o(\alpha t) \int_o^{\pi} \cos(M_1 t \cos\theta)d\theta$$

$$+ 2 \sum_{k=1}^{\infty} J_{2k}(\alpha t) \int_o^{\pi} \cos 2k\theta \cos(M_1 t \cos\theta)d\theta \Big\} \tag{c-13}$$

Again consulting [7] allows us to evaluate these integrals with the result that

$$H_m(t) = 2\pi t \Big\{ J_o(\alpha t) J_o(M_1 t) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\alpha t) J_{2k}(M_1 t) \Big\} \tag{c-14}$$

This is of the form described by Gegenbauer's Addition Theorem [6], which for the special case that the function being expanded is a zero th order Bessel function, becomes

$$J_o(pR) = J_o(pq)J_o(pr) + \sum_{k=1}^{\infty} \cos k\phi \; J_k(pq)J_k(pr) \tag{c-15}$$

where $R = [q^2 + r^2 - 2qr\cos\phi]^{1/2}$. If we let $p = t$, $q = \alpha$, $r = M_1$, and $\phi = \frac{\pi}{2}$, we find

$$R = [\alpha^2 + M_1^2]^{1/2} \tag{c-16}$$

Therefore, $H_m(t)$ in (c-14) becomes

$$H_m(t) = 2\pi t J_o[t(\alpha^2 + M_1^2)^{1/2}] \tag{c-17}$$

If for simplicity, we define the following quantity

$$h_m(\alpha,d) = \frac{M_1 \sin\frac{m\pi}{2} J_o(M_1 g)}{K_m^{(1)} \sin K_m^{(1)} b_1} \cdot \frac{(\cos kd - \cos K_{md}^{(1)})}{M_1^2 + \alpha^2} \tag{c-18}$$

the above results for I_4 and I_5 combine to give Z_2 as

$$Z_2 = \frac{-i\omega\mu k^2 I_o^2}{4a_1^2 \; I_t^2} \int_{-\infty}^{\infty} \left\{ \frac{L(\alpha)}{\zeta^2} \sum_{mo'} h_{m'}(\alpha,d) \sum_{mo} h_m(\alpha,d) J_o[t(\alpha^2 + M_1^2)^{1/2}] \right\} d\alpha \tag{c-19}$$

If we now recall that I_t, the total current at the aperture, is given by $-I_o \sin kd$, Z_2 becomes

$$Z_2 = \frac{-i\omega\mu k^2}{4a_1^2} \csc^2 kd \int_{-\infty}^{\infty} \left\{ \frac{L(\alpha)}{\zeta^2} \sum_{mo} h_{m'}(\alpha,d) \sum_{mo} h_m(\alpha,d) J_o[t(\alpha^2 + M_1^2)^{1/2}] \right\} d\alpha \tag{c-20}$$

Unfortunately, the α-integration is sufficiently complex that some numerical approach has to be used in evaluating this integral. Notice that for large m, since $d < b_1$, $h_m(\alpha,d)$ will contain an exponential decay according to $\exp(-\alpha(b_1 - d))$. Therefore, the integrand will decay rapidly with α and a numerical routine should not prove time consuming.

APPENDIX D

CHECKS OF R AND Z_1

This appendix will detail checks of two of the impedance contributions. First, the radiation resistance will be checked against a result given by Tippet [1], and second, Z_1 will be compared to the rectangular waveguide result given by Collin [3]. The radiation resistance R_e' of an electric dipole inside a TEM cell due to the TEM mode, as given by Tippet [1], is

$$R_e' = \frac{2Z_c}{|I|^2} \left(d\ell \, \frac{IE_o \cos\theta}{2V}\right)^2 \tag{d-1}$$

where Z_c is the characteristic impedance of an RCTL, I is the magnitude of the current, $d\ell$ is the effective length of the dipole, E_o is the TEM mode electric field distribution, θ is the angle between the dipole direction and the y axis, (for a vertical dipole $\theta = 0$), and V is the voltage associated with the TEM mode. Because the probe is an electrically short monopole ($kd << 1$) the radiation resistance should be nearly that of a dipole with the same current distribution. Expression (d-1) assumes an elementary dipole, however. We may replace $IE_o d\ell$ with an effective value defined by

$$(IE_o d\ell)_{eff} = \int_{b_1-d}^{b_1} I(y)E(y)dy \tag{d-2}$$

where $I(y) = \sin k(b_1-d-y)$, and $E(y)$, for a centrally located dipole, is found in [4] to be

$$E(y) = \frac{2V}{a} \sum_{mo} \frac{\cosh M(b_1-y)}{\sin h\, Mb_1} \sin\frac{m\pi}{2} \; J_o(Mg) \tag{d-3}$$

Integrating yields the following

$$(IE_o d\ell)_{eff} = \frac{2Vk}{a} \sum_{mo} \frac{(\cos kd - \cosh Md)}{\sin h\;\, Mb_1} \sin\frac{m\pi}{2} \; J_o(mg) \tag{d-4}$$

which, if we recall the definition of $h_m(\alpha,d)$, may be written

$$(IE_o d\ell)_{eff} = \frac{-2Vk}{a} \sum_{mo} h_m(k,d) \tag{d-5}$$

If we now normalize by the magnitude of the current, $|I| = \sin kd$, we find R_e' becomes

$$R_e' = \frac{2Z_c k^2}{a^2} \csc^2 kd \left(\sum_{mo} h_m(k,d) \right)^2 \tag{d-6}$$

This may be compared to the radiation resistance R due to the TEM mode given by

$$R = \frac{2Z_c k^2}{a^2} \csc^2 kd \sum_{mo} h_{m'}(k,d) \sum_{mo} h_m(k,d) J_o[t(k^2+M^2)^{\frac{1}{2}}] \tag{d-7}$$

The term $h_m(k,d)$ has a strong decay in M according to

$$h_m(k,d) \sim \frac{1}{M^2} e^{-M(b_1-d)} \tag{d-8}$$

Only a few terms will be significant. For these few terms J_o will remain near unity since we have assumed that $\pi t/2a << 1$. Thus, the radiation resistance may be approximated by

$$R \approx \frac{2Z_c k^2}{a^2} \csc^2 kd \left(\sum_{mo} h_m(k,d) \right)^2 = R_e' \tag{d-9}$$

and the results indeed do check.

Next the case of no gap will be considered. As $g \to 0$, $L(\alpha) \to 0$ so only Z_1 contributes. Collin discusses a rectangular waveguide with a short circuit at a distance ℓ from the probe [3]. Thus we need to modify his result by letting $\ell \to \infty$. Making the appropriate changes, the radiation resistance for a probe in a rectangular waveguide becomes

$$R = \frac{\eta_o}{2ab\ \beta_{10} k} \tan^2 \frac{kd}{2} \tag{d-10}$$

The equivalent reactance is given by

$$X = \frac{\eta_o}{2\pi bk} \tan^2 \frac{kd}{2} \left\{ \ell n\left(\frac{4a}{\pi t}\right) + \frac{k^2 a^2}{\pi^2} (.2072) - 2\left(1 - \frac{t}{a}\right) - 2k^2 \sum_{n=1} \left(1 - \frac{\sin^2 N_1 d/2}{\sin^2 kd/2}\right)^2 \frac{K_o[t(N_1^2 - k^2)^{\frac{1}{2}}]}{N_1^2 - k^2} \right\} \tag{d-11}$$

In order to compare our expression to that of Collin, we must return to Appendix B (b-4) and let k be larger than M ($a_1 = a_2 = a$). This will create a pole on the real axis when $\alpha = \beta_{10}$. Thus I_2 becomes

$$I_2 = \frac{-\pi i k^2}{2\beta_{10}} g_{10}\,(\bar{x}_t,\ \bar{x}_t')e^{i\beta_{10}|z|} + \pi \sum_{m,n=0}{}' \Delta_n \frac{k^2 - N_1^2}{\Gamma_{mn}^{(1)}} g_{mn}(\bar{x}_t,\bar{x}_t')e^{-\Gamma_{mn}^{(1)}|z|}, \tag{d-12}$$

where the primed summation indicates that the term $m = 1$, $n = 0$ is missing. Performing the current integrations yields

$$Z_1 = \frac{i\omega\mu\ I_o^2}{\pi ab\ I_t^2} \int_o^\pi \left\{ \frac{(1 - \cos kd)^2}{2ik^2\beta_{10}} \cos(\frac{\pi t}{2a} \cos\theta)e^{i\beta_{10}t \sin\theta} \right.$$

$$\left. + \sum_{mo,n=0}^{\infty}{}' \Delta_n \frac{(\cos kd - \cos N_1 d)^2}{k^2 - N_1^2} \frac{1}{\Gamma_{mn}^{(1)}} \cos(Mt\cos\theta)e^{-\Gamma_{mn}^{(1)}t \sin\theta} \right\} d\theta \tag{d-13}$$

The sum over m and n is precisely that encountered by Collin [3] with the result that

$$\sum_{mo,n=0}{}' \Delta_n \frac{(\cos kd - \cos N_1 d)}{k^2 - N_1^2} \frac{1}{\Gamma_{mn}^{(1)}} \cos(Mt\cos\theta)e^{-\Gamma_{mn}^{(1)}t \sin\theta}$$

$$= \frac{(1 - \cos kd)^2}{2k^2} \frac{a}{\pi} \left[\ell n(\frac{4a}{\pi t}) + \frac{k^2a^2}{2} (.2072) - 2(1 - \frac{t}{a}) \right]$$

$$+ \frac{a}{\pi} \sum_{n=1}^{\infty} \frac{(\cos kd - \cos N_1 d)^2}{k^2 - N_1^2} K_o[t(N_1^2 - k^2)^{\frac{1}{2}}] \tag{d-14}$$

The significant contribution for $\pi t/2a \ll 1$ may be approximated via

$$\cos(\frac{\pi t}{2a} \cos\theta\ )e^{i\beta_{10}t \sin\theta} \simeq 1 \tag{d-15}$$

Combining these results and integrating over θ , which simply introduces a factor of π, yields

$$Z_1 \simeq \frac{i\omega\mu\ I_o^2}{\pi ab\ I_t^2} \left\{ \frac{(1 - \cos kd)^2}{2ik^2\beta_{10}} \pi + \frac{a(1 - \cos kd)^2}{2k^2} \left[\ell n(\frac{4a}{\pi t}) + \frac{k^2a^2}{\pi^2} (.2072) \right.\right.$$

$$\left.\left. -2(1 - \frac{t}{a}) \right] + a \sum_{n=1}^{\infty} \frac{(\cos kd - \cos N_1 d)^2}{k^2 - N_1^2} K_o[t(N_1^2 - k^2)^{\frac{1}{2}}] \right\} \tag{d-16}$$

Finally, letting $I_t = -I_o \sin kd$, and separating Z_1 into real and imaginary parts R and X respectively, we obtain

$$R = \frac{\eta_o}{2abk\beta_{10}} \tan^2 \frac{kd}{2} \tag{d-17}$$

$$X = \frac{\eta_o}{2\pi bk} \tan^2 \frac{kd}{2} \left\{ \ln\left(\frac{4a}{\pi t}\right) + \frac{k^2a^2}{\pi^2} (.2072) - 2\left(1 - \frac{t}{a}\right) \right.$$
$$\left. - 2k^2 \sum_{n=1}^{\infty} \left(1 - \frac{\sin^2 N_1 d/2}{\sin^2 kd/2}\right)^2 \frac{K_o[t(N_1^2 - k^2)^{\frac{1}{2}}]}{N_1^2 - k^2} \right\} \tag{d-18}$$

which are the precise results found by Collin in (d-10) and (d-11).

APPENDIX E

SUBROUTINE ZIN

SUBROUTINE ZIN is used to compute the input impedance Z of a probe inserted into a TEM cell. ZIN calls upon four major subroutines. These are (1) SUBROUTINE ZC which computes the characteristic impedance of a section of RCTL according to (18); (2) SUBROUTINE HMSUM which sums $h_m(\alpha,d)$, and $h_m(\alpha,d)J_o[t(\alpha^2+M_1^2)^{1/2}]$; (3) SUBROUTINE XSUM which finds the reactance X given by (20), and (4) SUBROUTINE INTEGR which integrates a complex function over the real axis. INTEGR is called twice in order to evaluate the principal value integral defined by (21). The input to ZIN is mostly via a common block PARAM which contains the cell dimensions a_1, a_2, b_1, b_2, and g, the probe dimensions d and t, the constant π, and the TEM mode propagation constant k. Figure 6 gives a flow chart of SUBROUTINE ZIN. The only direct input to ZIN is ε which sets the desired accuracy. ZIN outputs Z_c, R, X, and δX as well as the total impedance Z.

The constant ε is used to set the upper summation limit in the various sums evaluated by the subroutines. The infinite integral in δX is also truncated according to ε. There are three basic sums; the sums inherent in $L(\alpha)$ and X, and the summation of the $h_m(\alpha,d)$. We will begin with the last. For large m, that is $M_1^2 >> k^2 - \alpha^2$, we have $K_m^{(1)} \sim iM_1$, and

$$h_m(\alpha,d) \sim \frac{\sin\frac{m\pi}{2}\cosh M_1 d}{M_1^2 \sinh M_1 b_1} \approx \frac{\sin\frac{m\pi}{2}}{M_1^2} e^{-M_1(b_1-d)} \tag{e-1}$$

We wish to truncate the summation so that the remainder is less than ε. Thus we need to look at the remainder sum

$$\sum_{m=K}^{\infty} \frac{4a_1^2(-1)^m}{(2m-1)^2\pi^2} e^{-(2m-1)\pi(b_1-d)/2a_1} < \varepsilon \tag{e-2}$$

Clearly the sum is dominated by the following much simpler sum.

$$R_K(x) = \sum_{m=K}^{\infty} (-1)^m e^{-(2m-1)x} \tag{e-3}$$

so we will set M by requiring that

$$\frac{4a_1^2}{\pi^2} R_K\left(\frac{\pi(b_1-d)}{2a_1}\right) < \varepsilon \tag{e-4}$$

Summing $R_K(x)$ is straightforward and yields

$$R_K(x) = \frac{2e^{-2(K-1)x}}{\cosh x} \tag{e-5}$$

Thus inserting this result into (e-4) and solving for K we find

$$K > \frac{a_1}{\pi(b_1-d)} \ln\left[\frac{8a_1^2}{\varepsilon\pi \cosh \frac{\pi(b_1-d)}{2a_1}}\right] + 1 \tag{e-6}$$

This condition will be used to truncate the $h_m(\alpha,d)$ summation.

Next consider the summation in $L(\alpha)$ given by

$$\frac{1}{a_j} \sum_{mo} \left[\frac{\cot K_m^{(j)}}{K_m^{(j)}} + \frac{1}{M_j}\right] \tag{e-7}$$

For large m the remainder will behave according to

$$\frac{2}{\pi} \sum_{m=K_j}^{\infty} \frac{1}{(2m-1)} \left(1 - \coth \frac{(2m-1)\ \pi b_j}{2a_j}\right) \tag{e-8}$$

Again we choose a simpler dominating sum. For large m, such that $(2m-1)\,\pi\, b_j/2aj >> 1$, the coth term may be replaced by

$$\coth \frac{(2m-1)\pi\, b_j}{2a_j} \approx 1 + 2e^{-(2m-1)\pi b_j/a_j} \tag{e-9}$$

Therefore we need to consider

$$\frac{4}{\pi} \sum_{m=K_j}^{\infty} \frac{1}{2m-1} e^{-(2m-1)\pi b_j/a_j} \tag{e-10}$$

Clearly we have

$$\frac{4}{\pi}\sum_{m=K_j}^{\infty}\frac{1}{2m-1}e^{-(2m-1)\pi b_j/a_j} < \frac{4}{\pi}\sum_{m=K_j}^{\infty}e^{-(2m-1)\pi b_j/a_j} \tag{e-11}$$

Summing the R.H.S. and requiring that the result is less than $\varepsilon/2$ yields

$$K_j > \frac{a_j}{2\pi b_j}\ln\left[\frac{16}{\varepsilon\pi\sinh\frac{\pi b_j}{a_j}}\right]+1 \tag{e-12}$$

This condition will be checked for both $j = 1$ and 2, and the maximum value K_j chosen.

We next consider the series contained in X. Because both kd and kt are small, the decay of the series will be primarily due to the N_1^2 in the denominator. Thus, we will consider the remainder series dominated by

$$\sum_{n=N}^{\infty}\frac{1}{N_1^2}, \quad (N_1 = n\pi/b_1) \tag{e-13}$$

This series in turn may be bounded by a simple integral

$$\sum_{n=N}^{\infty}\frac{1}{N_1^2} < \left(\frac{b_1}{\pi}\right)^2\int_{N-1}^{\infty}\frac{dx}{x^2} = \left(\frac{b_1}{\pi}\right)^2\frac{1}{N-1} \tag{e-14}$$

So requiring that the R.H.S. be bounded by ε implies that

$$N > \frac{b_1^2}{\varepsilon\pi}+1 \tag{e-15}$$

What remains is to bound the truncation of the principal value integral. In terms of α we see $h_m(\alpha,d)$ behaving roughly as α^{-2}. Therefore, the total integrand behaves approximately as α^{-6}. Thus, the truncation gives a remainder integral similar to

$$C\int_{x}^{\infty}\alpha^{-6}\,d\alpha = \frac{C}{5}x^{-5} \tag{e-16}$$

where $C = -\omega\mu k^2\csc^2 kd/4a_1^2$. If we bound this result by $\varepsilon/2$, x must satisfy

$$x > (2C/5\varepsilon)^{1/5}\ , \tag{e-17}$$

which is the criterion used to set the upper limit of integration.

A listing of the subroutines follows. They were run at the University of Colorado using the MNF compiler, Version 5.4. It should be stressed that ε sets a percentage error rather than an absolute error. That is $\varepsilon = 0.01$ implies 1% error rather than accuracy to two digits beyond the decimal point.

```
UNIVERSITY OF MINNESOTA FORTRAN COMPILER (VERSION 5.4 - 79/03/01) ON THE 6400 UNDER KRONOS 2.1.0 ON 81/07/28 AT 14.02
UNIVERSITY COMPUTING CENTER - UNIVERSITY OF COLORADO

                MNF.

 000000B          COMMON/PARAM/A1,A2,B1,B2,D,G,T,PI,KO
 002131B          REAL PI,KO
 002131B          COMPLEX Z
 002131B          A1=1.0
 002133B          A2=1.0
 002134B          B1=1.0
 002135B          B2=1.0
 002135B          G=0.2
 002136B          T=0.001
 002140B          PI=4.0*ATAN(1.0)
 002143B          F=3.0E+06
 002145B          C=2.99793E+08
 002146B          KO=2.0*PI*F/C
 002151B          EPS=0.01
 002152B          DO 10 M=34,39
 002155B          D=FLOAT(M)/40.0
 002156B          CALL ZIN(EPS,ZC,ZR,ZI,DELZI,Z)
 002162B     10   CONTINUE
 002164B          END

 000000B          SUBROUTINE ZIN(EPS,ZC,ZR,ZI,DELZI,Z)
 000000B
            C          SUBROUTINE ZIN COMPUTES THE INPUT IMPEDANCE Z OF A PROBE
            C     IN A TEM CELL. THE ONLY DIRECT INPUT TO ZIN IS EPS (LESS THAN
            C     ONE), THE DESIRED ACCURACY. THE GUIDE PARAMETERS ARE PASSED
            C     INTO ZIN VIA THE COMMON BLOCK PARAM WHICH CONTAINS THE FOLLOWING
            C          A1,A2 THE UPPER AND LOWER CHAMBER HALF-WIDTHS
            C          B1,B2 THE UPPER AND LOWER CHAMBER HEIGHTS
            C          D THE PROBE LENGTH
            C          G THE GAP WIDTH
            C          T THE PROBE RADIUS
            C          PI THE SIXTEENTH LETTER OF THE GREEK ALPHABET
            C          KO THE TEM MODE PROPAGATION CONSTANT
            C     THE OUTPUTS ARE
            C          ZC THE CHARACTERISTIC IMPEDANCE OF THE RCTL SECTION
            C          ZR THE REAL RADIATION RESISTANCE (19)
            C          ZI THE ORDINARY WAVEGUIDE REACTANCE (20)
            C          DELZI THE GAP PERTURBATION REACTANCE (21)
            C          Z THE TOTAL IMPEDANCE, Z=R+J*(ZI+DELZI)
 000000B
 000000B          COMMON/PARAM/A1,A2,B1,B2,D,G,T,PI,KO
 000000B          COMMON/CPARAM/ZERO,UNITY,J
 000000B          REAL PI,KO
 000000B          COMPLEX ZERO,UNITY,J,HM,HMBJ,VAL1,VAL2,Z
 000000B          LOGICAL GG
 000000B          EXTERNAL XOFA
 000000B          ZERO =(0.0,0.0)
 000011B          UNITY=(1.0,0.0)
 000014B          J=(0.0,1.0)
 000016B          NSTEP=1024
 000020B          X0=30.0*PI*KO*((KO/A1/SIN(KO*D))**2.0)
 000030B          X1=0.0
 000031B          X2=KO-EPS
 000032B          X3=KO+EPS
```

```
000033B        X4=(2.0*XO/EPS/5.0)**0.2
000041B        CALL ZONORM(EPS,ZC)
000045B        CALL HMSUM(KO,HM,HMBJ,EPS)
000050B        CALL XSUM(EPS,ZI)
000054B        CALL INTEGR(X1,X2,EPS,NSTEP,XOFA,VAL1,X,XRELTV,K,GG)
000057B        CALL INTEGR(X3,X4,EPS,NSTEP,XOFA,VAL2,X,XRELTV,K,GG)
000062B        ST1=KO/A1/SIN(KO*D)
000067B        ZR=ST1*ST1*ZC*REAL(HM*HMBJ)
000077B        DELZI=REAL(VAL1+VAL2)
000102B        Z=UNITY*ZR+J*(ZI+DELZI)
000113B        PRINT 10,A1,A2,B1,B2,G
000125B   10   FORMAT(1X,*THE GUIDE DIMENSIONS ARE AS FOLLOWS*/,1X,*      A1= *
              1F12.6/,1X,*      A2= *F12.6/,1X,*      B1= *F12.6/,1X,*      B2= *
              2F12.6/,1X,*      G= *F12.6//)
000125B        PRINT 11,D,T
000132B   11   FORMAT(1X,* THE PROBE DIMENSIONS ARE AS FOLLOWS*/,1X,*      D= *
              1F12.6/,1X,*      T= *F12.6//)
000132B        PRINT 12,ZC,ZR,ZI,DELZI,Z
000142B   12   FORMAT(1X,*THE RESULTING IMPEDANCES ARE AS FOLLOWS*/,1X,*      ZC=
              1*F12.6/,1X,*      ZR= *F12.6/,1X,*      ZI= *F12.6/,1X,*      DELZI=
              2*F12.6//,1X,*      THE TOTAL IMPEDANCE Z= *F12.6,* +J *F12.6,5/)
000142B        RETURN
000144B        END


000000B        SUBROUTINE ZONORM(EPS,X)
000000B        COMMON/PARAM/A1,A2,B1,B2,D,G,T,PI,KO
000000B        REAL PI,KO,KM1,KM2,QO,L1,L2,LIM
000000B        QO=0.0
000006B        X1=PI*B1/A1
000011B        X2=PI*B2/A2
000013B        X3=(EXP(X1)-EXP(-X1))/2.0
000022B        X4=(EXP(X2)-EXP(-X2))/2.0
000032B        L1=1.0+ALOG(16.0/EPS/PI/X3)/X3/2.0
000043B        L2=1.0+ALOG(16.0/EPS/PI/X4)/X4/2.0
000053B        LIM=AMAX1(L1,L2)
000055B        MLIM=INT(LIM)+1
000057B        DO 10 M=1,MLIM
000061B          KM1=(2.0*M-1.0)*PI/2.0/A1
000065B          KM2=(2.0*M-1.0)*PI/2.0/A2
000070B          ST1=KM1*B1
000072B          ST2=KM2*B2
000073B          ST3=1.0-1.0/TANH(ST1)
000100B          ST4=1.0-1.0/TANH(ST2)
000104B          ST5=FLOAT(2*M-1)
000106B          QO=QO+(ST3+ST4)/ST5
000112B   10     CONTINUE
000114B        ST6=8.0*SQRT(A1*A2)/PI/G
000122B        ST7=ALOG(ST6)+QO
000126B        X=120.0*PI*PI/ST7/8.0
000131B        RETURN
000133B        END


000000B        SUBROUTINE BJO(X,BJ)
000000B        T=X/3.
000001B        Z=3./X
000002B        Y=T*T
000003B        IF (X.GE.3.) GO TO 10
000004B        BJ=1.-Y*(2.2499997-Y*(1.2656208-Y*(.3163866-Y*(.0444479-
```

```
                        1Y*(.0039444-Y*.0002100)))))
000015B           RETURN
000020B   10      W=SQRT(X)
000023B           AF=.79788456-Z*(.00000077+Z*(.00552740+Z*(.00009512-Z*(.00137237
                 1-Z*(.00072805-Z*.00014476)))))
000034B           THETA=X-.78539816-Z*(.04166397+Z*(.00003954-Z*(.00262573
                 1-Z*(.00054125+Z*(.00029333-Z*.00013558)))))
000047B           BJ=AF*COS(THETA)/W
000053B           RETURN
000056B           END


000000B           SUBROUTINE HMSUM(ALPHA,HM,HMBJ,EPS)
000000B           COMMON/PARAM/A1,A2,B1,B2,D,G,T,PI,KO
000000B           COMMON/CPARAM/ZERO,UNITY,J
000000B           REAL PI,KO,M1,LIM
000000B           COMPLEX ZERO,UNITY,J,HM,HMBJ,KM1,ST2,ST3
000000B           HM=ZERO
000011B           HMBJ=ZERO
000013B           X1=PI*(B1-D)/2.0/A1
000017B           X2=(EXP(X1)+EXP(-X1))/2.0
000026B           X3=8.0*A1*A1/EPS/PI/X2
000032B           LIM=1.0+ALOG(X3)/X1/2.0
000037B           MLIM=INT(LIM)+1
000041B           DO 10 M=1,MLIM
000043B             M1=(2.0*M-1.0)*PI/2.0/A1
000047B             ST1=ALPHA*ALPHA+M1*M1
000051B             KM1=CSQRT(UNITY*(KO*KO-ST1))
000061B             ST2=M1*SIN(M1*A1)*(UNITY*COS(KO*D)-CCOS(KM1*D))
000107B             ST3=KM1*CSIN(KM1*B1)*ST1
000121B             ST4=M1*G
000123B             ST5=SQRT(ST1)*T
000126B             CALL BJO(ST4,BJG)
000131B             CALL BJO(ST5,BJT)
000133B             HM=HM+ST2*BJG/ST3
000150B             HMBJ=HMBJ+ST2*BJG*BJT/ST3
000167B   10        CONTINUE
000172B           RETURN
000174B           END


000000B           SUBROUTINE BJKO(X,Y)
000000B           IF (X.GE.2.0) GO TO 10
000002B           Z=X/2.0
000003B           V=Z*Z
000004B           T=X/3.75
000005B           W=T*T
000006B           U=1.0+W*(3.5156229+W*(3.0899424+W*(1.2067492+W*(0.2659732+
                 1W*(0.0360768+W*0.0045813)))))
000017B           Y=-ALOG(Z)*U-0.57721566+V*(0.4227842+V*(0.23069756+V*(0.0348859+
                 1V*(0.00262698+V*(0.0001075+V*0.0000074)))))
000036B           RETURN
000040B   10      Z=2.0/X
000041B           U=1.25331414-Z*(0.07832358-Z*(0.02189568-Z*(0.01062446-Z*(
                 10.00587872-Z*(0.0025154-Z*(0.0005328))))))
000053B           Y=U/SQRT(X)/EXP(X)
000062B           RETURN
000065B           END
```

```
000000B        SUBROUTINE XSUM(EPS,X)
000000B        COMMON/PARAM/A1,A2,B1,B2,D,G,T,PI,KO
000000B        REAL PI,KO,N1,LIM
000000B        SUM=0.0
000002B        LIM=1.0+B1*B1/EPS/PI
000006B        NLIM=INT(LIM)+1
000010B        DO 10 N=1,NLIM
000012B          N1=FLOAT(N)*PI/B1
000014B          ST1=SIN(N1*D/2.0)/SIN(KO*D/2.0)
000030B          ST2=(1.0-ST1*ST1)**2.0
000033B          ST3=N1*N1-KO*KO
000036B          ST4=SQRT(ST3)*T
000041B          CALL BJKO(ST4,BKT)
000044B          SUM=SUM+ST2*BKT/ST3
000047B   10     CONTINUE
000052B        ST5=4.0*A1/PI/T
000055B        ST6=A1*KO/PI
000057B        ST7=ALOG(ST5)+ST6*ST6*4.207175-2.0*KO*KO*SUM
000065B        ST8=TAN(KO*D/2.0)
000072B        X=30.0*ST8*ST8*ST7/B1/KO
000076B        RETURN
000100B        END

000000B        SUBROUTINE LALPHA(ALPHA,X,EPS)
000000B        COMMON/PARAM/A1,A2,B1,B2,D,G,T,PI,KO
000000B        COMMON/CPARAM/ZERO,UNITY,J
000000B        REAL PI,KO,M1,M2,L1,L2,LIM
000000B        COMPLEX ZERO,UNITY,J,KM1,KM2,SUM,ST1,ST2,X
000000B        SUM=ZERO
000020B        ZETA2=KO*KO-ALPHA*ALPHA
000023B        X1=PI*B1/A1
000026B        X2=PI*B2/A2
000030B        X3=(EXP(X1)-EXP(-X1))/2.0
000040B        X4=(EXP(X2)-EXP(-X2))/2.0
000050B        L1=1.0+ALOG(16.0/EPS/PI/X3)/X3/2.0
000061B        L2=1.0+ALOG(16.0/EPS/PI/X4)/X4/2.0
000071B        LIM=AMAX1(L1,L2)
000073B        MLIM=INT(LIM)+1
000075B        DO 10 M=1,MLIM
000077B          M1=FLOAT(M)*PI/2.0/A1
000102B          M2=FLOAT(M)*PI/2.0/A2
000104B          KM1=CSQRT(UNITY*(ZETA2-M1*M1))
000115B          KM2=CSQRT(UNITY*(ZETA2-M2*M2))
000125B          ST1=CCOS(KM1*B1)/CSIN(KM1*B1)/KM1+UNITY/M1
000163B          ST2=CCOS(KM2*B2)/CSIN(KM2*B2)/KM2+UNITY/M2
000222B          SUM=SUM+ST1/A1+ST2/A2
000234B   10     CONTINUE
000240B        ST3=8.0*SQRT(A1*A2)/PI/G
000246B        X=UNITY/(UNITY*ALOG(ST3)+PI*SUM/2.0)
000271B        RETURN
000275B        END

000000B        SUBROUTINE XOFA(ALPHA,Y,EPS)
000000B        COMMON/PARAM/A1,A2,B1,B2,D,G,T,PI,KO
000000B        REAL PI,KO
000000B        COMPLEX LA,HM,HMBJ,Y
000000B        ZETA2=KO*KO-ALPHA*ALPHA
000010B        CALL LALPHA(ALPHA,LA,EPS)
```

```
000014B          CALL HMSUM(ALPHA,HM,HMBJ,EPS)
000020B          ST1=K0/A1/SIN(K0*D)
000025B          ST2=-60.0*PI*K0*ST1*ST1
000031B          Y=LA*HM*HMBJ*ST2/ZETA2
000042B          RETURN
000045B          END

000000B          SUBROUTINE INTEGR(A,B,EPS,NSTEP,F,VALUE,X,XRELTV,K,G)
000000B          COMPLEX FCNA,FCNB,FCNXI,T,SUM,QX1,QX2,VALUE,Q(16)
000000B          LOGICAL G
000000B          QX1=0.0
000056B          QX2=0.0
000060B          H=B-A
000062B          CALL F(A,FCNA,EPS)
000067B          CALL F(B,FCNB,EPS)
000073B          T=(FCNA+FCNB)*H/2.0
000100B          NX=1
000101B          N=1
000102B    10    K=2**N
000104B          H=H/2.0
000106B          SUM=(0.0,0.0)
000107B          DO 20 I=1,NX
000112B            XI=2.0*FLOAT(I)-1.0
000114B            XA=A+XI*H
000116B            CALL F(XA,FCNXI,EPS)
000122B            SUM=SUM+FCNXI
000125B    20      CONTINUE
000130B          T=T/2.0+H*SUM
000137B          Q(N)=(T+H*SUM)*2.0/3.0
000145B          IF (N-2) 100,30,30
000151B    30    FX=4.0
000152B          DO 40 J=2,N
000155B            I=N+1-J
000156B            FX=FX*4.0
000160B            Q(I)=Q(I+1)+(Q(I+1)-Q(I))/(FX-1.0)
000171B    40      CONTINUE
000173B          IF (N-3) 90,50,50
000174B    50    X=0.0
000175B          XRELTV=0.0
000175B          XREAL=ABS(REAL(Q(1)-QX2))+ABS(REAL(QX2-QX1))
000206B          XIMAG=ABS(AIMAG(Q(1)-QX2))+ABS(AIMAG(QX2-QX1))
000217B          CR=CABS(Q(1))
000222B          IF (CR.EQ.0.0) GO TO 60
000223B          XR=AMAX1(XREAL,XIMAG)/CR
000227B          GO TO 70
000230B    60    XR=0.0
000230B    70    XRELTV=AMAX1(XR,XRELTV)
000233B          X=AMAX1(X,XREAL,XIMAG)
000237B          COMPA=X-3.0*EPS
000242B          COMPR=XRELTV-3.0*EPS
000244B          IF (COMPA.LE.0.0.OR.COMPR.LE.0.0) 110,80
000247B    80    IF (NSTEP-K) 110,110,90
000252B    90    QX1=QX2
000254B   100    QX2=Q(1)
000257B          NX=NX*2
000261B          N=N+1
000262B          GO TO 10
000263B   110    VALUE=Q(1)
000265B          G=NSTEP.LT.K
```

```
53.   000270B        RETURN
54.   000273B        END

THE GUIDE DIMENSIONS ARE AS FOLLOWS
     A1=      1.000000
     A2=      1.000000
     B1=      1.000000
     B2=      1.000000
     G=        .200000

 THE PROBE DIMENSIONS ARE AS FOLLOWS
     D=        .850000
     T=        .001000

THE RESULTING IMPEDANCES ARE AS FOLLOWS
     ZC=     62.640845
     ZR=      2.289899
     ZI= -3132.962182
     DELZI=      -.122230

     THE TOTAL IMPEDANCE Z=      2.289899 +J -3133.084413



THE GUIDE DIMENSIONS ARE AS FOLLOWS
     A1=      1.000000
     A2=      1.000000
     B1=      1.000000
     B2=      1.000000
     G=        .200000

 THE PROBE DIMENSIONS ARE AS FOLLOWS
     D=        .875000
     T=        .001000

THE RESULTING IMPEDANCES ARE AS FOLLOWS
     ZC=     62.640845
     ZR=      2.447836
     ZI= -3049.147672
     DELZI=      -.130126

     THE TOTAL IMPEDANCE Z=      2.447836 +J -3049.277799



THE GUIDE DIMENSIONS ARE AS FOLLOWS
     A1=      1.000000
     A2=      1.000000
     B1=      1.000000
     B2=      1.000000
     G=        .200000
```

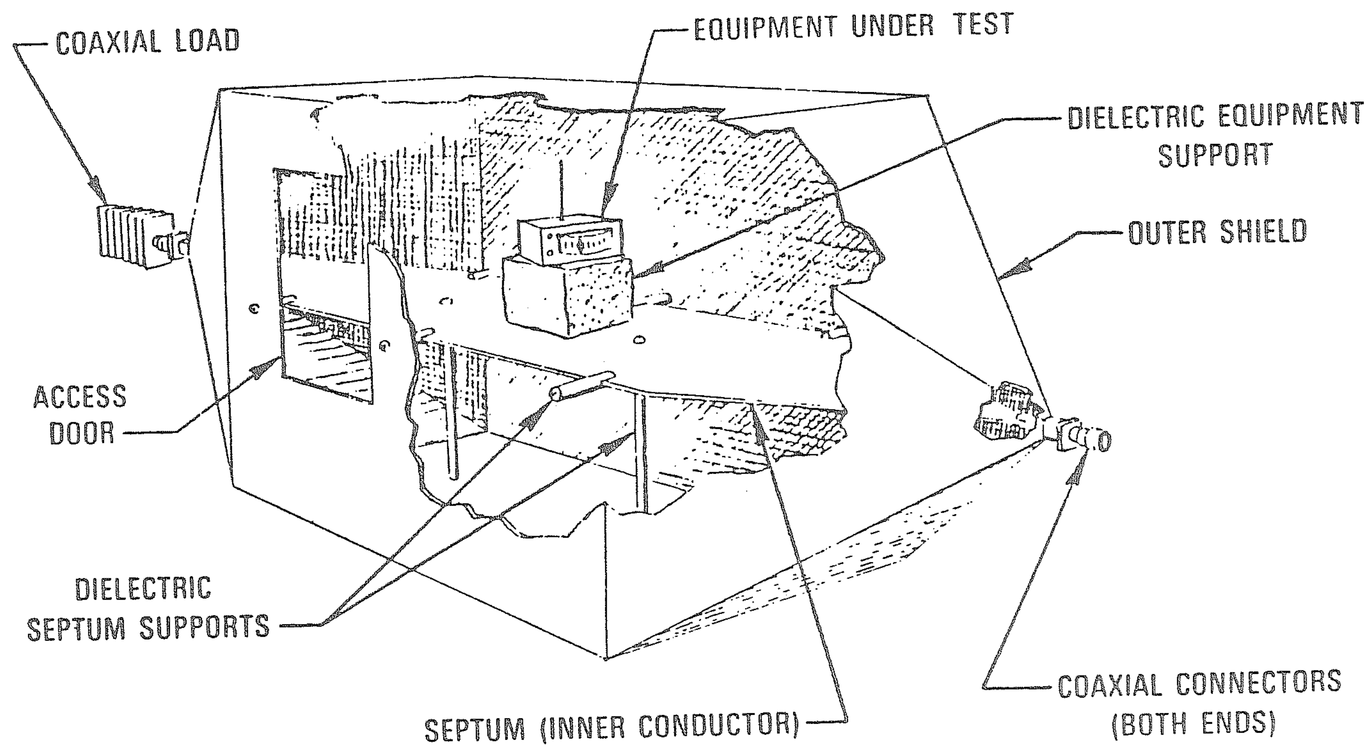

Figure 1. An NBS TEM Cell

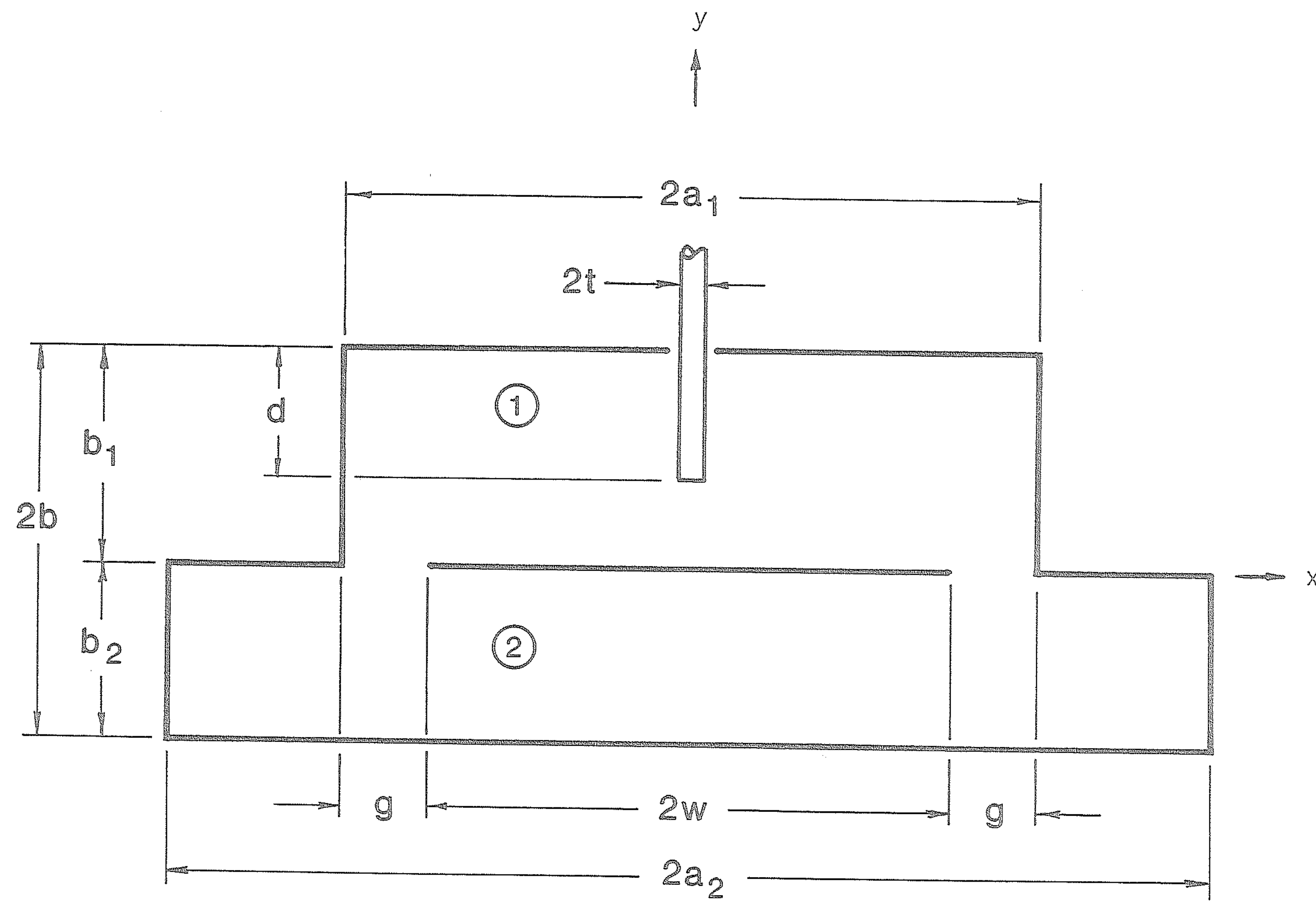

Figure 2. A RCTL with coaxial-line probe

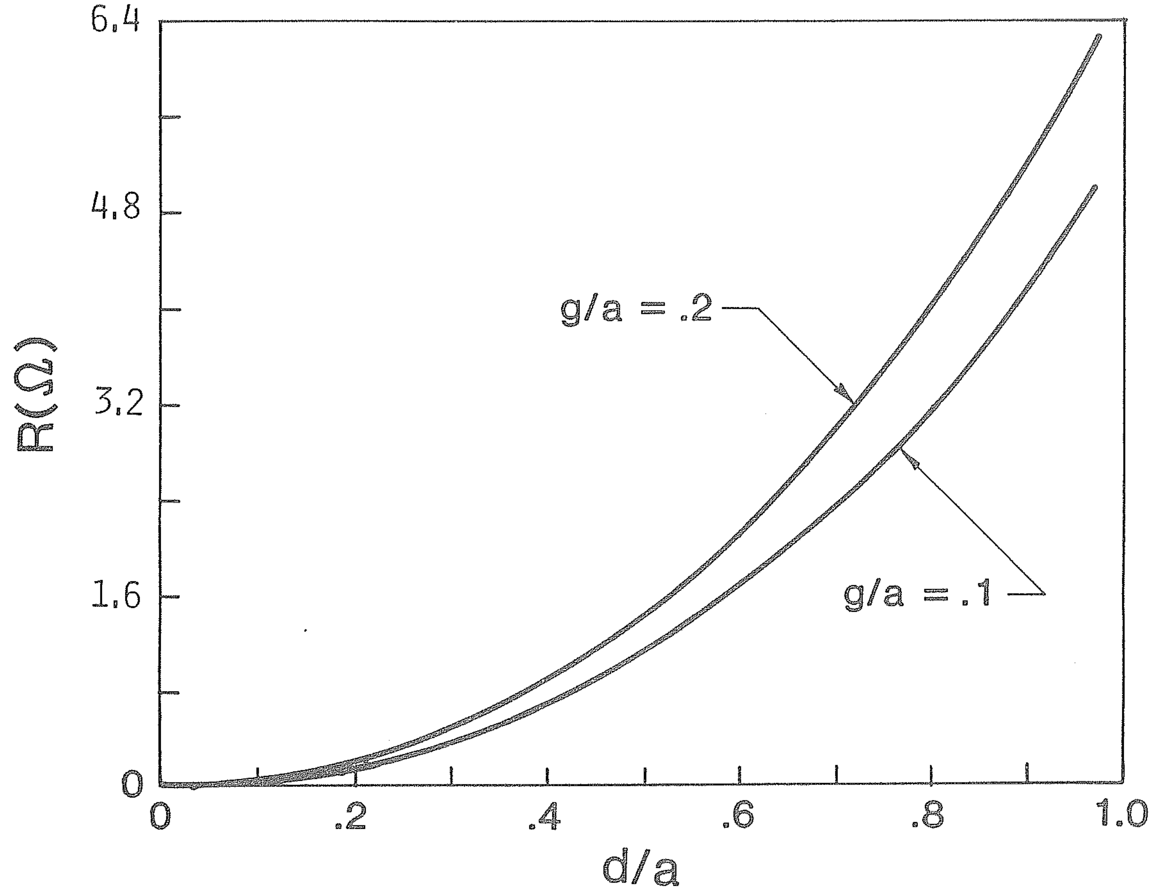

Figure 3. Radiation resistance for $a_1 = a_2 = b_1 = b_2 = 1$ m, $t = 1$ mm, and $f = 1$ MHz

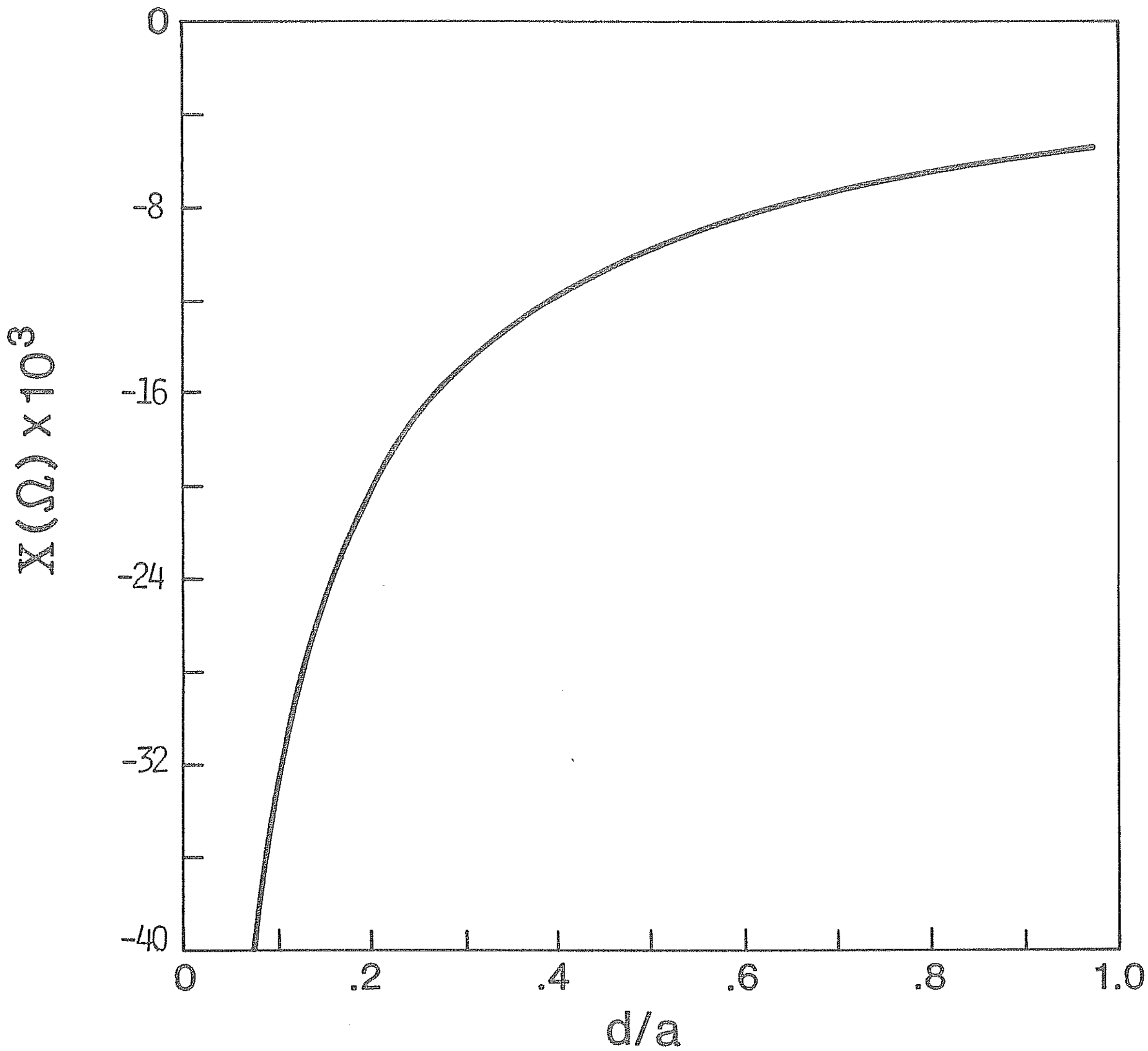

Figure 4. Reactance for $a_1 = a_2 = b_1 = b_2 = 1$ m, $t = 1$ mm, and $f = 1$ MHz

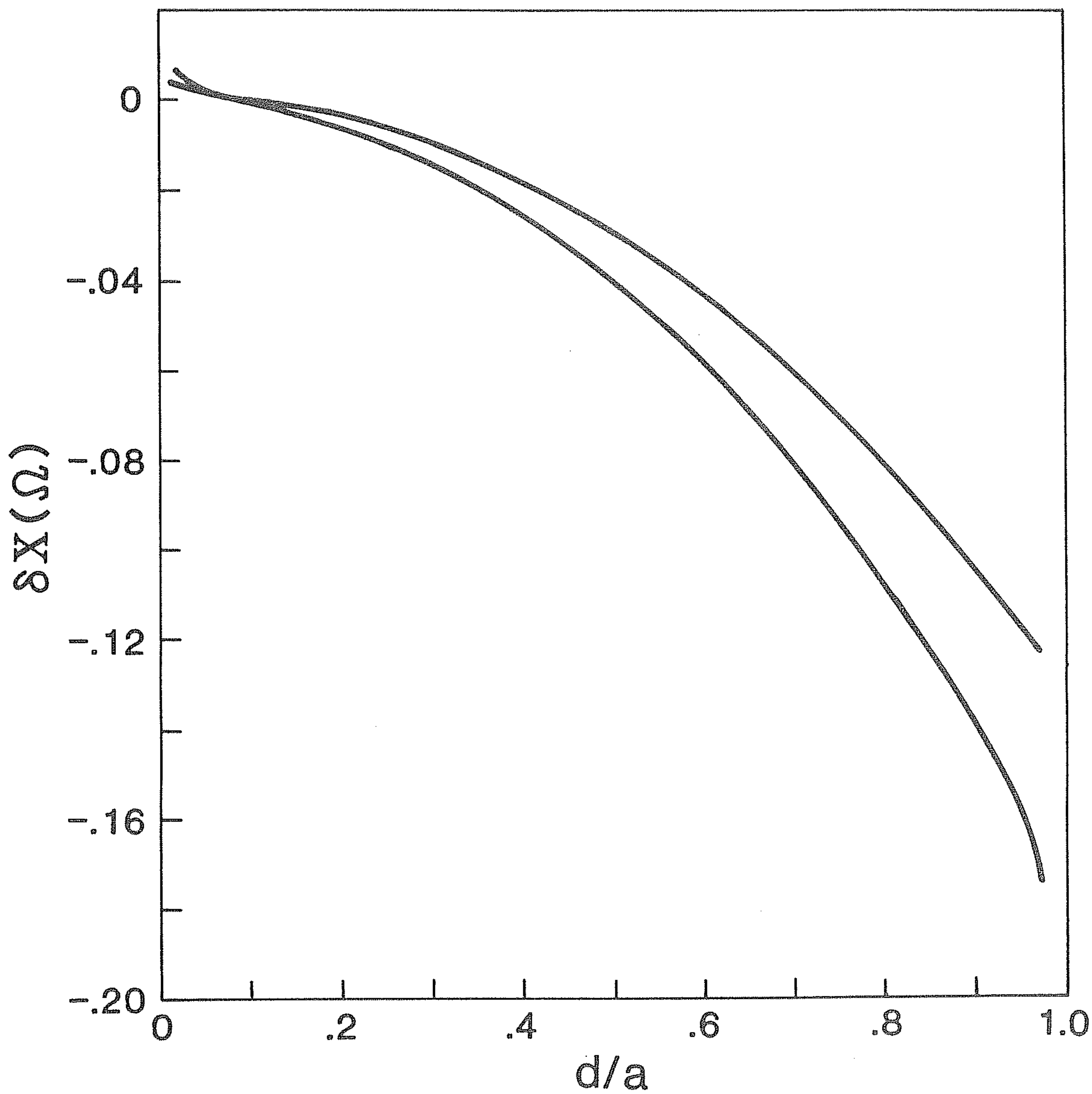

Figure 5. Perturbation reactance for $a_1 = a_2 = b_1 = b_2 = 1$ m, $t = 1$ mm, and $f = 1$ MHz

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11. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)*

The input impedance of a probe antenna exciting a transverse electromagnetic (TEM) transmission line cell is formulated by a variational approach. The formulation also utilizes the results from a previous work on the field distribution inside a TEM cell excited by a vertical electrical Hertzian dipole. The final result of impedance is shown to consist of two distinct terms, which are respectively contributed by the ordinary rectangular waveguide and the gap perturbation. Numerical results for both the real and imaginary parts of the impedance are given. The resistive part is found to be proportional to the square of the probe length, and the reactive part largely capacitive.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

Green's function, input impedance; probe antenna; radiation resistance, rectangular coaxial transmission line; TEM cell; variational method.

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