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STATISTICAL UNCERTAINTY ANALYSIS OF CCEM-K2 COMPARISONS OF RESISTANCE STANDARDS

N. F. Zhang [§], N. Sedransk[§], and D. G. Jarrett[#]
National Institute of Standards and Technology
Gaithersburg, MD 20899-8980

Abstract

Details of the statistical uncertainty analysis applied to key comparison CCEM-K2 are reported. Formulas were derived to determine the uncertainty of the combined difference between the measurements of multiple artifacts by an NMI and the corresponding predictions based on pilot lab measurements. In addition, the uncertainties of the reference value of the key comparison and the degrees of equivalence between two NMI's are obtained.

Background

An international comparison of dc resistance at 10 MΩ and 1 GΩ was organized under the auspices of the Consultative Committee for Electricity and Magnetism (CCEM) and piloted by the National Institute of Standards and Technology (NIST) with 14 other national metrology institutes (NMIs) participating [1].

In this key comparison, three 10 MΩ wirewound resistors and three 1 GΩ film-type resistors were used as the traveling standards. Each participating NMI was requested to complete their measurements of the traveling standards within a two-month period. During the comparison, the traveling standards were measured at the pilot lab, NIST, for seven separate periods by two different measurement systems that are used on a regular basis to calibrate customer high resistance standards [2], [3].

Data Analysis

For each traveling standard at the 10 MΩ and 1 GΩ resistance levels, a simple linear regression line was fitted to the NIST measurements of resistors using the mean date of each period of measurements as the predictor variable in the regression. For a fixed traveling standard (the j^{th}, $j=1,2,3$) and each of the non-NIST NMIs, the difference between its measurement and the corresponding prediction or the reference value is

$$D_i(j) = x_i(j) - x_{i,p}(j),$$

where $x_i(j)$ is the measurement for j^{th} traveling standard made by the i^{th} ($i=1,2,\ldots,15$) NMI and $x_{i,p}(j)$ is the prediction of the measurement of the i^{th} NMI based on the j^{th} regression. Then, the differences for each of the three traveling standards are combined as a weighted mean:

$$D_{i,comb} = \sum_{j=1}^{3} w(j) D_i(j),$$

where the weights, $w(j)$, are based on the reciprocals of the residual variances of the regressions. $D_{i,comb}$ represents the difference between the measurements made by the i^{th} NMI and the prediction for this NMI based on the measurements made by the pilot lab NIST. For a non-pilot lab NMI, the variance of $D_{i,comb}$ is obtained as

$$\sigma^2_{D_{i,comb}} = \sigma^2_{x,B,i} + \sigma^2_{x,A,i} + \frac{\sum_{j=1}^{3} \frac{1}{\sigma_r^4(j)}}{\left(\sum_{j=1}^{3} \frac{1}{\sigma_r^2(j)}\right)^2} + \frac{1 + \frac{1}{n} + \frac{(t_i - \bar{t}_{NIST})^2}{\sum_{k=1}^{n} (t_{NIST,k} - \bar{t}_{NIST})^2}}{\sum_{j=1}^{3} \frac{1}{\sigma_r^2(j)}},$$

where $\sigma_{x,A,i}$ and $\sigma_{x,B,i}$ are the root-sum-squares of the Type A and Type B uncertainties of the i^{th} NMI based on the NMI's uncertainty budget, respectively. $\sigma_r(j)$ is the standard deviation of the residuals from the j^{th} linear regression corresponding to the j^{th} resistor ($j=1,2,3$) and n (in this case $n=7$) is the number of periods of time when the resistors were measured by the pilot lab. t_i is the date when the measurements were made by the i^{th} NMI and $t_{NIST,k}$ is the date for the k^{th} period when the measurements were made by the pilot lab. $\bar{t}_{NIST}$ is the

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[§] Statistical Engineering Division, Gaithersburg, MD

[#] Electricity Division, Gaithersburg, MD.