Interconnection Transmission Line Parameter Characterization[1]

Roger B. Marks and Dylan F. Williams
National Institute of Standards and Technology
325 Broadway, Boulder, CO 80303

Abstract

This paper introduces a new method for the characterization of transmission lines fabricated on lossy or dispersive dielectrics. The method, which is more accurate than conventional techniques, is used to examine the resistance, inductance, capacitance, and conductance per unit length of coplanar waveguide transmission lines fabricated on lossy silicon substrates.

Introduction

In this paper we introduce a new procedure, which we call the calibration comparison method, for estimating the characteristic impedance Z_o of planar transmission lines fabricated on lossy or dispersive dielectrics. The new method, which overcomes many of the limitations of other methods, estimates the reference impedance of a thru-reflect-line (TRL) calibration, which equals Z_o [1].

For transmission lines on nondispersive, low-loss substrates, Z_o can be determined accurately by the method of [2]. In the more general case, an alternative estimate Z_o^r, suggested in [3], can be derived from the reflection coefficient of a small resistive load. This estimate is based on the approximation that the impedance of a small resistive load is real, constant, and equal to its dc resistance. This approximation, however, holds well only at low frequencies [4].

The conventional method [5,6] for the measurement of Z_o is based upon the measurement of the scattering parameters (S-parameters) of a single section of the transmission line. The measured S-parameters are equated to the S-parameters of an ideal transmission line that is electrically described solely by a characteristic impedance Z_o^e and a propagation constant. In certain specialized cases, such as with a uniform coaxial line in which a length of the dielectric has been replaced by a different dielectric, this procedure is exact and forms the basis of a number of methods for the determination of electromagnetic material properties. More commonly, however, the electrical discontinuity at the connection to the transmission line to be characterized cannot be described simply by the change of impedance. Therefore, the impedance Z_o^e determined by the conventional method is only an estimate of Z_o. In fact, as we show here, even small electrical discontinuities can lead to large errors in the estimate Z_o^e, particularly when the line impedance differs greatly from the calibration reference impedance or when the line length is near a multiple of a half wavelength.

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The calibration comparison method introduced here is based on an approach we suggested for determining the reference impedance of a calibration [7]. In [7] the reference impedance of an S-parameter calibration is determined from the "error box" relating it to a second S-parameter calibration of known reference impedance Z_r. Here the calibration of unknown reference impedance is a TRL calibration, for which the reference impedance $Z_r=Z_o$ [1].

Below we present an error analysis which indicates that the electrical discontinuity introduces a much smaller error in the calibration comparison method than in the conventional method. We also test the two methods with lines of known Z_o to demonstrate the improved accuracy and show that the calibration comparison method overcomes the low-frequency limitation of the estimate Z_o^r. Finally, we apply the calibration comparison method to the electromagnetic characterization of coplanar waveguide transmission lines fabricated on lossy silicon substrates.

Determination of Z_o

The conventional method [5,6] estimates Z_o from the measured cascade matrix T of a single transmission line. T is a cascade of matrices

$$T = A\,Q\,L\,\overline{Q}\,\overline{B}, \tag{1}$$

where A and B represent electrical discontinuities at the interface between the test port and the transmission line. Q is an impedance transformer given by [1]

$$Q \equiv \frac{1}{2Z_o}\left|\frac{Z_o}{Z_r}\right|\sqrt{\frac{Z_r}{Z_o}}\begin{bmatrix} Z_o+Z_r & Z_o-Z_r \\ Z_o-Z_r & Z_o+Z_r \end{bmatrix}, \tag{2}$$

where Z_r is the reference impedance of the calibration. L is the cascade matrix of the line

$$L = \begin{bmatrix} e^{-\gamma l} & 0 \\ 0 & e^{+\gamma l} \end{bmatrix}, \tag{3}$$

where γ is its propagation constant. The reverse cascade matrix of a matrix Y is given by

$$\overline{Y} \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} Y^{-1} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{4}$$

In the conventional method for determining Z_o, T is equated with $QL\overline{Q}$. This approximation yields the estimate [5,6]

$$Z_o^e = Z_r\sqrt{\frac{(1+S_{11})^2-S_{21}^2}{(1-S_{11})^2-S_{21}^2}}, \tag{5}$$

where the S_{ij} are the measured S-parameters of the line. However, the use of (5) introduces and error due to neglect of the electrical discontinuities A and B at the transition to the line. A linear analysis shows that the relative error is given approximately by

$$\frac{Z_o^e - Z_o}{Z_o} \approx \frac{1}{Z_o\, Z_r\, D_1} \left[(Z_r^2 + Z_o^2)\, D_2 + (Z_r^2 - Z_o^2)\, D_3 \right] , \tag{6}$$

where

$$D_1 = (1 - S_{11}^o)^2 - (S_{21}^o)^2 , \tag{7}$$

$$D_2 = \epsilon_{11}^A + (\epsilon_{12}^A + \epsilon_{21}^A)\, S_{11}^o + \epsilon_{22}^B\, (S_{21}^o)^2 + \epsilon_{22}^A\, (S_{11}^o)^2 , \tag{8}$$

$$D_3 = \epsilon_{11}^A\, S_{11}^o + (\epsilon_{12}^A + \epsilon_{12}^B)(S_{11}^o)^2 + \epsilon_{22}^A\, (S_{11}^o)^3 - \epsilon_{22}^A\, S_{11}^o\, (S_{21}^o)^2 - (\epsilon_{21}^A + \epsilon_{12}^B)(S_{21}^o)^2 , \tag{9}$$

$$S_{11}^o = \frac{(Z_o^2 - Z_r^2)\ \sinh\gamma l}{2 Z_o Z_r \cosh\gamma l + (Z_o^2 + Z_r^2)\ \sinh\gamma l} , \tag{10}$$

and

$$S_{21}^o = \frac{2\, Z_o\, Z_r}{2 Z_o Z_r \cosh\gamma l + (Z_o^2 + Z_r^2)\ \sinh\gamma l} . \tag{11}$$

The S_{ij}^o are the S-parameters corresponding to $QL\bar{Q}$. The ϵ_{ij} are related to the deviations of A and B from the identity and are defined by

$$S^A \equiv \begin{bmatrix} \epsilon_{11}^A & 1+\epsilon_{12}^A \\ 1+\epsilon_{21}^A & \epsilon_{22}^A \end{bmatrix} ; \quad S^B \equiv \begin{bmatrix} \epsilon_{11}^B & 1+\epsilon_{12}^B \\ 1+\epsilon_{21}^B & \epsilon_{22}^B \end{bmatrix} , \tag{12}$$

where S^A and S^B are the S-parameters corresponding to the cascade matrices A and B. Six of the eight elements ϵ_{ij} contribute to the error in Z_o and the denominator D_1 in (6), which is small when the line is near some multiple of a half wavelength long, magnifies the effects of the ϵ_{ij}.

In the calibration comparison method, the matrices AQ and BQ, the "error boxes" relating the two calibrations, are determined directly. This gives two independent estimates of Z_o, as described in [7]. Linear analysis shows that, in this case, the linearized relative error in the characteristic impedance estimate Z_o^A, as determined from AQ, is given by

$$\frac{Z_o^A - Z_o}{Z_o} \approx \epsilon_{11}^A - \epsilon_{22}^A \tag{13}$$

and that in Z_o^B, as determined from BQ, is given by

$$\frac{Z_o^B - Z_o}{Z_o} \approx \epsilon_{11}^B - \epsilon_{22}^B . \tag{14}$$

Small deviations of A and B from the identity matrix do not result in large errors in the estimates

Z_o^A and Z_o^B. Furthermore, the method is insensitive to errors in the transmission coefficients of A and B. Finally, the linear error term vanishes when the matrices A or B are symmetric. In our measurements, we report the average result $Z_o^c = (Z_o^A + Z_o^B)/2$.

Comparison of the Methods

The relative accuracy of the two techniques is illustrated by the simulated results of Fig. 1, in which Z_o was 85 Ω and the values of the ϵ_{ij} were about 0.01. The peaks in the measurement errors predicted for the conventional method occur when the line tested is near some multiple of a half wavelength long. The figure illustrates that even small electrical discontinuities at the test port give rise to large errors in the conventional method. In contrast, the simulated errors in the calibration comparison method are of the order of the ϵ_{ij}.

We also applied the methods to a coplanar line fabricated on quartz, with $Z_o \approx 85$ Ω, roughly corresponding to the case analyzed in Fig. 1. In the experiments, the S-parameters of these lines were determined with respect to a 50 Ω GaAs TRL coplanar waveguide probe-tip calibration, as described in [8]. We used the broadband multi-line TRL calibration of Marks [9] throughout. Figures 2a and 2b compares the real and imaginary parts of Z_o^e from the conventional method using the measured S-parameters of a 7.115 mm long line, Z_o^c from the calibration comparison method, Z_o^r from the reflection coefficient of a square 50 Ω resistor, and Z_o from direct measurement [2]. The figure illustrates the large errors associated with the estimate Z_o^e from the conventional method and the degradation with increasing frequency of the imaginary part of the estimate Z_o^r due to neglect of the reactance associated with the resistor [4]. These contrast to the well behaved and more accurate estimate Z_o^c from the calibration comparison method. We even found that when we eliminated all of the lines in the second TRL calibration except the thru line and the 7.115 mm line, which might be expected to degrade the accuracy of the estimate near the points at which the line was nearly a multiple of a half wavelength, the estimate Z_o^c did not change appreciably from the values plotted in the figure.

Application to Coplanar Waveguide Fabricated on Silicon

To further illustrate the utility of the calibration comparison method, we applied it to coplanar waveguide lines fabricated on lossy silicon substrates. We estimated the inductance L, resistance R, capacitance C, and conductance G per unit length of the lines from Z_o^c and γ by

$$j\omega C + G \equiv \frac{\gamma}{Z_o} \approx \frac{\gamma}{Z_o^c} \tag{15}$$

and

$$j\omega L + R \equiv \gamma Z_o \approx \gamma Z_o^c . \tag{16}$$

The propagation constant γ was determined from the multiline TRL calibration [9] performed with the silicon lines. In Fig. 3 we plot R and ωL of the silicon lines alongside their values for coplanar lines of the same geometry fabricated on gallium arsenide. The figure illustrates the expected independence of R and L on the substrate. At low frequencies R approaches the measured dc resistance R_{dc} indicated in the figure. L depends weakly on frequency and is nearly equal to the quasi-static value computed for lossless conductors except at low frequencies where the internal inductance due to field penetration in the metal is significant. In Figs. 4 and 5 we plot C and G for the silicon lines. The quasi-static capacitance per unit length C_o for a coplanar line on a substrate with the dielectric constant of 11.7, that of pure silicon, is shown for comparison. Figure 4 shows that C is nearly independent of frequency and nearly equal to C_o for the lines on high-resistivity silicon but increases at the low frequencies on the silicon of low resistivity. This increase may be due to the formation of a Schottky barrier at the interface between the metal conductors and the silicon substrate. Figure 5 shows that, for the most part, G increases with increasing substrate conductivity.

Conclusion

An error analysis shows that the calibration comparison method for determining characteristic impedance is more accurate than the conventional method [5,6] or the method based on the reflection coefficient of a small lumped resistor. We tested the method on a coplanar waveguide transmission line of known characteristic impedance. The results illustrate the superiority of the calibration comparison method.

We also applied the method to coplanar lines fabricated on silicon substrates and determined R, L, C, and G from the characteristic impedance and propagation constant. R and L were seen to be nearly independent of the substrate while C and G depended strongly on the substrate. This independence of electrical parameters and material parameters suggests that surface impedance or other metal parameters might be accurately extracted from measurements of R and L while substrate dielectric constant or loss tangent might be accurately extracted from measurements of C and G.

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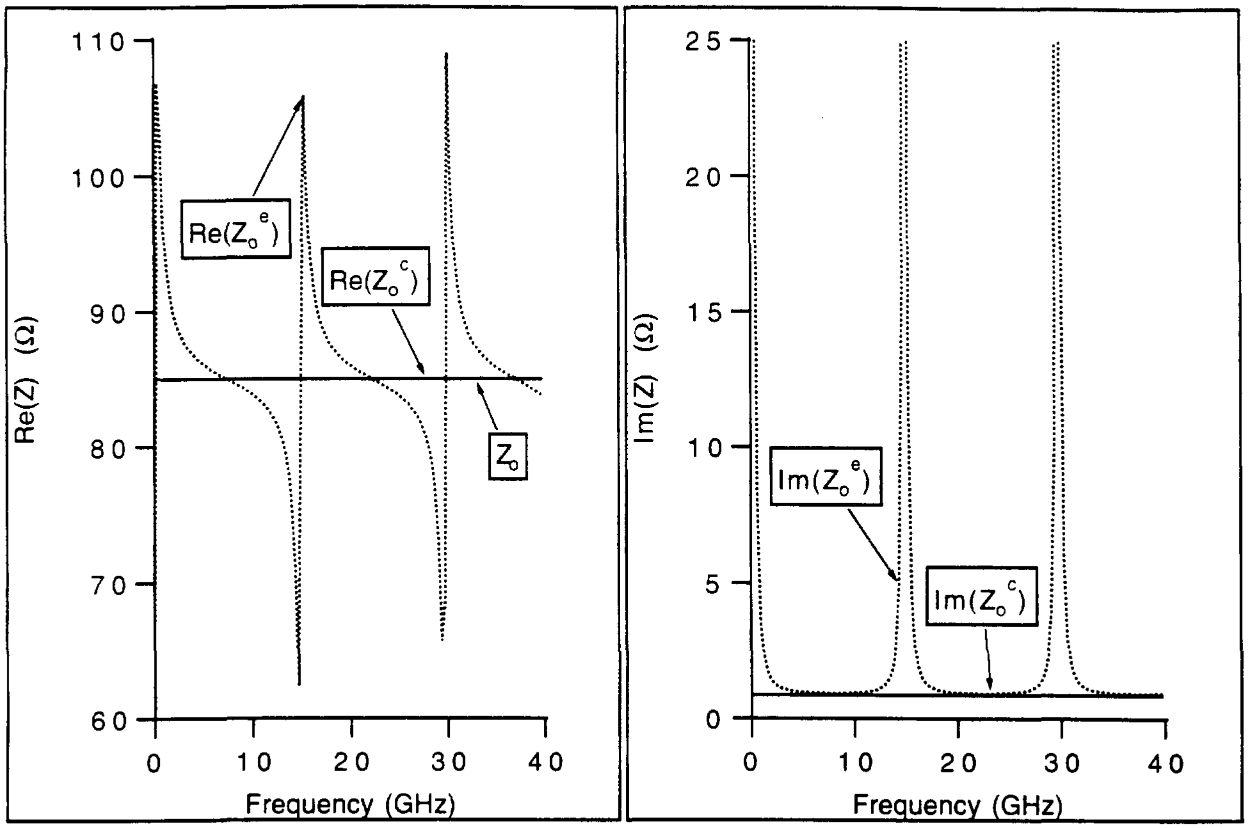

Figures 1a and 1b. Real and imaginary parts of simulated estimates. $Z_o=85\ \Omega$, $Z_r=50\ \Omega$, $\epsilon_{ij}=0.01j$ except $\epsilon_{22}=0$. The parameters of the line correspond to the quartz line of Figure 2.

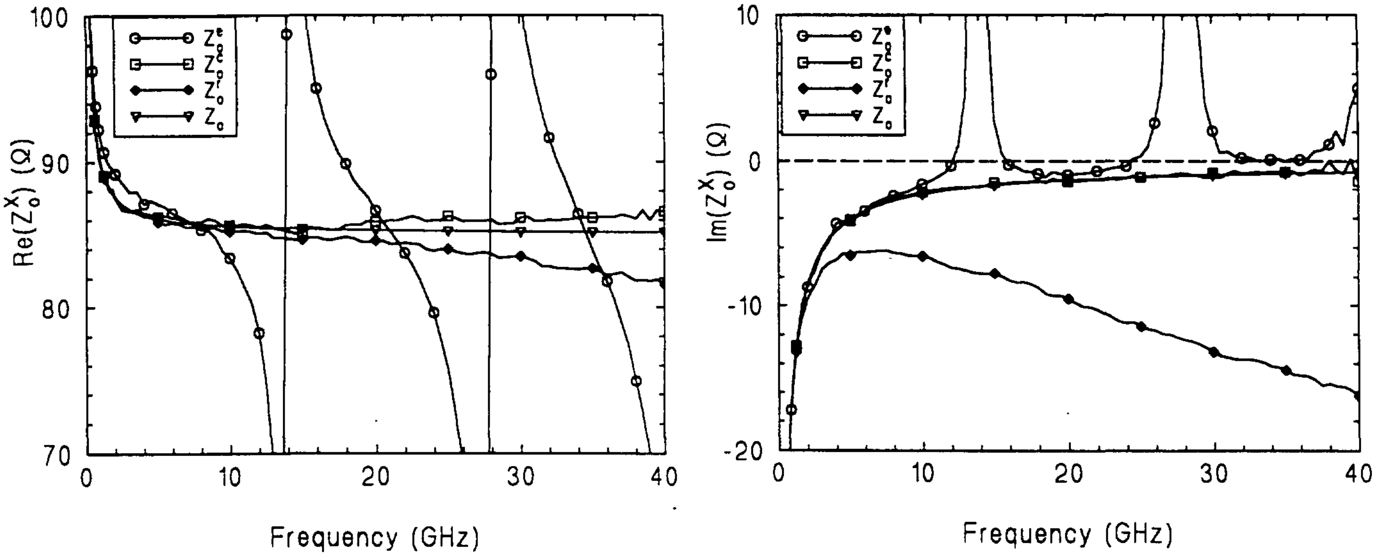

Figure 2a and 2b. The real and imaginary parts of Z_o and its estimates for the CPW line fabricated on quartz.

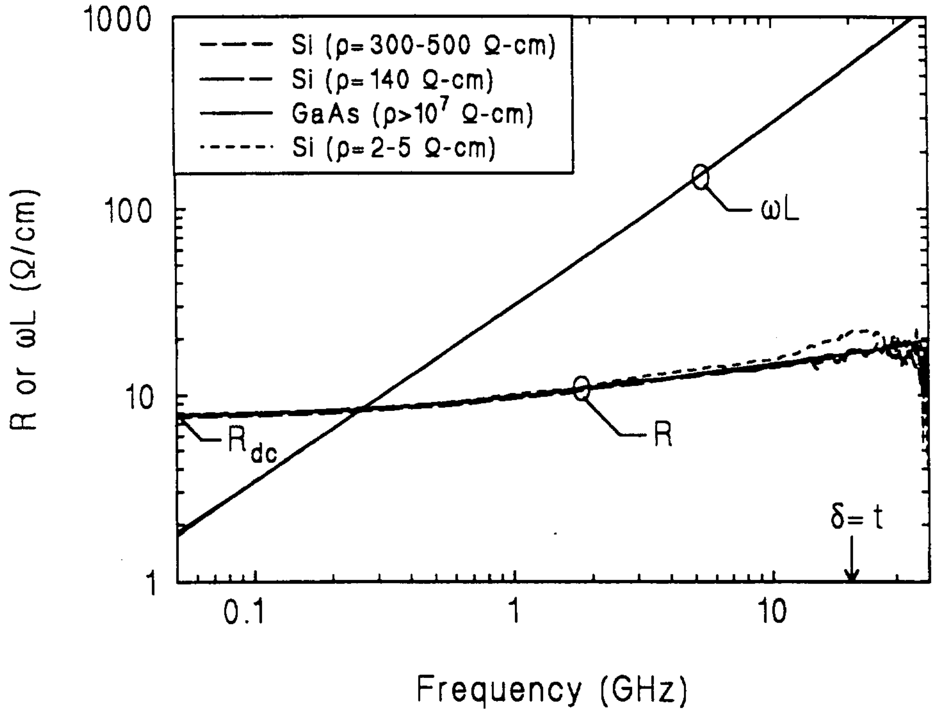

Figure 3. *R* and *ωL* of coplanar lines fabricated on lossy silicon and semi-insulating gallium arsenide substrates. ρ is the approximate resistivity of the substrate indicated by the manufacturer. R_{dc} is the measured dc resistance per unit length of the transmission lines, which were fabricated with identical conductor geometry and metal thickness. The point at which the skin depth δ is equal to the conductor thickness t is marked for reference.

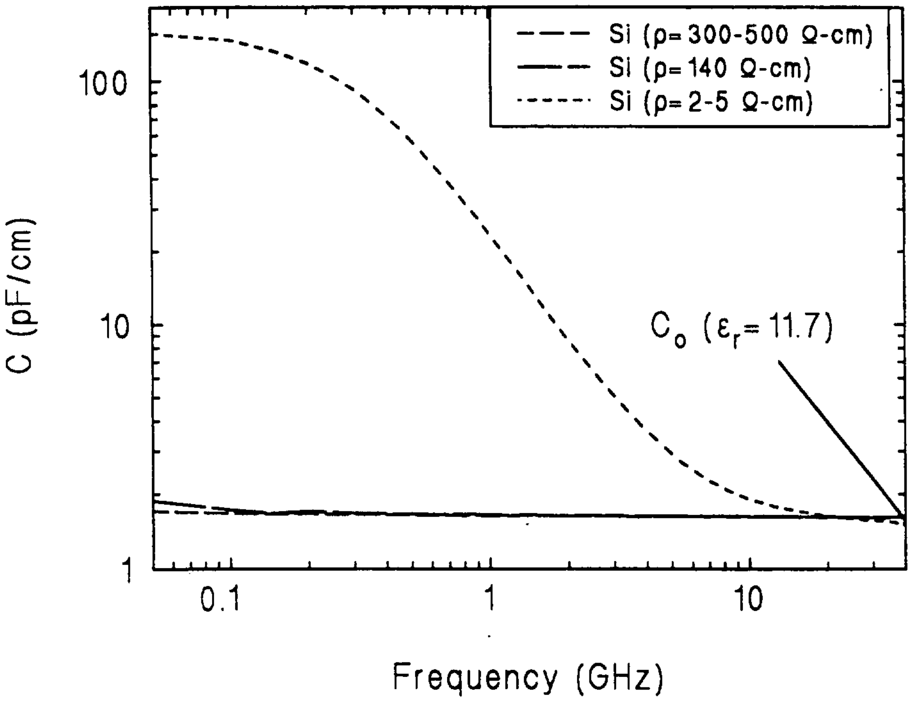

Figure 4. *C* for the coplanar lines fabricated on lossy silicon substrates of Fig. 3. C_o indicates the quasi-static capacitance per unit length of the lines calculated using a relative substrate dielectric constant is 11.7, approximately that of pure silicon.

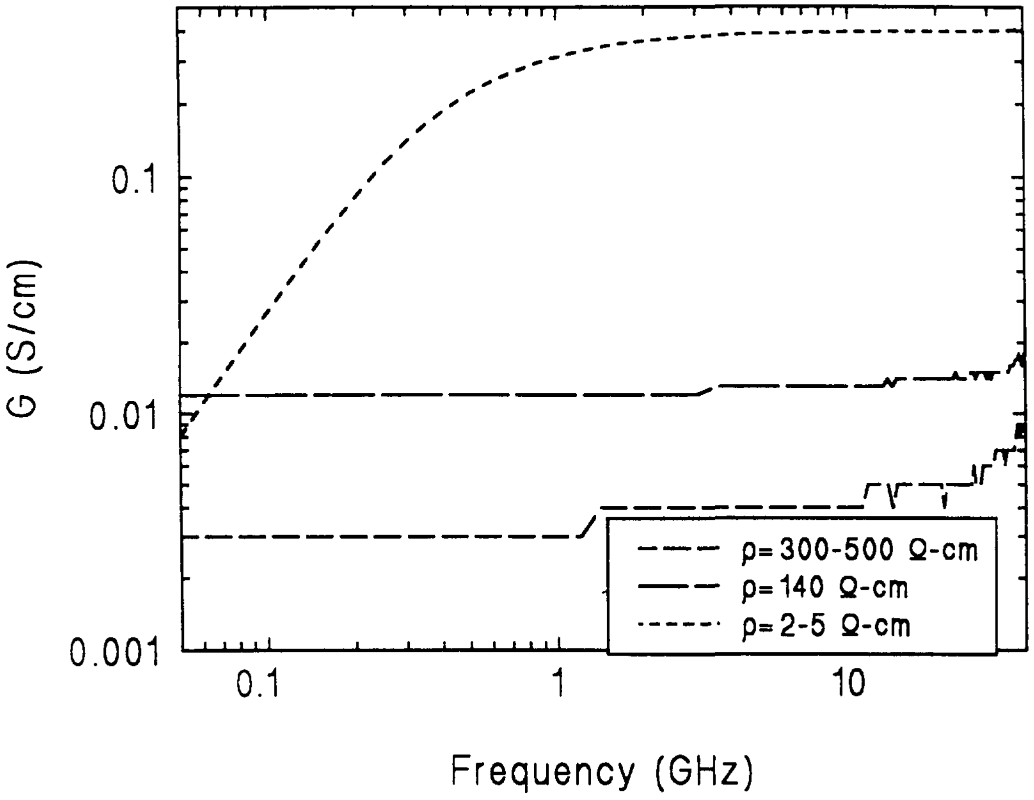

Figure 5. *G* for the coplanar lines fabricated on lossy silicon substrates of Fig. 3.