

n-VALUE AND SECOND DERIVATIVE OF THE SUPERCONDUCTOR VOLTAGE-CURRENT CHARACTERISTIC

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Abstract--We studied the n-value ($V \propto I^n$) and second derivative (d^2V/dI^2) of the voltage-current curve of high and low temperature superconductors and superconductor simulators. We used these parameters for diagnosing problems with sample heating and data acquisition, and as indicators of the superconducting-to-normal state transition. The superconductor simulator may be useful in testing the measurement system integrity and reducing measurement variability since its characteristics are highly repeatable.

I. INTRODUCTION

The voltage-current (V-I) curve of a superconductor can often be modeled by the empirical equation

$$V = V_0 (I/I_0)^n - V_1, \quad (1)$$

where n reflects the abruptness of the transition from the superconducting to the normal state, with typical values ranging from 10 to 100. The value of V_1 is usually taken as zero, but under high voltage conditions with small values of n , this parameter may be nonzero. The superconductor simulator is an electronic circuit that emulates the V-I curve.

Both the n-value and the second derivative (d^2V/dI^2) characteristic of a superconductor can be used as a qualitative figure of merit [1-4]. The d^2V/dI^2 characteristic usually has the shape of an asymmetric bell curve, with a peak that occurs in the transition region from the superconducting to the normal state. Ideally, the characteristic would have a sharp peak near the critical current of the conductor, with a small width. The two figures of merit (n and d^2V/dI^2) are related by:

$$n = (1 + I \cdot (d^2V/dI^2) / (dV/dI)). \quad (2)$$

The second derivative measurement has some disadvantages: it is difficult to compute numerically, is very susceptible to noise, and high voltage data are needed to determine the characteristic's prominent features. High voltages lead to sample heating problems: the contact resistance (I^2R) and the voltage drop (V-I) become factors. The d^2V/dI^2 characteristic is obtained by ramping the current to higher levels than the critical-current measurement. It is

necessary to obtain a characteristic that includes the peak in the second derivative so that the relative noise and prominent features in the data are apparent. The n-value characteristic is often easier to measure than the second derivative and its features are apparent without high voltage data.

A basic premise of this paper is that the continuous V-I curve has been discretized by obtaining a set of representative V-I pairs. These data points can be obtained with low uncertainty by using the stepped dc method, discussed in detail in references [5, 6]. Other methods such as the ac lock-in technique [5, 7] are available for obtaining the d^2V/dI^2 characteristic, but they are nonstandard for high current applications.

The simplest method of computing the second derivative of a discrete set of data points is to compute the slope between pairs of adjacent data points and then compute the slope between these resultant slopes. Neither the raw data nor the dV/dI data are smoothed or filtered. All second derivative data obtained for this paper were computed using two points for dV/dI and three points for d^2V/dI^2 . Thus, the data were not filtered to a high degree. Optimal selection of the distribution of V-I data points so that the second derivative curve reflects structure but not noise becomes important.

The current setpoints used for the d^2V/dI^2 measurements presented in this paper were obtained so that a subset of them fell past the peak in the curve, to the point where the characteristic was less than about 20% of the maximum value.

Warnes and Larbalestier [1] give a comprehensive review of the historical background of the second-derivative characteristic as the distribution of the superconductor's critical current density. Umeda [4] extends these models to include superconducting filament irregularity. In addition to these interpretations, the second derivative characteristic can be used as a diagnostic aid for evaluating the overall shape of the superconducting-to-normal transition as well as data acquisition and stability problems in critical-current measurements. Irregular features in the second-derivative and n-value characteristic are indicators of these problems.

II. OBTAINING SMOOTH CHARACTERISTICS

The V-I data points needed to obtain a full (not necessarily smooth) d^2V/dI^2 characteristic must span a large

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range of current and voltage. This voltage range could be as large as from 0 to 1000 $\mu\text{V}/\text{cm}$, which is much larger than the voltage range needed to obtain a critical-current measurement. Ideally, to obtain a relatively symmetric characteristic the setpoints could be determined using an algorithm that computes currents corresponding to equally spaced points along the d^2V/dI^2 characteristic.

There is a tradeoff between smoothing the d^2V/dI^2 characteristic and losing the structure of the characteristic. Increasing the number of setpoints would increase the structure of the characteristic, but at a loss of smoothness. Similarly, decreasing the number of points would decrease the structure of the characteristic with an increase in smoothness.

The stepped dc method was used to obtain low noise data for the d^2V/dI^2 characteristics. These setpoints are chosen so that the voltages are approximately equally spaced on a logarithmic scale, with an increased point density at higher voltages. This second-derivative setpoint algorithm (SDSA) relies on properly choosing points to reduce the necessity of smoothing the curve. Another approach to obtain a smooth d^2V/dI^2 curve explored by Warnes and Larbalestier [1] is to apply digital filtering techniques to the second-derivative data to obtain a smooth curve. These two approaches pose different computational problems.

Figure 1 shows two second derivative curves that were generated using a superconductor simulator [8] that has an adjustable n-value and critical current. The setpoints for each curve were computed using SDSA. Each curve is normalized to 1, and the upper curve is displayed shifted by 0.5 units from the lower curve for clarity. Notice that the curves are smooth, with virtually no noise. The low-noise behavior is a property of the superconductor simulator and the SDSA, and is an idealized version of what would be observed on an actual superconductor. Each curve corresponds to a different n-value: the upper curve has an approximate peak n-value of 20,

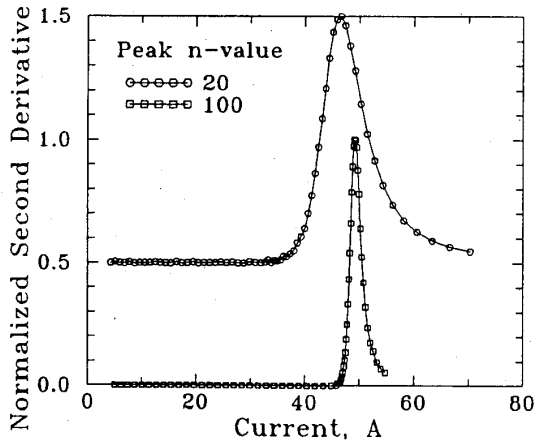


Fig. 1 Shifted, normalized d^2V/dI^2 versus current for the active simulator for two n-values.

while the lower curve has a peak n-value of about 100. These two curves illustrate the fact that increasing the n-value results in a smaller width at the half maximum point. The peak second derivative is a strong function of the critical current, n-value, and normal-state resistance. However, the normalized shape is appropriate for comparison.

The simulators provide an ideal test facility to develop data acquisition algorithms such as the SDSA, since they have low noise (1 nV is possible), highly reproducible critical currents (less than 0.06% ΔI_c in a three month period), and reproducible n-values. The setpoints developed for the second-derivative curve using the simulators can be applied to a real superconductor with minor modifications.

III. COMPUTING n-VALUE WITH DERIVATIVES

The n-value measurement can be performed without going to the high current and voltage needed for the second derivative measurement. The conventional method used to compute the n-value of a V-I curve is to perform a local linear fit of the logarithms of the V-I pairs. In this method, it is assumed that the value of V_1 in Eqn. 1 is zero. We can also use the second derivative data to compute n-values in accordance with Eqn. 2. Eqn. 2 indicates that the n-value will approach 1 as the second derivative approaches zero. Often, the V-I curve of a superconductor near the normal state transition can be approximated by the linear V-I relationship:

$$V = r \cdot (I - I_1) \quad (3)$$

Eqn. 3 is simply Eqn. 1 with $V_1 = r \cdot I_1$ and $n=1$. The superconductor simulator follows Eqn. 3 at high voltages. The d^2V/dI^2 characteristic will approach zero as the local V-I curve approaches a linear relationship. Thus $n=1$ according to Eqn. 2. However, in this region, the n-value calculated as the slope of the log V versus log I curve does not yield an n-value of 1 since the term $r \cdot I_1$ (which corresponds to V_1 in Eqn. 1) is nonzero. This fact is illustrated in Figs. 2 and 3. The two methods of calculating the n-values of the V-I curve yield different results especially at high voltages.

Figure 2 shows normalized n-value as a function of sample current, where the n-value is computed using the second derivative method outlined above. When not normalized, these curves approach 1 at high sample currents. Figure 3 shows the same data with n-value computed the three point logarithmic fit method. In this case, when the n-values are not normalized, the curves do not approach 1; they are significantly higher. Similar differences between n-values calculated by the two methods on superconductor samples were observed.

Using Eqn. 2 to determine n-values of a given V-I curve yields results that have a higher noise level compared to n-values calculated using the logarithmic version of Eqn. 1, since Eqn. 2 contains a division term of dV/dI .

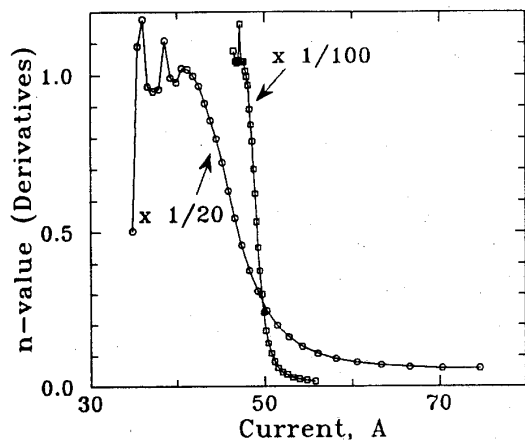


Fig. 2 Normalized n -value versus current for the active simulator computed using derivatives.

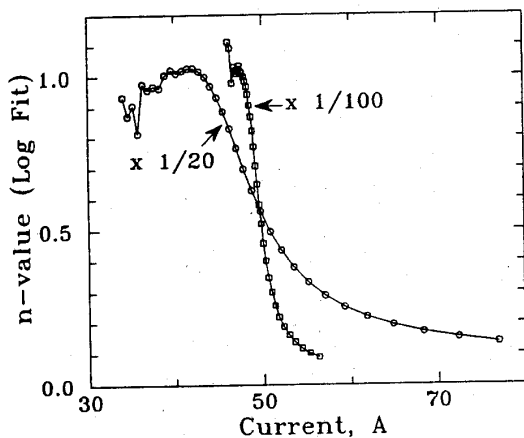


Fig. 3 Normalized n -value versus current for the active simulator computed using a three point logarithmic fit.

IV. d^2V/dI^2 AND n -VALUE OF SUPERCONDUCTORS

Figure 4 is a data plot of the second derivative characteristic for stabilized monofilament Cu/Nb-Ti superconductor for various magnetic fields at a temperature of 4 K in liquid helium. This sample was stabilized by soldering to copper foil. Each curve on the plot is shifted by 0.1 unit for clarity. The V-I characteristics of this conductor without stabilization were too unstable at fields below 4 T to be useful. The stabilization technique did not significantly change the relevant features of the d^2V/dI^2 characteristic at high magnetic fields.

The peak of the curve occurs at lower currents for higher magnetic fields, since the transition region from the superconducting to normal state occurs in this current region. The curves become smoother and more well defined at high

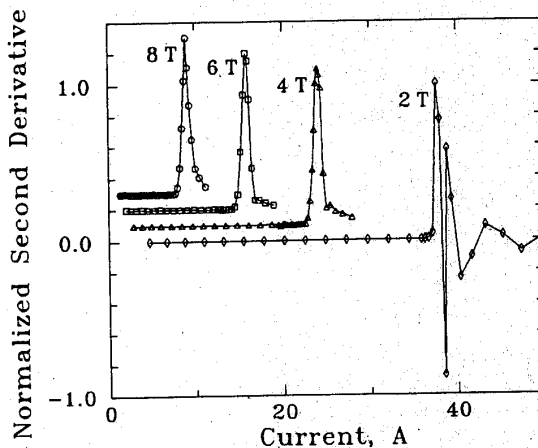


Fig. 4 Shifted, normalized d^2V/dI^2 versus current taken on a monofilament Cu/Nb-Ti in liquid helium.

magnetic fields. In fact, they approach the second derivative characteristic of an active simulator with a high n -value. The data at 2 T in Fig. 4 illustrate the use of the second derivative characteristic to diagnose insufficient stability or cooling. There is a corresponding irregular feature in the n -value characteristic at 2 T. At magnetic fields below approximately 1 T, the V-I curve approaches a step function, indicating a very large n -value. The second derivative thus has a high positive spike followed by a high negative spike. A monofilament conductor is an extreme example since its critical current is highly dependent on the sample temperature and there is no difficulty in current redistribution compared to its multi-filament counterpart. Also, a monofilament conductor may be susceptible to flux-jump instability.

The second derivative measurement provides a rich source of information concerning the superconducting-to-normal transition. Figure 5 shows the d^2V/dI^2 characteristic

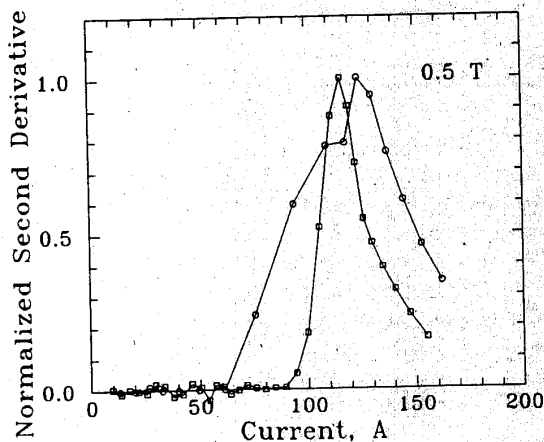


Fig. 5 Normalized d^2V/dI^2 versus current for two Bi-based oxide/Ag tape samples.

of two different Bi-based oxide/Ag tape samples [9]. These two samples had similar critical currents, but the second derivatives are quite different. This method of characterizing samples may provide useful information about the processing of conductors. One curve plateaus before it reaches its peak. Although it may seem as though this plateau is a random occurrence, the curves in Fig. 6, taken on the same sample at higher magnetic fields show that this is most probably the beginnings of a double peak. If more data points were taken in the plateau region, the double peak structure might become more obvious.

Each curve in Fig. 6 shows the double peak for the same sample at magnetic fields of 1, 4, and 12 T. The left-most curve shows increased noise near the first peak. This apparent noise in the second derivative is most likely due to the large number of data points in that current interval. Writing a general algorithm that is applicable to a variety of superconductors that optimally distributes data points along the second derivative curve is difficult.

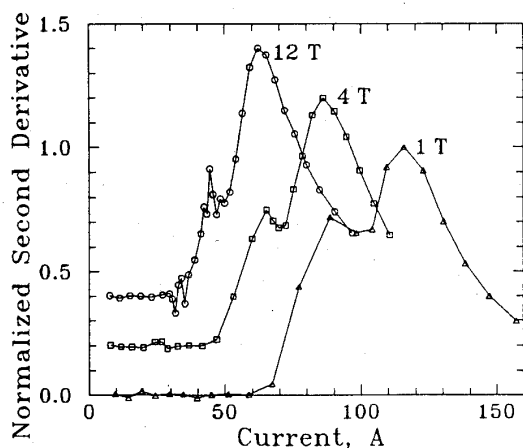


Fig. 6 Shifted, normalized d^2V/dI^2 versus current for a Bi-based oxide/Ag tape sample at three magnetic fields.

V. SUMMARY

The second derivative of the V-I curve is difficult to obtain, since it is extremely sensitive to noise and the algorithm used to compute the setpoints. However, it is a viable figure of merit of the conductor. An effective algorithm for computing the data points for this measurement is to choose them so that the voltages are approximately equally spaced on a logarithmic scale, with an increased point density at higher voltages. The interesting features of the second derivative occur at high current and voltage. Thus, these measurements are susceptible to sample heating.

The n-value as a function of current can be calculated from the V-I curve obtained in the critical-current measurement without having to obtain data at the high current

and voltage needed for the d^2V/dI^2 measurement. Thus, it can be used as a diagnostic in critical-current measurement.

The n-value and the second derivative characteristics provide information about the shape of the transition from the superconducting to the normal state, and about the noise and sample heating that are present during the measurement. The second derivative provides another way to compute the n-value of the voltage-current curve as opposed to the conventional method of computing a logarithmic fit of the voltage-current data.

The superconductor simulator is a useful tool for investigating data acquisition patterns for n-value and the second derivative measurement. Because it is an idealized superconductor, data acquisition algorithms can be designed and tested using the simulator, and then applied to actual superconducting specimens with minor modifications.

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