Internal fields in magnetic materials and superconductors

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This Paper reviews some of the concepts needed for the correct analysis of magnetization data, both for magnetic materials and superconductors. Demagnetization factors, initial susceptibilities and hysteresis losses are discussed.

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Magnetization measurements have acquired an important function in the determination of a.c. losses in superconductors. Many of the traditional analysis methods used for magnetic materials apply to superconductors with the assumption of perfect diamagnetism. Some refinement is necessary for type-II and multifilamentary superconductors.

Demagnetization factors

The magnetization curve characteristic of a magnetic material is obtained by plotting magnetization, M, *versus* internal field, H_{int}. H_{int} is obtained by correcting the external applied field, H_{ext}, by the demagnetization field, $H_{demag} \equiv DM$, where D is the geometric demagnetization factor for a uniformly magnetized material. In SI units

$$H_{int} = H_{ext} - DM \quad (1)$$

For a sphere, D = 1/3. For a long cylinder, with H_{ext} applied axially, D = 0; for H_{ext} applied transversely, D = 1/2. H_{int} corresponds to the field H in the equation $B = \mu_0(H+M)$, where B is the flux density and μ_0 is the permeability of free space. H_{int} is sometimes referred to as the Maxwell field. Here we ignore any fields arising from a.c. effects such as eddy currents.

Initial susceptibilities

Often the initial slope of the magnetization curve is used for calibration of magnetometers. We define the internal susceptibility, χ_{int}, as dM/dH_{int} and the external susceptibility, χ_{ext}, as dM/dH_{ext}. Thus, from Equation (1)

$$\chi_{int} = \chi_{ext}/(1 - D\chi_{ext}) \quad (2)$$

$$\chi_{ext} = \chi_{int}/(1 + D\chi_{int}) \quad (3)$$

Let us consider the case of a soft ferromagnet with a very large χ_{int}. In a small applied field H_{ext}, we would measure M such that, from Equation (3), $\chi_{ext} = dM/dH_{ext} = 1/D$.

We can treat a simple type-I superconductor as a perfectly diamagnetic material: $\chi_{int} = -1$. In an applied field H_{ext}, from Equation (3)

$$\chi_{ext} = dM/dH_{ext} = 1/(D-1) \quad (4)$$

Thus, for a sphere, $\chi_{ext} = -3/2$. For a long cylinder axially, $\chi_{ext} = -1$ and transversely, $\chi_{ext} = -2$.

An interesting case is that of an infinitely thin sheet of finite dimensions. For H_{ext} normal to the surface, D = 1. From Equation (4), χ_{ext}, and thus M, will approach infinity. While this seems unphysical at first, we note that M is calculated as magnetic moment per unit volume, V. M gets infinitely large only as V approaches zero. Furthermore, for $H_{ext} > 0$, flux immediately begins to penetrate the sheet in this 'intermediate' state; full penetration occurs at $H = H_c$, the critical field.

For the above cases[1], we plot M *versus* H_{ext} in *Figure 1*. Note that M *versus* H_{int} would be characteristic of the material, independent of geometry. The plot of M *versus*

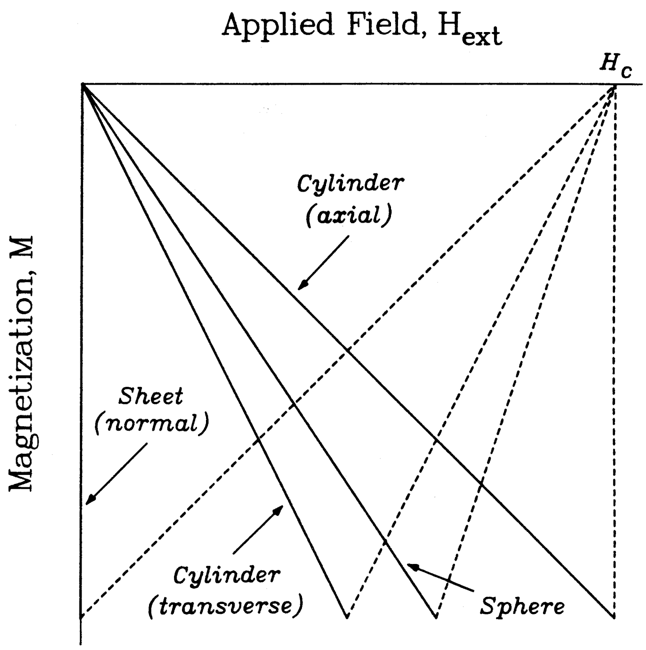

Figure 1 Magnetization *versus* external applied field for an ideal type-I superconductor of various geometries. The external susceptibilities are given by the slopes of the solid lines, —. Flux penetration for the intermediate states are indicated by the broken lines, ---

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H_{int} would be the same as that for the axial cylinder ($D = 0$).

The same *initial* χ_{int} would be obtained for a type-II superconductor. The lower critical field, H_{c1}, is signalled when the initial M *versus* H_{int} curve deviates from linearity, indicating flux penetration and the 'mixed' state. (The intermediate state is also possible in type-II superconductors.)

Hysteresis loss

We now consider hysteresis loss in superconductors. Theoretical loss expressions are usually given in terms of the applied field, H_{ext}. (For type-II superconductors, the theoretical equations depend on the direction of H_{ext}.) While a hysteresis loop plotted as M *versus* H_{int} is generally shaped differently from one plotted as M *versus* H_{ext}, the areas enclosed by the loops are the same. This can be seen by integrating H_{int} as a function of M and referring to Equation (1), as we now show.

Consider a hysteresis loop plotted as M *versus* H_{int}. Let the end points of the loop be denoted as 1 and 2; the two points are joined by curves A and B. Using the usual convention of integrating H as a function of M, the area of the loop enclosed by curves A and B is

$$W = \int_1^2 H_{int}^{A} dM - \int_1^2 H_{int}^{B} dM \qquad (5)$$

$$= \int_1^2 H_{ext}^{A} dM - D\int_1^2 M dM - \int_1^2 H_{ext}^{B} dM + D\int_1^2 M dM$$

$$= \int_1^2 H_{ext}^{A} dM - \int_1^2 H_{ext}^{B} dM \qquad (6)$$

Thus, the experimental loop areas are the same for both M *versus* H_{int} and M *versus* H_{ext}.

Multifilamentary superconductors

The fields in the somewhat special case of a multifilamentary superconductor have been analysed by Carr [2–4]. In the equations, M now refers to the magnetic moment per unit volume of superconductor, M_{sc}, not including matrix material. Often, however, M is expressed as magnetic moment per total volume, M_{tot}, and the volume fraction of superconductor, λ, is carried in the equations, where $M_{tot} = \lambda M_{sc}$.

For hysteresis calculations, the local field, H_{loc}, applied to each filament is used rather than H_{ext} applied to the composite conductor. H_{loc} is sometimes referred to as a Lorentz or cavity field

$$H_{loc} \equiv H_{int} + DM \qquad (7)$$

For H_{ext} applied axially to the conductor (and filaments), $H_{loc} = H_{ext} = H_{int}$.

For H_{ext} applied transversely, $H_{loc} = H_{int} + M/2$. For the special case when H_{ext} is changed slowly and there is no transport current, Equation (1) applies and $H_{int} = H_{ext} - M/2$; thus, $H_{loc} = H_{ext}$. Therefore, we can continue to use H_{ext} in the hysteresis loss calculations for this special case. Similarly, χ_{loc}, equal to dM/dH_{loc} for multifilamentary conductors, will equal χ_{ext}. For the general transverse case, however, shielding effects prevent us from using H_{loc} and H_{ext} interchangeably.

We summarize the equations for χ_{int} and χ_{ext} when there is no shielding and the volume fraction of superconductor, λ, is carried in the equations [2,4]. We have, for H_{ext} applied axially

$$\chi_{int} = \chi_{ext} = -\lambda \qquad (8)$$

The expressions for H_{ext} applied transversely are

$$\chi_{int} = -2\lambda/(1+\lambda) \qquad (9)$$

and

$$\chi_{ext} = -2\lambda \qquad (10)$$

Equations (9) and (10) are related by Equations (2) and (3).

Effective demagnetization factor for coils

Sumiyoshi *et al.* derived an effective demagnetization factor, D_{eff}, for a single-layer coil of multifilamentary superconductor parallel to the coil axis[5]. That is, H_{ext} is applied transversely to the conductor.

$$D_{eff} = [1 - \cos(\pi r/d)]/[\cosh(\pi r/d) - \cos(\pi r/d)] \qquad (11)$$

Here, r is the radius of the filament bundle and $2d$ is the centre-to-centre separation of the bundles. D_{eff} takes into account the interaction between the turns of the coil. We assume that there is no current in the conductor.

In the limiting case where $d >> r$, $D_{eff} = 1/2$, as for isolated transverse cylinders. In the other limit, $d = r$, $D_{eff} = 0.1588$. Thus from Equation (4), χ_{ext} can range from -2 to -1.189.

The effect of coil-turn interaction has also been discussed by Carr *et al.*[6], Zenkevitch and Romanyuk[7], and Campbell[8].

Conclusion

The correct analysis of magnetization data requires that one account for demagnetization factors and internal fields. They are important for the determination of susceptibility of strong diamagnets (superconductors), superparamagnets (stainless steels), and ferromagnets. The use of internal fields is not necessary for the measurement of hysteresis loss.

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