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AC LOSSES IN Nb-Ti MEASURED BY MAGNETIZATION AND COMPLEX SUSCEPTIBILITY

R. B. Goldfarb and A. F. Clark
Electromagnetic Technology Division
National Bureau of Standards
Boulder, Colorado

ABSTRACT

DC magnetization and complex ac susceptibility were measured at 4 K as functions of longitudinal dc field for a multifilamentary Nb-Ti superconductor with no transport current. Minor hysteresis loops were obtained in the dc measurements. The full-penetration field, H_p, a function of applied field, H, was deduced directly for each minor loop. The values for H_p were fit to the Kim-type equation, $H_p(H) = H_p(0)/(1+H/H_k)$, where $H_p(0)$ and H_k are constants. The minor hysteresis-loop areas gave losses that were in excellent agreement with Carr's theoretical critical-state equation, $W = (4\mu_0 H_0 H_p/3)(1-H_p/2H_0)$, where H_0 is the maximum applied field for each loop.

An expression was obtained for the ideal reversible differential susceptibility: $\chi_{rev} = \phi_0/8\pi\mu_0(H-H_{c1})\lambda^2$, where ϕ_0 is the flux quantum, H_{c1} is the lower critical field, and λ is the penetration depth. H_{c1} and λ for the sample were deduced from the shape of the major hysteresis loop. Clem's theoretical expressions for the real (χ') and imaginary (χ'') components of ac susceptibility are functions of χ_{rev}, H_p, and ac field amplitude, h. The predicted susceptibilities based on these expressions were in good agreement with measured curves of χ' and χ'' as functions of h and H. The measured χ' and χ'' were independent of frequency up to 1 kHz, as expected when bulk hysteresis is the primary loss mechanism.

INTRODUCTION

In an earlier work[1] we discussed the relationship between dc magnetization and ac susceptibility in a type-II superconductor. That work described how magnetic measurements provide information on hysteresis losses. In this paper, as in Ref. 1, we examine magnetization and susceptibility for longitudinal fields and no transport current. Here, however, we obtain the full-penetration fields, H_p, directly from the hysteresis loops rather than estimate H_p from measurements of critical current density, J_c. Also, we derive an expression for the reversible susceptibility, χ_{rev}, rather than using the experimental susceptibility, χ_{dc}, as an approximation. A superconducting wire whose low-field magnetization approached a reversible curve was selected for study.[2] Several minor hysteresis loops were obtained in addition to the major loop. The susceptibility curves were more nearly reversible and virtually independent of frequency, as expected from theory.

Table 1. Characteristics of Nb-Ti Wire

Cross section bare wire:	0.63 × 0.88 mm
Twist length:	1.7 cm
Cu/Nb-Ti volume ratio:	0.91
Number of filaments:	240
Filament radius:	19.3 μm
Density of Nb-Ti alloy:	6.20 g/cm^3

EXPERIMENT

The magnetization measurements were made with a vibrating-sample magnetometer (VSM) at 4 K. Magnetization was computed as magnetic moment per unit volume of Nb-Ti. The volume of the Cu matrix was not included.

It is useful to compare the advantages of the VSM method vis-à-vis the popular integration method of Fietz.[3] The VSM method (1) is useful for small samples, (2) is a dc measurement, not sensitive to coupling losses, (3) does not require precise pick-up coil balance when the applied field is stepped, and (4) is not subject to integrator drift. The integration method (1) measures frequency dependences, (2) detects flux jumps, and (3) is easily adaptable to measurements with transport current.

The experimental methods are more fully described in Ref. 1. The sample in this study is different and its length is 3.0 rather than 1.5 cm. The purpose of a longer sample was to avoid significant end effects. The characteristics of the wire are given in Table 1. This is the same wire as sample 8 in Ref. 2.

DC HYSTERESIS LOOP

Full-Penetration Field

The major and minor hysteresis loops are shown in Fig. 1. The full-penetration field, H_p, a function of applied field, H, may be estimated directly for each minor loop.[2] As the loops are traversed, the filaments go from full penetration in one direction to full penetration in the other direction. Therefore, at a high-field end of a loop, H_p is approximately one-half the field required to reverse the magnetization. The reversal field, $2H_p$, for the major loop is shown in Fig. 1.

The well-known Kim model for critical current density is[4]

$$J_c(H) = J_c(0)/(1+H/H_k) , \qquad (1)$$

where $J_c(0)$ and H_k are constants. In the critical-state model, for a field applied axially,

$$H_p = J_c r , \qquad (2)$$

where r is the filament radius. We did a linear least-squares fit of each loop's H_p to the expression

$$H_p(H) = H_p(0)/(1+H/H_k) , \qquad (3)$$

where $H_p(0)$ is a constant equal to $J_c(0)r$. The fit was excellent. We obtained $H_p(0)$ = 140 kA/m (1.76 kOe) and H_k = 1.33 MA/m (16.7 kOe).

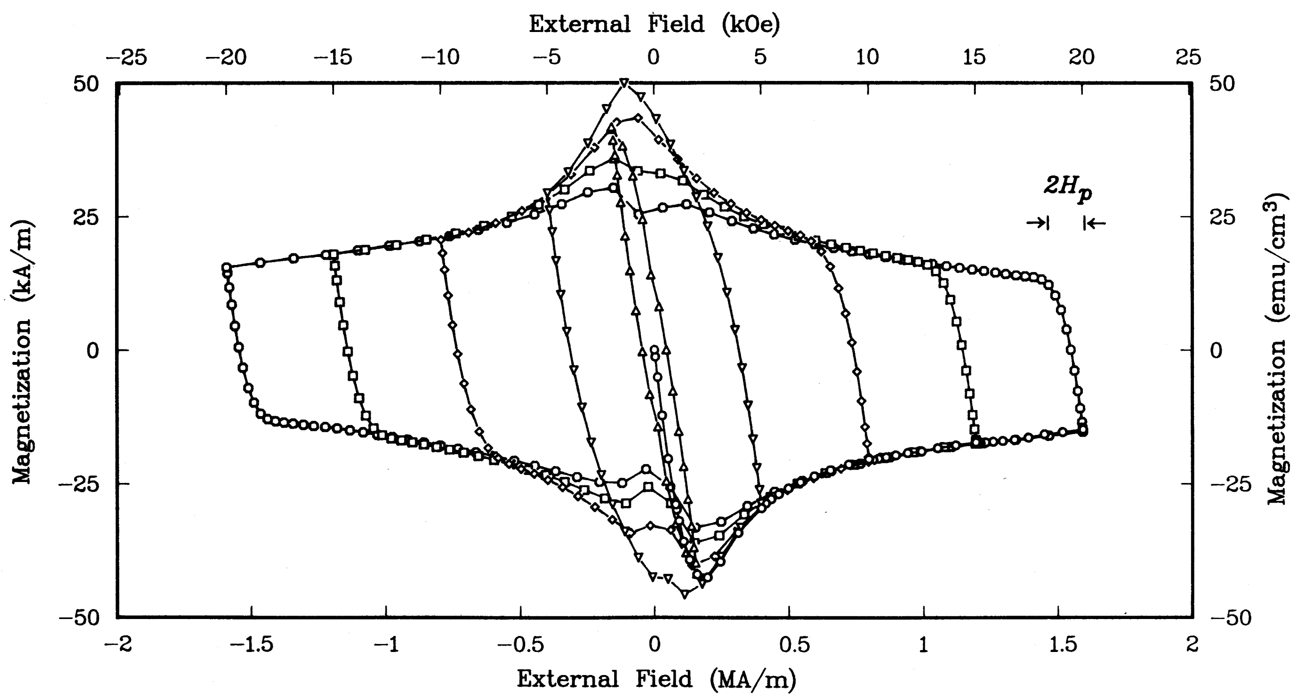

Fig. 1. Major and minor hysteresis loops for a multifilamentary Nb-Ti superconductor. DC magnetization is plotted vs. longitudinal applied field.

Table 2. Comparison of Losses Obtained from Eq. 4 and Loop Area.

H_0 (kA/m)	H_p	Eq. 4 (kJ/m³)	Area
1591	63	165	159
1194	74	143	136
796	87	110	105
398	108	62	58

Hysteresis Loss

For the case when the peak applied field, H_0, is greater than H_p, Carr derived an expression for the loss, assuming J_c to be spatially constant and field independent in the London-Bean approximation.[5] In SI units,

$$W = (4\mu_0 H_0 H_p/3)(1-H_p/2H_0) \ . \qquad (4)$$

Clearly H_0 is much larger than H_p for the loops in Fig. 1. Losses calculated by numerically integrating the loops are compared in Table 2 to losses obtained from Eq. 4. They agree to within 6%.

REVERSIBLE SUSCEPTIBILITY χ_{rev}

An exact equation for the ideal, reversible magnetization-versus-field curve of a type-II superconductor, $H > H_{c1}$, is not obtainable in analytic form. The usual procedure is to divide the field into three regions: immediately above H_{c1} ($H_{c1} \lesssim H$), the middle region ($H_{c1} < H < H_{c2}$), and immediately below H_{c2} ($H \lesssim H_{c2}$), and make judicious approximations.

One tractable case is $H_{c1} < H < H_{c2}$, when the separation between flux vortices is larger than the coherence length, ξ, but much smaller than the penetration depth, λ. Thus the Ginzburg-Landau parameter $\kappa \equiv \lambda/\xi$ is much greater than 1. For Nb-Ti at 4 K,[6] $\lambda \cong 394$ nm and $\xi \cong 5$ nm, and this case is applicable. To first order, the magnetization as a function of magnetic field strength M(H) is[7]

$$M = (\phi_0/8\pi\mu_0\lambda^2)\ \{\ln[4\pi\mu_0(H-H_{c1})\lambda^2/\phi_0] + \alpha\} - H_{c1} \ , \qquad (5)$$

where ϕ_0 is the magnetic flux quantum = $h/2e = 2.068 \times 10^{-15}$ Wb (T·m²) and α is a constant on the order of unity. The functional dependence of M(H) can be seen by dropping the α term and using the relationship[8]

$$H_{c1} = (\phi_0/4\pi\mu_0\lambda^2)\ \ln(\lambda/\xi) \ . \qquad (6)$$

Thus,

$$M = -(\phi_0/8\pi\mu_0\lambda^2)\ \ln[\phi_0/4\pi\mu_0(H-H_{c1})\xi^2] \ . \qquad (7)$$

The volume magnetic susceptibility $\chi = dM/dH$ can be obtained from either Eq. 5 or 7 and is simply

$$\chi = \phi_0/8\pi\mu_0(H-H_{c1})\lambda^2 \ . \qquad (8)$$

We take this χ to be equal to the ideal reversible susceptibility χ_{rev} in the range $H_{c1} < H < H_{c2}$. For $H < H_{c1}$, $\chi_{rev} = -1$. χ_{rev} represents the susceptibility under conditions of thermodynamic equilibrium.

Our goal is to obtain numerical values for χ_{rev} for our wire. The differential slope of the third-quadrant branch of the major magnetization curve (Fig. 1), from 0.2 to 1.6 MA/m (2.5 to 20 kOe), was fit to Eq. 8 using linear least squares. While this portion of the magnetization curve itself is not reversible, we expect that its slope will simulate χ_{rev}. For the adjustable parameters H_{c1} and λ, we obtain 92 kA/m (1 kOe) and 78 nm, respectively. These values are not too unreasonable, though this fit risks overestimating H_{c1} with values closer to $H_p(0)$. Substituting into Eq. 8 we get χ_{rev} for this wire for $H_{c1} < H < H_{c2}$ (SI units):

$$\chi_{rev} = 10772/(H-91751) \ . \tag{9}$$

This equation will be used in the next section to calculate χ' and χ''. An alternate approach would be to use textbook values of H_{c1} and λ in Eq. 8.[9] This would give slightly different values of χ' and χ'' for $H \gtrsim H_{c1}$. For larger fields, the differences are negligible.

MEASURED AND IDEAL χ' AND χ''

The real and imaginary components of susceptibility, χ' and χ'', were measured as functions of frequency (10, 100 and 1000 Hz), sinusoidal ac field amplitude, h [11 and 56 kA/m (140 and 700 Oe)], and dc bias field, H [0 - 1.6 MA/m (0 - 20 kOe)]. The susceptibilities were independent of frequency to within 3%. Since χ'' is a measure of the losses and hysteresis is known to be frequency independent, bulk hysteresis is probably the primary loss mechanism. Frequency-dependent eddy-current and coupling losses are likely insignificant for this sample.

Figures 2 and 3 show χ' and χ'' measured at 10 Hz. The initial and decreasing-field branches are plotted. Some irreversibility may be seen. In Fig. 3, h is on the order of H_p. As shown by Clem,[10] χ' and χ'' as functions of H may be predicted from h, H_p, and χ_{rev}. For our cases, $h < H_p$, the equations are:[1,10]

$$\chi' = (1+\chi_{rev})(h/H_p-5h^2/16H_p^{\,2})-1 \ , \tag{10}$$

$$\chi'' = (1+\chi_{rev})(4h/H_p-2h^2/H_p^{\,2})/3\pi \ . \tag{11}$$

Using the expressions for H_p and χ_{rev} (Eqs. 3 and 9), we computed the theoretical curves of χ' and χ'' for this wire for ac field amplitudes of 11 and 56 kA/m (140 and 700 Oe). They are shown in Figs. 4 and 5. They compare quite favorably with the actual curves in Figs. 2 and 3. For $H < H_{c1}$, $\chi' = -1$ and $\chi'' = 0$. There are discontinuities at H_{c1}, as expected. In the actual curves these are rounded owing to flux pinning.

CONCLUSION

DC magnetization curves and measurements of complex susceptibility provide information on hysteretic losses in multifilamentary superconductors such as Nb-Ti. These measurements may be performed on small samples. Interpretation of the data yields information on full-penetration field, critical current density, lower critical field, and penetration depth. The frequency independence of susceptibility suggests bulk hysteresis as the primary loss mechanism.

ACKNOWLEDGMENT

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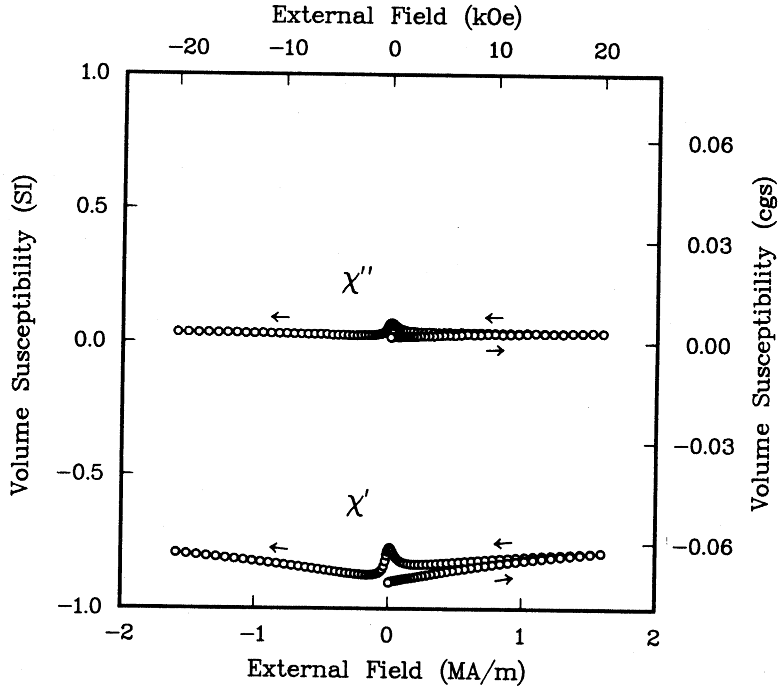

Fig. 2. Complex ac susceptibility as a function of dc bias field. The ac field amplitude is 11 kA/m (140 Oe) at 10 Hz.

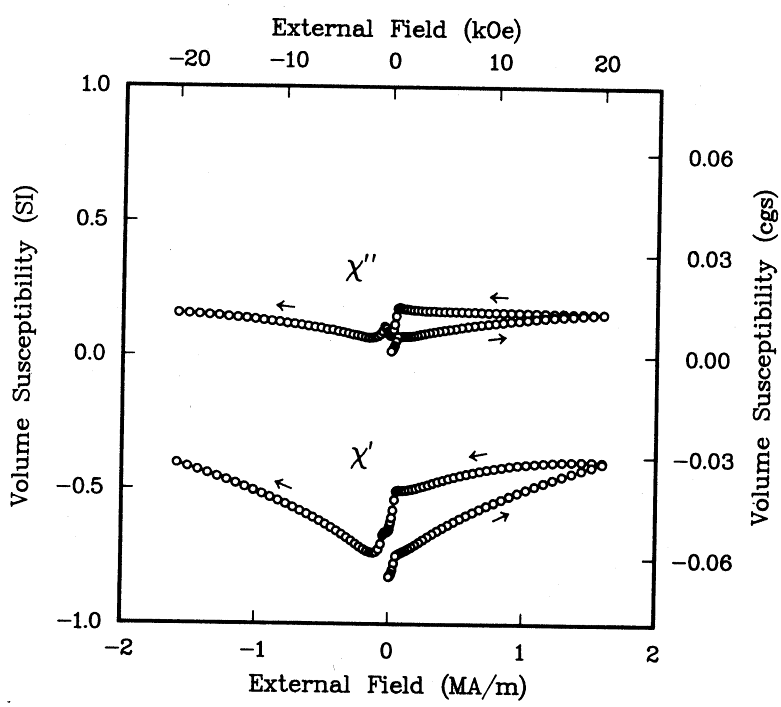

Fig. 3. Complex ac susceptibility as a function of dc bias field. The ac field amplitude is 56 kA/m (700 Oe) at 10 Hz.

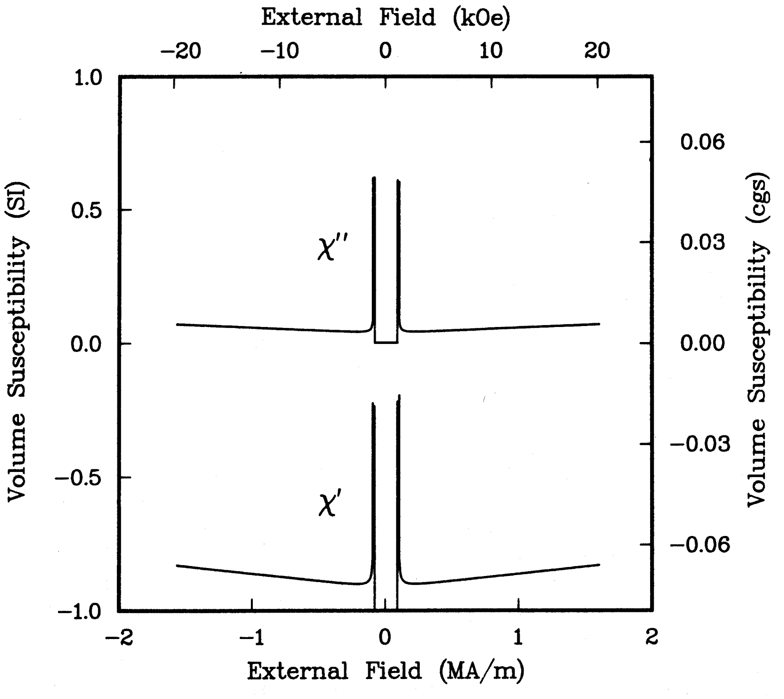

Fig. 4. Theoretical complex susceptibility vs. dc bias field for an ac field amplitude of 11 kA/m (140 Oe).

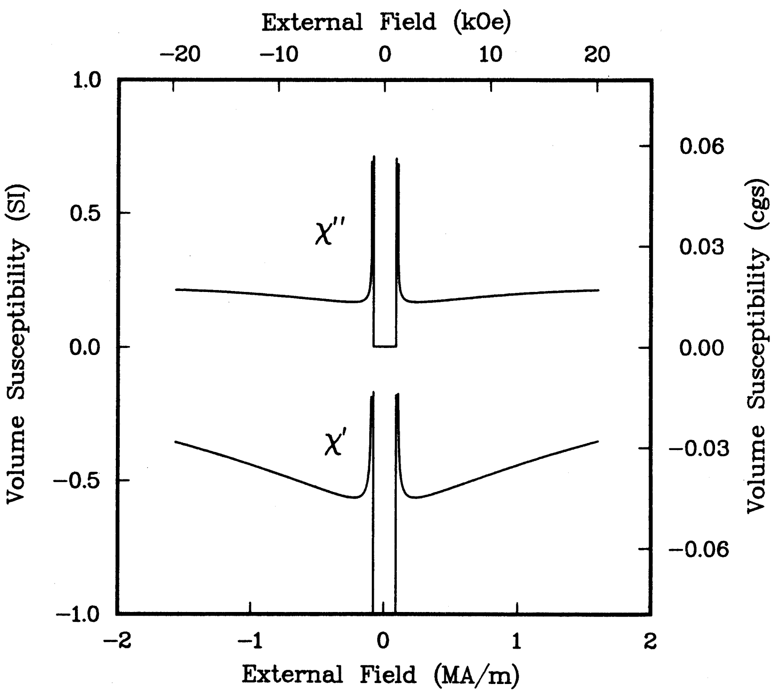

Fig. 5. Theoretical complex susceptibility vs. dc bias field for an ac field amplitude of 56 kA/m (700 Oe).

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