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Demagnetizing Factors

Demagnetizing (or demagnetization) factors are used to correct data on particular magnetic samples to give results intrinsic to the magnetic material. The demagnetizing correction adjusts the applied field H_a by the demagnetizing field H_d to yield the internal magnetic field strength

$$H = H_a + H_d$$

Magnetic susceptibility, permeability and the shape of the magnetization curve as a function of field are affected by the demagnetizing correction. The demagnetizing field H_d is related to the magnetization M of the sample by the geometric demagnetizing factor N:

$$H_d = -NM$$

Sometimes the symbol D is used for N (see *Measurements in Magnetic Materials*). In SI units, $0 \leqslant N \leqslant 1$; in cgs units, $0 \leqslant N \leqslant 4\pi$. The demagnetizing correction is important in materials with large magnetic susceptibilities (or permeabilities) or in samples whose relative dimension parallel to the applied field is small.

This article examines theoretical demagnetizing factors for ellipsoids of revolution and right circular cylinders. The applied field is assumed to be uniform and along a principal axis. Empirically, an "effective" demagnetizing factor of a sample of arbitrary geometry at a given state of magnetization may be deduced from the measured external susceptibility $\chi_{ext} = dM/dH_a$ provided that the internal susceptibility $\chi = dM/dH$ for the material is known:

$$N = \chi_{ext}^{-1} - \chi^{-1}$$

Demagnetizing factors should not be confused with the process of reducing the remanent magnetization of samples, similarly known as demagnetization.

1. Ellipsoids

The source of the demagnetizing field is usually taken to be fictitious magnetic poles (sometimes termed free poles) on the surface of a magnetized specimen. In ellipsoids, the poles are distributed in such a way that the fields H_a, H_d and H, and the magnetization M, are uniform. For ellipsoids of revolution (spheroids), N_z for the axis of symmetry (longitudinal or polar axis) is a function of the aspect ratio γ of the ellipsoid and independent of susceptibility χ (Osborn 1945, Stoner 1945):

$$N_z = (1-\gamma^2)^{-1}[1-\gamma(1-\gamma^2)^{-1/2}\cos^{-1}\gamma] \qquad (\gamma < 1, \text{oblate})$$

$$N_z = \tfrac{1}{3} \qquad (\gamma = 1, \text{sphere})$$

$$N_z = (\gamma^2-1)^{-1}[\gamma(\gamma^2-1)^{-1/2}\cosh^{-1}\gamma - 1] \qquad (\gamma > 1, \text{prolate})$$

Here, γ is the ratio of the longitudinal (polar) axis to the transverse (equatorial) axis. These formulae occasionally appear in different, but algebraically equivalent, forms in terms of the ratio of the long axis to the short axis, equal to γ^{-1} for oblate spheroids or in terms of the eccentricity, equal to $(1-\gamma^2)^{1/2}$ for oblate spheroids and $(\gamma^2-1)^{1/2}/\gamma$ for prolate spheroids.

For the general ellipsoid,

$$N_x + N_y + N_z = 1$$

where the subscripts x, y and z refer to the three semiaxes. For ellipsoids of revolution, with the z axis equal to the axis of symmetry,

$$N_x = N_y = \tfrac{1}{2}(1 - N_z)$$

Representative values of N_z and N_x are listed in Table 1. In the limit of small γ an oblate spheroid approximates a circular film with $N_x = N_y \simeq \frac{1}{4}\pi\gamma - \gamma^2$ and $N_z \simeq 1 - \frac{1}{2}\pi\gamma + 2\gamma^2$.

2. Cylinders

There are two types of demagnetizing factors commonly used for cylinders. The magnetometric demagnetizing factor N_m is the ratio $-\langle H_d\rangle_v/\langle M\rangle_v$, where $\langle\ \rangle_v$ indicates an average over the volume of the cylinder. The fluxmetric (or ballistic) demagnetizing factor N_f is the ratio $-\langle H_d\rangle_s/\langle M\rangle_s$, where $\langle\ \rangle_s$ indicates an average over the center plane of the cylinder. Magnetometric factors are used with mag-

Table 1
Longitudinal and transverse demagnetizing factors N_z and N_x as functions of γ for ellipsoids of revolution

γ	N_z	N_x
0	1	0
0.25	0.7036	0.1482
0.5	0.5272	0.2364
1	0.3333	0.3333
2	0.1736	0.4132
4	0.07541	0.4623
∞	0	0.5

netometers that sense the entire sample volume, such as vibrating-sample magnetometers, SQUID magnetometers and ac susceptometers. Fluxmetric factors are used when magnetization is measured ballistically, with a short search coil closely wrapped around the center of a long sample.

The demagnetizing factors N_m and N_f depend on γ and χ. Susceptibility χ is assumed to be constant, independent of both H and position in the sample. For nonlinear or hysteretic materials, N may be taken to be a function of H, with the corresponding susceptibility approximated by $\chi(H) = dM/dH$. With H_a along the cylinder axis, M and H_d are both nonuniform except in two cases. When $\chi = 0$, M is uniform. The approximation $\chi = 0$ is used for saturated ferromagnets, diamagnets and paramagnets. When $\chi \rightarrow \infty$, H_d is uniform and equal to $-H_a$. The condition $\chi \rightarrow \infty$ applies to soft ferromagnetic materials. When $\chi = -1$, $(M + H_d)$ is uniform and equal to $-H_a$. That is, flux density $B = \mu_0(H_a + H_d + M) = 0$, as for superconductors in the shielding state.

2.1 Zero Susceptibility

The magnetometric demagnetizing factor for a uniformly magnetized cylinder ($\chi = 0$) with ratio of length to diameter γ is (Joseph 1966)

$$N_{mz} = 1 - [4/(3\pi\gamma)]\{(1 + \gamma^2)^{1/2}[\gamma^2 F(\kappa) + (1 - \gamma^2)E(\kappa)] - 1\}$$

where $F(\kappa)$ and $E(\kappa)$ are the complete elliptic integrals of the first and second kind of modulus κ, which is defined by

$$\kappa = (1 + \gamma^2)^{-1/2}$$

As is true for ellipsoids (Brown 1962), $N_{mx} + N_{my} + N_{mz} = 1$ and $N_{mx} = N_{my} = \frac{1}{2}(1 - N_{mz})$. Several values of $N_{mz}(0)$ and $N_{mx}(0)$ are listed in Table 2. The fluxmetric (ballistic) demagnetizing factor is (Joseph 1966)

$$N_{fz} = 1 - (2/\pi)(\gamma/k)[F(k) - E(k)]$$

where the modulus k is defined by

$$k = (1 + \tfrac{1}{4}\gamma^2)^{-1/2}$$

Table 2
Axial and radial magnetometric demagnetizing factors N_{mz} and N_{mx} as functions of γ for cylinders with $\chi = 0$, ∞ and -1

γ	$N_{mz}(0)$	$N_{mx}(0)$	$N_{mz}(\infty)$	$N_{mx}(\infty)$	$N_{mz}(-1)$	$N_{mx}(-1)$
0	1	0	1	0	1	0
0.25	0.6346	0.1827	0.5712	0.1618	0.6764	0.2136
0.5	0.4745	0.2628	0.4111	0.2371	0.5258	0.2928
1	0.3116	0.3442	0.2590	0.3154	0.3692	0.3669
2	0.1819	0.4091	0.1409	0.3829	0.2341	0.4237
4	0.09835	0.4508	0.06635	0.4319	0.1361	0.4596
∞	0	0.5	0	0.5	0	0.5

Table 3
Axial fluxmetric (ballistic) demagnetizing factors N_{fz} as functions of γ for cylinders with $\chi = 0$ and ∞

γ	$N_{fz}(0)$	$N_{fz}(\infty)$
0	1	1
0.5	0.4221	0.3769
1	0.2322	0.2275
2.5	0.06544	0.09260
5	0.01889	0.03988
10	0.004927	0.01530
25	0.0007981	0.003766
50	0.0001999	0.001211
∞	0	0

For the fluxmetric demagnetizing factors, $N_{fx} + N_{fy} + N_{fz} < 1$. Although they are seldom used, several values of $N_{fz}(0)$ are listed in Table 3.

2.2 Nonzero Susceptibilities

Axial and radial magnetometric demagnetizing factors N_{mz} and N_{mx} for $\chi \rightarrow \infty$ and $\chi = -1$ may be deduced from published values of electric and magnetic polarizability of perfectly conducting cylinders as described by Chen et al. (1991). They are listed in Table 2. For $\chi > 0$, $N_{mx} + N_{my} + N_{mz} < 1$, and for $\chi < 0$, $N_{mx} + N_{my} + N_{mz} > 1$. Fluxmetric demagnetizing factors N_{fz} for $\chi \rightarrow \infty$ have been evaluated precisely by Templeton and Arrott (1987) (Table 3). The radial fluxmetric factors N_{fx} are rarely needed. A complete treatment of N_{mz} and N_{fz} for cylinders, as functions of γ and χ, is given by Chen et al. (1991).

See also: Design with Magnetic Materials; Measurements in Magnetic Materials; Measurements in Superconducting Materials; Shields, Magnetic

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