Broadband Josephson Voltage Standards[1]

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ABSTRACT — A Josephson junction is a perfect frequency-to-voltage converter, that is, $V = f / K_{J-90}$ where $K_{J-90} = 483597.9$ GHz/V. This unique property has been used to convert a narrow band (1 Hz) 75 GHz reference frequency to a dc voltage standard. Josephson standards use arrays of thousands of junctions to raise the voltage to 10 V. More recently, the broadband capability of Josephson devices is being exploited at NIST to create a new ac voltage standard. In this case, the Josephson junction is a pulse generator that can be triggered at frequencies from dc to 20 GHz, and that produces short (~50 ps) voltage pulses with a time integral of exactly $1/K_{J-90}$. A delta-sigma algorithm with a high oversampling ratio is used to define a digital pulse sequence for any desired output waveform. The sequence is programmed into a digital code generator that triggers the Josephson device. The result is a replication of the desired waveform with a time-dependent amplitude that is exactly calculable from a knowledge of the pulse code, the sampling frequency, and the number of Josephson junctions in the array.

I. INTRODUCTION

In 1962, Brian Josephson derived equations for the current and voltage across a junction consisting of a thin insulating barrier separating two superconductors[1]. His equations predicted that if a junction is driven at frequency f, then its current-voltage (I-V) curve will develop regions of constant voltage at the values $nhf/2e$, where n is an integer and e/h is the ratio of the elementary charge e to the Planck constant h. This prediction was verified experimentally by Shapiro [2] in 1963 and has become known as the ac Josephson effect. It found immediate application in metrology because it relates the volt to the second through a proportionality involving only fundamental constants. Initially, this led to an improved value of the ratio h/e. Today it is the basis for voltage standards around the world.

Josephson's equation for the supercurrent through a superconductive tunnel junction is given by

$$I = I_c \sin[(4\pi e/h)\int V dt],\qquad (1)$$

where I is the junction current, I_c is the critical current, and V is the junction voltage. When a dc voltage is applied across the junction, Eq. (1) shows that the current will oscillate at a frequency $f_j = 2eV/h$, where $2e/h \approx 484$ GHz/mV. If an ac current at frequency f is applied to the junction, the junction oscillation tends to phase-lock to the applied frequency. Under this phase lock, the average voltage across the junction equals $hf/2e$. This effect, known as the ac Josephson effect, is observed as a constant-voltage step at $V = hf/2e$ in the voltage-current (I-V) curve of the junction. It is also possible for the junction to phase-lock to harmonics of f. This results in a series of steps at voltages $V = nhf/2e$, where n is an integer, as shown in Fig. 1a.
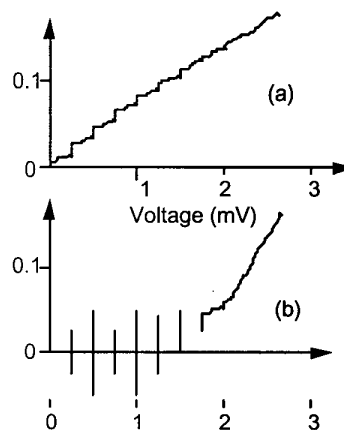

Fig. 1 Constant-voltage steps in the I-V curve of a junction driven with microwave radiation for (a) a low capacitance junction and (b) a high capacitance junction.

Further study of the behavior of these steps shows that by proper selection of the junction's length, width, critical current, and drive frequency, the steps would cross the zero-current axis as shown in Fig. 1b [3]-[4]. This was a critical discovery because it made possible very large series arrays of junctions. When biased at zero current,

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every junction is guaranteed to be on a step so that the equation $V = nhf/2e$ applies for the entire array. Modern arrays using up to 20 000 junctions and a drive frequency f = 75 GHz generate over 150 000 steps [5], [6]. In voltage standards based on these arrays, both the frequency and the step number are selected to generate any desired voltage in the range ±12 V. For the purposes of voltage metrology, a defined value $K_{J-90} = 2e/h$ = 483597.9 GHz/V is used by all standards laboratories.

Figure 2 is a semilog plot that illustrates how typical differences in dc voltage measurements among National Measurement Institutes (NMIs) have decreased over the last 70 years [7]. The two major improvements coincide with the introduction of single-junction Josephson standards in the early 1970s and the introduction of series-array Josephson standards beginning in 1984.
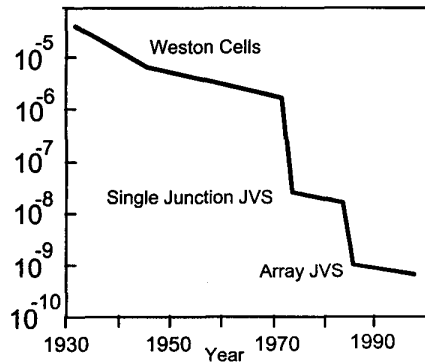

Fig. 2. The approximate level of agreement in dc voltage measurements among standards laboratories through the years 1930 to 2000.

II. PROGRAMMABLE VOLTAGE STANDARDS

Josephson voltage standards (JVS) based on the zero-crossing steps of series arrays of hysteretic junctions have two important disadvantages: (1) the step number n cannot be quickly set to a desired value, and (2) noise may cause spontaneous transitions between steps. The step-transition problem requires that the bandwidth of all connections to the chip be severely restricted in order to filter out noise. In the case of classical dc measurements, these are minor inconveniences that are easily resolved with software. However, the step stability, step selection, and bandwidth problems preclude measurements such as the rapid automated analysis of analog-to-digital (A/D) and digital-to-analog (D/A) converters and the synthesis of ac waveforms with a computable rms value.

A. Binary Weighted Programmable Arrays

To make possible these broader applications, a new type of Josephson voltage standard has been developed in

which the output voltage $V = nf/K_J$ is defined by digitally programming the step number n [8]. The key to this new Josephson standard is the use of junctions that are designed to be nonhysteretic, that is, the junction voltage is a single-valued function of the junction current, as in Fig. 1a. This is achieved by using small area junctions with a resistive shunt.

The circuit for this new standard uses an array of nonhysteretic junctions that is divided into a binary sequence of array segments, as shown in Fig. 3a. The microwave excitation for each junction is set to equalize the amplitude of the n = 0 and n = ±1 steps, as shown in Fig. 3b. Each segment of the array can be set to the n = -1, 0, or +1 step by applying a bias current ($-I_s$, 0, $+I_s$) at the appropriate nodes. The combined step number N for the whole array can thus be set to any integer value between $-M$ and $+M$, where M is the total number of junctions in the array. For example, to select step 5 we would set $I_3 = I_1 = +I_s$, $I_2 = I_o = -I_s$. This would bias the single junction and the set of four junctions on the n = 1 step and leave all other junctions on the n = 0 step.
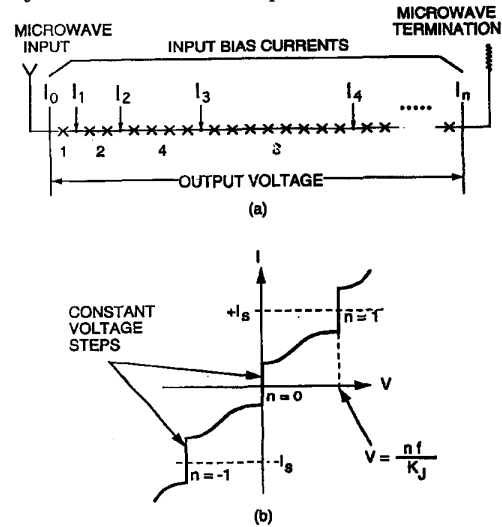

Fig. 3 (a) The schematic design of a programmable voltage standard based on a set of binary weighted arrays, and (b) the I-V curve of a single junction with the microwave power set to equalize the amplitude of the n = 0 and n = ±1 steps.

The rapid settling time and inherent step stability of the JVS in Fig. 3 make it potentially superior to a conventional JVS for dc measurements. (We define a dc measurement to be one in which the transient associated with changing N can be excluded from the measurement.) Such measurements include calibration of dc reference standards and digital voltmeters, and the characterization of A/D and D/A converters. The circuit of Fig. 3 can also

generate a staircase approximation to a sine wave by selecting appropriate step numbers in rapid succession. In theory, the resulting waveform has a computable rms value and might be used to confirm the ac-dc difference of a thermal voltage converter and for other ac measurements. In the case of ac measurements, however, the transient waveform during step transitions is included in the rms value and usually leads to an unacceptably large uncertainty.

B. Pulse-Driven Josephson Arrays

Thus far, we have discussed ways to program the voltage of a Josephson array by changing the step number n in the equation $V = nf/K_J$. It is clear that the same result might be achieved by changing f. Unfortunately, in the case of a sine-wave excitation, the step amplitudes collapse rapidly to zero as the frequency decreases. This means that it is practical to control the voltage via the frequency only over a small range near the optimum frequency $f_c = I_cRK_J$, where I_c is the junction's critical current and R is the normal resistance of the junction. However, simulations show that if the sine-wave excitation is replaced with a pulse excitation, then the step amplitude is independent of the pulse repetition frequency for all frequencies below f_c. The optimum pulse width is $\tau = 1/(2\pi f_c)$. Figure 4 shows the result of a calculation of the $n = 1$ step boundaries for a junction driven with a sine wave (shaded area) and a pulse train (black area). Note that for a pulse drive, the step amplitude is large, symmetric around zero, and independent of frequency all the way to zero frequency. In fact, if the pulse's polarity is reversed, then the array can generate both positive and negative voltages.

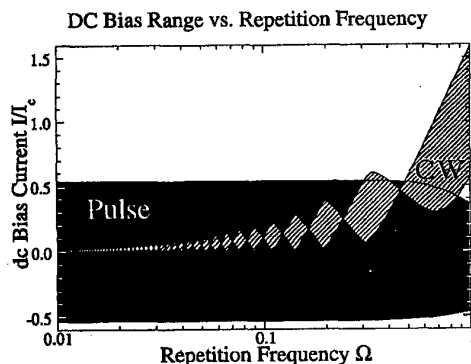### DC Bias Range vs. Repetition Frequency

Fig. 4. Comparison of the normalized $n = 1$ step boundaries for a junction driven with a pulse train (black) and a continuous sine wave (shaded) as a function of normalized frequency $\Omega = f/f_c$.

A programmable voltage source based on this idea consists of a single large array of N junctions distributed along a high bandwidth transmission line [9]. A pulse train at frequency f propagating down the line generates an average voltage Nf/K_J across the ends of the array. A complex output waveform can be generated by modulating the pulse train with a digital word generator. For example, using a clock frequency of $f_c = I_cRK_J = 10$ GHz, the pulse sequence 1111100000111110000... creates an output square wave of amplitude of Nf_c/K_J and frequency of 1 GHz.

Figure 5 is a block diagram of the process that is used to generate an accurate sine wave of frequency f_1 or any other periodic waveform from quantized Josephson pulses. The modulator algorithm block is a computer program that digitizes an input signal $S(t)$ at a sampling frequency f_s. The algorithm is typically a second-order delta-sigma modulator that optimizes the signal-to-quantization-noise ratio over a desired frequency band. For a repetitive waveform, the code generated by the modulator is calculated just once and stored in the circulating memory of a digital code generator. When the digital code generator is clocked at the sampling frequency, it recreates an approximation to the original signal as an output voltage $S_D(t)$. It has been shown that combining $S_D(t)$ with a sine wave bias at $(3/2) f_s$ to drive the Josephson array makes possible bipolar operation and results in a factor of six improvement in voltage range over that from driving the array with $S_D(t)$ alone [10]. The function of the array is to perfectly quantize the input pulses, thus greatly reducing amplitude noise in $S_D(t)$ within the signal band. A reduction in noise on the order of 60 dB can be achieved with this method. When the ratio f_s/f is large, (e.g. >10^4) the in-band component of the voltage across the array is an almost perfect reproduction of the input signal. An equally important feature of the quantized voltage pulses generated by each junction is that their time integral is exactly equal to a single flux quantum $h/2e = 1/K_J = 2.067834$ mV-ps This means that a knowledge of the digital code, the sampling frequency, and the number of junctions in the array is sufficient to exactly compute the spectrum and the rms value of $S'(t)$.

The pulse-driven Josephson array has the potential to be both a dc voltage standard and an ac voltage standard with a bandwidth of 1 MHz or more. The realities are that: (1) this idea has been demonstrated to voltage amplitudes of only about 150 mV, and (2) a great deal of work is required to increase the voltage range and to prove that every pulse specified by the digital code is faithfully reproduced by every junction in the array. This can happen only if the transmission path to every junction is reasonably independent of frequency from dc to about 18 GHz, a very stringent requirement. Fortunately, there are a variety of simple tests for "lost pulses." For example, the dc output of the circuit of Fig. 5 is proportional to n_1 −

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n_0, where n_1 is the number of ones in the code and n_0 is the number of zeros in the code. Exactly the same dc voltage should result from all codes that have the same n_1 and n_0 regardless of the distribution of ones and zeros within the code. Another obvious test is to compare the computed rms voltage of a synthesized sine wave with the best available ac voltmeter. Not only should the agreement fall within the uncertainty of the voltmeter, but the measured voltage must have a "flat spot" when subjected to small variations in the amplitude of the digital code, the amplitude of the sine wave drive, and their relative phase and offsets.
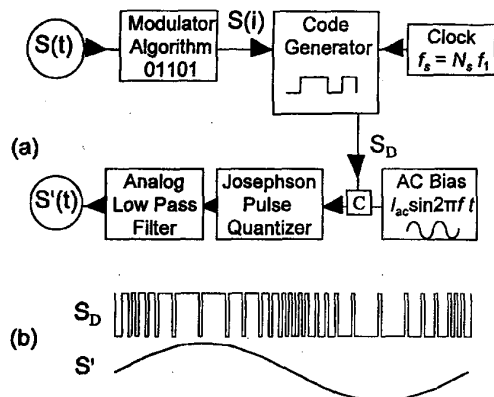

Fig. 5. (a) Josephson array pulse quantizer and (b) a block diagram of a delta-sigma digital-to-analog converter based on pulsed Josephson junctions. S(t) is the desired waveform and $S'(t)$ is the output waveform.

A grounded-array circuit design and improved broadband voltage-tap filters resulted in the first measurements of both dc and ac synthesized voltages that have a "flat spot" for all input bias parameters [11]. DC voltages in the range −6 mV to +6 mV were measured with a high-accuracy voltmeter and found to agree within the manufacturer's 66 nV analog-to-digital linearity specifications. Similarly, ac voltages from 1 kHz to 50 kHz with 3.7 mV rms amplitude were synthesized and shown to have a "flat spot" for all input parameters. However, low voltages and input-output coupling limited the uncertainty of the measurements.

A new technique uses ac coupling to split the broadband input into separate low-frequency and high-frequency bias components [12]. This technique allows arrays with more junctions to be used and results in a 43 mV rms amplitude, more than 10-times larger than previous circuits. AC voltage comparisons had an uncertainty at least ten times lower than that for previous circuits with smaller voltages. However, input-output

coupling still dominates the uncertainty for frequencies greater than 50 kHz.

New circuits are being fabricated that should reduce the input-output coupling and further improve the uncertainty for ac voltages. Higher-frequency filters are being investigated for the bias taps to further increase the output voltage per junction. Further increases in the total output voltage will be attempted by adding the output voltage of multiple arrays in series. With these improvements it is hoped that amplitudes larger than 0.25 V will be demonstrated in the near future.

REFERENCES

[1] B.D. Josephson, "Possible new effects in superconductive tunneling," Physics Letters, vol, 1, pp. 251-253, 1962.
[2] S. Shapiro, "Josephson currents in superconducting tunneling: the effect of microwaves and other observations," Physical Review Letters, vol. 11 pp. 80-82, 1963.
[3] M.T. Levinson, R.Y. Chiao, M.J. Feldman and B.A. Tucker, "An inverse ac Josephson effect voltage standard," Applied. Physics Letters, vol. 31 pp. 776-778, 1977.
[4] R.L. Kautz, "Noise, chaos, and the Josephson voltage standard," Reviews of Progress in Physics, vol. 59, pp. 935-992, 1996.
[5] F.L. Lloyd, C.A. Hamilton, J.A. Beall, D. Go, R.H. Ono and R.E. Harris, "A Josephson array voltage standard at 10 V," IEEE Electron Device Lett., vol. 8, pp. 449-450, 1987.
[6] R. Pöpel, J. Niemeyer, W Meier L Grimm, F.W. Dünschede, "Nb/Al₂O₃/Nb-Josephson voltage standard at 1V and 10 V," IEEE Trans. Instrum. Meas. vol. 40, pp. 298-300, 1991.
[7] C. A. Hamilton "Josephson Voltage Standards", Review of Scientific Instruments, vol. 71, Oct. 2000.
[8] C. A. Hamilton, C. J. Burroughs, and R. L. Kautz, "Josephson D/A Converter with Fundamental Accuracy", IEEE Transactions on Instrumentation and Measurement, vol. 44, pp. 223-225, April 1995.
[9] S.P. Benz and C.A. Hamilton, "A Pulse-driven Programmable Josephson Voltage Standard", Applied Physics Letters, vol. 68, no 22 pp. 3171-3173, May 1996.
[10] S. P. Benz, C. A. Hamilton, C. J. Burroughs, and T. E. Harvey, IEEE Trans. Instrum. Meas., vol. 48, pp. 266-268, 1999.
[11] S.P. Benz, C.J. Burroughs, P.D. Dresselhaus, and L.A. Christian, "AC and DC Voltages from a Josephson Arbitrary Waveform Synthesizer,"to appear in IEEE Transactions on Instrumentation and Measurement, Vol. 50, (April 2001).
[12] S.P. Benz, C.J. Burroughs, and P.D. Dresselhaus, "AC coupling technique for Josephson waveform synthesis," to appear in IEEE Transactions on Applied Superconductivity Vol. 11, (June 2001).