AC Josephson Voltage Standard Error Measurements and Analysis

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***Abstract*—The Josephson arbitrary waveform synthesizer can be used as a precision voltage source for both ac and dc signals. Recent improvements in circuit designs have resulted in output voltages greater than 100 mV rms for ac metrology measurements in the frequency range from 1 to 50 kHz. In this paper, we present measurements of several significant error sources that must be considered in order to create an intrinsic ac voltage standard at these frequencies. These results and the corresponding theoretical analysis allow us to move toward creating a practical ac Josephson voltage standard.**

***Index Terms*—AC metrology, Josephson voltage standard, precision ac source.**

I. INTRODUCTION

Research at NIST on pulse-driven Josephson arrays for ac and dc metrology has been ongoing since 1996. We have achieved higher voltage output and improved operating margins by implementing different biasing techniques and circuit improvements, including bipolar waveform generation using a combination of gigahertz frequency digital codes and sinusoidal rf-drive [1], referencing the Josephson output voltage to ground using an ac-coupling technique [2], and designing appropriate on-chip filters for the low-frequency output lines and bias connections [3]. The circuits described in this paper combine all these techniques, allowing us to quantitatively evaluate the performance of the Josephson arbitrary waveform synthesizer as a voltage source for ac metrology.

Our goal is to develop a quantum-based voltage source that delivers precisely calculable rms values for both ac (sinewaves and arbitrary waveforms) and dc voltage signals. The ac Josephson source uses a well-known digital-to-analog conversion technique called delta–sigma modulation [4], which allows any arbitrary waveform within the usable bandwidth (dc to 10 MHz) to be generated by using a predetermined sequence of pulses clocked at a much higher frequency (10 Gbit/s). Josephson junctions are ideally suited for a delta–sigma modulator because their voltage pulses are perfectly quantized. Knowledge of the number of pulses and their position in time is sufficient to precisely determine the time-dependent voltage of any synthesized waveform and, in particular, its rms voltage.

II. POTENTIAL SOURCES OF ERROR

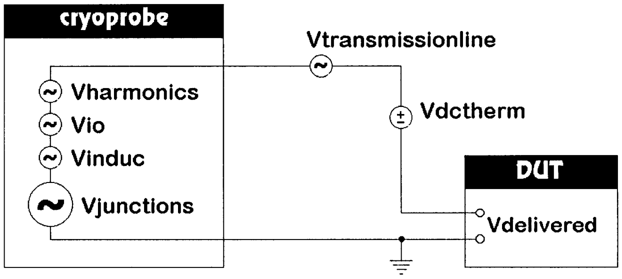

Fig. 1. Schematic diagram of the total voltage delivered by the ac Josephson voltage source to a device under test (DUT), including potential sources of error.

Since we operate these devices with a wide output bandwidth (dc to GHz), there are many sources of error that need to be considered when computing the actual rms voltage delivered by the Josephson source to a room-temperature device under test (DUT). The most significant sources of error and uncertainty are summarized in Fig. 1, where $V_{\mathrm{junctions}}$ is the quantum-mechanically accurate voltage produced by the Josephson junctions, typically a pure sinewave at fundamental frequency f. V_{induc} is the voltage generated by the on-chip inductance at the fundamental. V_{io} is the input/output coupling from the drive current to the output leads at the fundamental. $V_{\mathrm{harmonics}}$ represents voltages from all undesired signals at frequencies other than the fundamental. $V_{\mathrm{transmissionline}}$ is the effect of the transmission line connecting the chip to the DUT. V_{dctherm} is the sum of the dc thermal voltages that are generated at the metal-to-metal interfaces of the output wiring. In order to calculate the exact rms voltage delivered to the DUT, all of these terms must be either precisely calculable or directly measurable

$$\boldsymbol{V}_{\mathrm{delivered}} = \boldsymbol{V}_{\mathrm{junctions}} + \boldsymbol{V}_{\mathrm{induc}} + \boldsymbol{V}_{\mathrm{io}} + \boldsymbol{V}_{\mathrm{harmonics}} + \boldsymbol{V}_{\mathrm{transmissionline}} + \boldsymbol{V}_{\mathrm{dctherm}}. \quad (1)$$

To determine the rms voltage delivered to the DUT, we must combine the Josephson array voltage and dominant sources of error illustrated in (1). This equation combines all the dominant sources of error, and is basically a summation of vectors because both the magnitude and phase of each term must be included. It is not a simple rms sum, however, because we must also consider the frequencies for certain terms (primarily $V_{\mathrm{harmonics}}$ and V_{dctherm}). Furthermore, the frequency and phase relationships between the various error terms change for different fundamental frequencies of $V_{\mathrm{junctions}}$. For example, when $V_{\mathrm{junctions}}$ is an ac waveform, some of the terms always combine as a simple RSS (root-sum-squares) summation, such as $V_{\mathrm{harmonics}}$ and V_{dctherm}. The effect of such error terms is relatively small as illustrated in Table I. Similarly, the term V_{induc} is 90° out of phase with $V_{\mathrm{junctions}}$, so it combines using RSS as well. However, other terms are more complex, such as V_{io}, because they are difficult to measure independently without the

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TABLE I
CONTRIBUTION OF AN INDIVIDUAL ERROR TERM THAT COMBINES USING RSS (SUCH AS A SINGLE HARMONIC), AND THE CONTRIBUTION OF 1000 HIGHER HARMONICS OF THE SAME AMPLITUDE

Amplitude of an individual harmonic	Contribution of the single harmonic to final rms voltage	Contribution of 1000 such higher harmonics to final rms voltage
-77 dBc	+10 nV/V	+10 µV/V
-87 dBc	+1 nV/V	+1 µV/V
-97 dBc	+0.1 nV/V	+0.1 µV/V
-107 dBc	+0.01 nV/V	+0.01 µV/V

TABLE II
CONTRIBUTION OF AN INDIVIDUAL ERROR TERM THAT COMBINES DIRECTLY (IN PHASE, NOT IN QUADRATURE) WITH $V_{\mathrm{junctions}}$

Amplitude of error signal	Contribution to final rms voltage
-77 dBc	+141 µV/V
-87 dBc	+45 µV/V
-97 dBc	+14 µV/V
-107 dBc	+4.5 µV/V
-117 dBc	+1.4 µV/V

effects of other signals at the same frequency. Accurate knowledge of V_{io} is critical, because the part of this error that is in phase with $V_{\mathrm{junctions}}$ adds directly (not in quadrature) and therefore has a dramatic effect on the rms voltage. The contributions of such in-phase error signals are illustrated in Table II.

To analyze the individual error sources more fully, we first consider unwanted on-chip signals that may be generated in the superconducting integrated circuit. The drive signal is comprised of two basic components—the low-frequency drive (which is a current of many mA at exactly the fundamental frequency), and the high-frequency drive (which is the high-speed pulse train that is coupled through a 10 MHz high-pass filter). The sum of these two currents flows through the Josephson junctions and generates the desired perfectly quantized voltage signal, $V_{\mathrm{junctions}}$, as well as higher—frequency harmonics from the delta–sigma modulation process that are included in $V_{\mathrm{harmonics}}$. Unfortunately, this same current creates one of the main challenges for pulse-driven arrays in that it also flows through the on-chip inductance (of the transmission line between the junctions and the on-chip low-pass filters) and generates the voltage of the second term in (1), V_{induc}. Fortunately, V_{induc} has a phase angle of 90° with respect to $V_{\mathrm{junctions}}$, so it combines in quadrature with $V_{\mathrm{junctions}}$. However, the next term, the input/output coupling voltage V_{io}, causes more difficulty because it can have a phase angle of 0° or 180° with respect to $V_{\mathrm{junctions}}$, and as such adds directly (not in quadrature) to the array voltage. Thus V_{io} must be orders of magnitude smaller than the other error terms in order to make a useful voltage standard. $V_{\mathrm{transmissionline}}$ and V_{dctherm} are not dominant errors at this time, and have been discussed in previous publications [5].

TABLE III
MEASURED AND CALCULATED VALUES OF THE DOMINANT ERROR TERMS V_{induc} AND V_{io}, CREATED BY THE DRIVE CURRENT USED TO BIAS A SINGLE ARRAY OF 3750 JOSEPHSON JUNCTIONS TO PRODUCE A 62 MV RMS SINEWAVE

Frequency	Measured Error (V_{induc} and V_{io})	Calculated Error (V_{induc} only)
2.67 kHz	8.2 µV rms	5.6 µV rms
53.4 kHz	169 µV rms	112 µV rms

III. MEASUREMENTS

Since the two terms V_{induc} and V_{io} are due primarily to the low-frequency drive current, we can directly measure them by simply turning off the high-frequency drive. In this case, the junctions do not pulse $(V_{\mathrm{junctions}} = 0)$ and the output from the chip measured by the spectrum analyzer is primarily V_{induc} and V_{io}. Table III shows measured values for these combined terms at two different frequencies, and compares them to the expected value of V_{induc} calculated from estimates of the on-chip inductance and the drive current. (V_{io} is not calculable). Since V_{induc} should be much larger than V_{io} at these frequencies, V_{induc} dominates the measurement because the phase difference between them is 90° and they combine in quadrature. Notice that the magnitude of the V_{induc} error scales with frequency as expected.

If V_{io} were zero, these measured values of V_{induc} would increase the rms value of $V_{\mathrm{delivered}}$ by approximately 0.5 μV/V at 2.67 kHz, and by about 4 μV/V at 53.4 kHz. At the present time, it is difficult for us to measure V_{io} independently since it is much smaller than V_{induc}. However, V_{io} presents a significant challenge since it must be less than 310 nV to keep the corresponding error less than 5 μV/V when $V_{\mathrm{junctions}} = 62$ mV.

The fourth term in (1), $V_{\mathrm{harmonics}}$, represents the contribution to the total rms voltage made by all signals, including harmonics, that are not at the desired fundamental frequency, f. Since the high-frequency drive signal is four to six orders of magnitude higher than f, the fundamental is generated with a very high degree of spectral purity. In order to see the effect of the term $V_{\mathrm{harmonics}}$ in a controlled manner, we made a special waveform in which we deliberately included 990 harmonics with amplitudes at 77.34 dB below the fundamental signal [i.e., -77.34 dBc (carrier)]. This value was chosen so that $V_{\mathrm{harmonics}}$ would contribute *exactly* 10 μV/V to the total rms voltage, and the results of these measurements are summarized in line one of Table IV. In a similar measurement, line two of Table IV shows the results for the same 990 harmonics added at the -100 dBc level. The significance of this second measurement is that the chosen values represent typical Josephson array performance (-100 dBc or lower) which places $V_{\mathrm{harmonics}}$ from the Josephson chip well below the noise floor of our rms voltage-measuring instrument (in this case an ac/dc transfer standard).

IV. ERROR CORRECTION CIRCUIT

To increase our understanding of the error terms V_{induc} and V_{io}, we added a correction circuit shown in Fig. 2 to remove these measured error signals. The correction circuit consists of a 10-Ω series resistor driven by a separate synchronized sinewave signal generator with variable frequency, phase, and amplitude control. With the low-frequency drive on and the high-frequency drive off, we used a spectrum analyzer to directly measure the combined voltage from V_{induc} and V_{io}. Then we applied a sinewave with the same magnitude but opposite phase to exactly cancel these terms to the noise floor of the spectrum analyzer, which was about -120 dBc. When the high-frequency drive is turned back on, the Josephson array operates normally and the errors due to V_{induc} and V_{io} are

TABLE IV
MEASURED AND CALCULATED VALUES OF THE ERROR TERM $V_{\text{harmonics}}$ FOR A SINEWAVE OF 62 MV (RMS) WITH 990 INTENTIONALLY ADDED HARMONIC TONES AT TWO DIFFERENT AMPLITUDES

Power level for each of the 990 tones	Measured $V_{\text{harmonics}}$ (rms contribution)	Calculated $V_{\text{harmonics}}$ (rms contribution)
-77.34 dBc	+12.2 µV/V	+10.0 µV/V
-100 dBc	Below noise floor	+0.05 µV/V

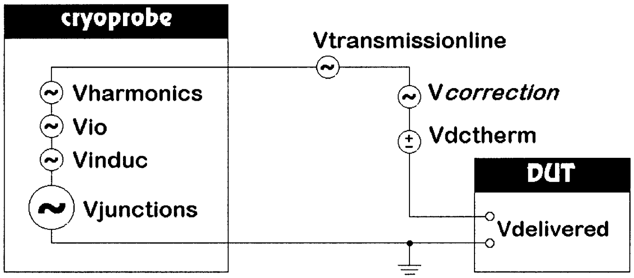

Fig. 2. Schematic diagram that includes the error correction circuit, which adds an ac voltage of the appropriate amplitude and phase to cancel error terms V_{induc} and V_{io}.

TABLE V
MEASUREMENT OF THE AC–AC DIFFERENCE COMPARING THE RMS VALUE OF A SINEWAVE AT 2.67 KHZ TO THE SAME WAVEFORM AT HIGHER FREQUENCIES, WITH AND WITHOUT THE ERROR CORRECTION SIGNAL (FOR V_{induc} AND V_{io})

Frequency	Uncorrected ac-ac difference	Corrected ac-ac difference
8.1 kHz	+23 µV/V	+21 µV/V
53.4 kHz	-176 µV/V	-32 µV/V

greatly reduced. This is illustrated in Table V, where we used our rms voltage-measuring instrument to measure sinewaves at several frequencies with and without the error correction signal. The corrected signal at 53 kHz is more significant because the contribution of the error signal to the total rms voltage is much larger than at 8 kHz. The 20 to 30 μV/V difference after correction may be due to either insufficient calibration of the transfer standard, or the presence of $V_{\text{harmonics}}$ at signals greater than 10 MHz that are not attenuated by the bandpass of our output voltage leads from the array.

Although this error correction technique is helpful for demonstrating the significance of V_{induc} and V_{io}, the best way to reduce these error terms is to improve the design of the circuits, both on the chip and in the output leads. However, the error correction technique will always be useful at higher frequencies because it enables an increased operating bandwidth by providing a way to decrease error signals at higher frequencies by measuring and canceling them.

V. CONCLUSION

We have measured and characterized the dominant sources of error and uncertainty for the pulse-driven Josephson voltage standard, and compared those results with the expected errors. We have also demonstrated a correction circuit that decreases the uncertainty in the rms voltage delivered to the device under test by measuring and canceling certain error voltages. The results of these experiments show great promise for creating an intrinsic ac Josephson voltage standard, but at the same time demonstrate that circuit improvements are necessary to increase the output bandwidth and to deliver overall uncertainties of parts in 10^6 or better.

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