Calculable Coaxial Resistors for Precision Measurements

Randolph E. Elmquist
National Institute of Standards and Technology
Gaithersburg, MD, USA†

Abstract

Coaxial straight-wire resistors have been constructed with the purpose of improving comparisons between resistors, capacitors, and inductors in the audio frequency range. The design is based on the principle that a coaxial line with a cylindrical shield can be described by relatively simple equations for the real and imaginary parts of the impedance. The resistors, with values above 100 Ω, will be used as primary transfer standards for characterization of frequency dependence.

1. Introduction

The National Institute of Standards and Technology (NIST) for many years maintained the units of resistance and capacitance based on cross-capacitance measurements made with a Thompson-Lampard calculable capacitor. Calculable cross-capacitors were constructed and used at several national laboratories, beginning in the 1960's, with four or more symmetrically mounted cylindrical bars which were partially shielded from one another by a central, moveable guard electrode. This design allows the farad to be related to the International System (SI) units through measurements of length, and the SI ohm can be determined from the calculable capacitor using a quadrature bridge based on SI frequency standards.

Adoption of the quantized Hall resistance (QHR) as a reference standard in 1990 allowed the ohm to be maintained independently of the farad, although the SI unit is still derived from the calculable capacitor [1]. A series of experiments using the calculable capacitor has allowed NIST to maintain the consistency between ohm and farad units.

The working standards for resistance and capacitance at NIST are maintained with a bank of 1 Ω Thomas-type resistors and a bank of 10 pF fused-silica capacitors, respectively. The complete measurement process from the calculable capacitor to the fused-silica standards and to the 1 Ω Thomas-type resistors or to the QHR standard is known as the calculable capacitor chain. Two calculable coaxial resistors were built in 1969 by Cutkosky and Haddad [2] in order to transfer the SI ohm to the 1 Ω resistance standards. Haddad's straight-wire 1000 Ω coaxial resistor with calculable frequency dependence from $\omega = 0$ to 10^5 rad/s (15 920 Hz) [2] is still used to determine the resistance difference between alternating current (ac) and direct-current (dc) measurements with an uncertainty of about 0.01 μΩ/Ω. This resistor consists of a single length of fine Evanohm[1] wire about 23 cm long. This paper describes the construction of new coaxial resistors of a similar design, certain tests that help to characterize the ac and dc behavior of these resistors, and the uses for calculable ac/dc resistance standards.

2. Frequency Dependence

Bridge measurements based on the four-terminal-pair definition of admittance [3] ensure that the current into the inner member of each coaxial connector is equal to the current out of its outer member. (A terminal-pair consists of inner and outer conductors of a connector to the four-terminal-pair standard.) The four-terminal-pair configuration also defines distributed capacitance and inductance in the measurement system and standard so that there is a smaller effect on the precision of the ac measurement. Since resistors can be precisely defined in such a bridge, the same value can be obtained when the standard is placed in a different bridge.

The effects of capacitance, inductance, non-ideal current distributions, and eddy currents on the frequency dependence of the coaxial resistor must be calculated. These theoretical contributions to frequency dependence at $\omega = 10^5$ rad/s sum to about −0.03 μΩ/Ω at 1000 Ω and

† The author is with the Electricity Division, Electronics and Electrical Engineering Laboratory, Technology Administration, U.S. Department of Commerce, Gaithersburg, MD 20899-8114 USA.

[1] Evanohm is a registered trademark of the Wilbur B. Driver Company. Certain commercial equipment and materials are identified in this paper in order to specify the experimental procedure. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment are necessarily the best available for the purpose.

0-7803-5276-9/99/$10.00 © 1999 IEEE

about −0.24 μΩ/Ω at 100 Ω. In the next section, the effects of certain changes in the resistor design will be described.

3. Construction of the Resistors

The resistors consist of a straight resistance wire surrounded by a coaxial brass case. Each case was assembled with silver solder and then split lengthwise into two equal halves by the electrical discharge method. The cases have chlorotriflouroethylene insulators at each end. Evanohm rods were clamped in the insulator and attached via Pt wires to current and potential terminations. Coaxial resistor elements were installed by spot-welding to the coaxial rods. Once the wire was mounted, by adjusting the position of the insulators the wire was brought as close as possible to the axis without applying tension. The design of the ac four-terminal-pair bridge measurement ensures that all of the ac current enters through one terminal and returns through the shield of the same terminal. Thus the case forms the return current conductor in the ac bridges, and the case resistance must be well defined. The cases were sealed by soldering at the four electrical termination points, leaving a 1.0 mm gap along the sides of the case. Copper rings serve as clamps at the terminations to reduce the differences in the resistance of the joints. This helps to equalize the current distribution between the two halves of the case. Table 1 gives several characteristics of these new resistors.

In the original Haddad resistors the mounting rods were 1 mm diameter copper rods. Alternating current changes the resistance of the mounting rods due to the skin effect, and this source of resistance change is especially important at higher audio frequencies. Instead, 2.1 mm diameter Evanohm rods are used in the newer NIST resistors. The higher resistivity reduces the ac skin effect in the rods, and at $\omega = 10^5$ rad/s the 100 Ω resistor ac/dc frequency difference should improve to about −0.15 μΩ/Ω.

The dc measurement properties of the straight-wire resistors depend on the composition of the alloy, annealing and heat treatment of the wire, and on the mounting process. Spot welding of similar alloys at the junctions should reduce the Peltier effect in the low frequency and dc measurements. There are two types of Evanohm alloy, nominally composed of about 75% Ni, 20% Cr, 2.75% Al, and 2% Cu. The Evanohm-R and -S alloys differ in composition by the addition of a small amount of silicon to the -S alloy. Four spools of wire of each type were obtained, with resistance values ranging from about 4 Ω/cm to about 50 Ω/cm. The different values of wire resistance will allow us to make resistors with values of 100 Ω, 1 kΩ, 129.06 Ω, and 1290.6 Ω.

The wire was annealed and heat-treated by the manufacturer to reduce the temperature coefficient of

Table 1

Coaxial resistor	SCA	SFA	SGA
Nominal value (Ω)	1000	1000	100
Wire diameter (mm)	0.020	0.019	0.064
Wire length (mm)	233	210	237
Case resistance (mΩ)	0.192	0.192	0.196

resistance (T_C) to +5 ±3 (μΩ/Ω)/°C. In order to lower the T_C of the resistance wire to within ±0.5 (μΩ/Ω)/°C, pieces were spot-welded to temporary mounts and heated in air using a regulated oven. The variations in T_C of adjacent sections of wire from a single spool were considerable for both the alloys, presumably because of inhomogeneous composition. Thus each resistance element was heat treated and monitored individually, after the length of the wire was adjusted to be about 1% longer than required. We found that the Evanohm-S alloy, treated at 390 °C to 405 °C, resulted in the most reproducible changes in T_C and the three resistors constructed so far were made using this alloy.

Four-terminal resistance was used to monitor the heat treatment process. One end of a resistance element is shown in Fig. 1 attached to a supporting Evanohm rod. The resistor wire is attached by spot welding, using an intermediate ribbon made from Evanohm wire. A micrometer was used to position the wire for low-power spot-welding after mounting, in order to adjust the resistance to within about 50 μΩ/Ω of the nominal value.

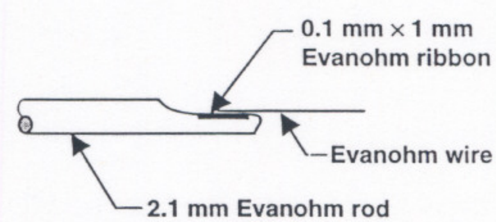

Figure 1. Spot-welded Evanohm wire

4. Measurements

Ac techniques have been used to characterize one of the new 1000 Ω four-terminal coaxial resistance standards. Using a four-terminal-pair bridge, ac resistors can be compared with a total relative uncertainty of about 0.01 μΩ/Ω. Resistance bridge measurements can determine variations in resistance and phase angle as a function of frequency [4]. The calculated changes due to the sum of effects such as the series inductance and eddy current losses can be compared to the measured variations. A 100:1 ratio equal-power resistance bridge [1] was used to compare the 1000 Ω coaxial resistor

Table 2. Electrical characteristics of resistive elements in four calculable coaxial resistors.

Coaxial resistor	R304	SCA	SFA	SGA
Nominal value (Ω)	1000	1000	1000	100
Resistance correction at 25 °C (μΩ/Ω)	−91	+35	−76	−67
First order T_c at 25 °C (μΩ/Ω)/°C	−1	−0.44	−0.18	+0.20
Loading at 4 mW power level (μΩ/Ω)	−0.30	−0.22	−0.41	−0.03
L_c at 4 mW power level (μΩ/Ω)/mW	−0.07	−0.03	−0.08	−0.01
Convection coefficient (μΩ/Ω)/mA	−0.38	−0.14	−0.37	−0.07

made by Haddad (R304) with one new 1000 Ω coaxial resistor (SCA). Each was compared against the same well-characterized 100 kΩ resistor at frequencies of 1592 Hz and 1000 Hz. The difference in the bridge readings (α) at each frequency is given below for each coaxial resistor:

$$\alpha_{304}\,(1592\text{ Hz}) - \alpha_{304}\,(1000\text{ Hz}) = 5.894\ \mu\Omega/\Omega,$$

$$\alpha_{SCA}(1592\text{ Hz}) - \alpha_{SCA}(1000\text{ Hz}) = 5.899\ \mu\Omega/\Omega.$$

The differences agree and this indicates that the two coaxial resistors have nearly the same frequency dependence. The leads were interchanged, switching the high potential to the opposite end of the resistor, and no changes in α_{SCA} larger than 0.005 μΩ/Ω were seen. The phase angle (δ) was measured using the same bridge by comparing SCA against two 100 kΩ resistors for which phase angles are known. The value δ_{SCA} = −10 μrad ± 1 μrad agrees with the value δ_{304} = −9.4 μrad ± 1 μrad obtained for the Haddad resistor, and with the theoretical result, δ_{1000} = −9.4 μrad ± 0.6 μrad.

Dc comparisons generally involve either 1:1 bridge ratios or current comparator bridge measurements at ratios of 10:1 or higher. A 1:1 bridge was operated in the ac laboratory with 2 mA measurement current, producing a 4 mW load on each coaxial resistor. Measurements using the 100:1 ac bridge were made with the same load before and after these dc measurements. The resistance of each coaxial case was subtracted from the ac resistance, and the ac and dc results differed by 0.02 μΩ/Ω ± 0.03 μΩ/Ω. These last measurements are tedious; however, they need to be repeated to improve the precision.

Dc comparisons show that the three new resistors have drift rates within ± 1 (μΩ/Ω)/y. The approximate resistance offset from the nominal value is given in Table 2. The overall T_C of each of the three new resistors has been characterized at dc, by repeated measurements in a controlled oil bath at temperatures between 23 °C and 27 °C. Both 1000 Ω resistors have a purely first-order T_C and the second-order T_C of the the 100 Ω resistor is quite small (+0.015 (μΩ/Ω)/(°C)^2) at 25 °C.

The coaxial resistors were measured at several of the above oil bath temperatures using power levels of between 0.25 mW and 16 mW for the 1000 Ω resistors, and between 1.6 mW and 30 mW for the 100 Ω resistor. Joule heating produces a significant rise in the wire temperature, and a resulting change in resistance (loading effect) occurs. In many standard resistors the load coefficient of resistance (L_C) is proportional to the T_C value. The coaxial resistor design provides little direct heat-sinking for the wire element. The change in resistance due to the loading effect is also complicated by the fact that the T_C of the element varies over short lengths of wire.

In the simple static approximation, power dissipation from the wire takes place by conduction into the mounting rods and outward into the surrounding oil. Near the mounting rods there is a temperature gradient in the wire when power is applied. Thus the change in resistance will be more closely related to the T_C of the warmer, central part of the wire than to the T_C of regions near the ends. This may explain why, in Table 2, L_C and T_C values of some of the resistors show only weak correspondence. The values given in Table 2 have remained constant over several weeks or longer.

Any bulk motion of the oil will increase the rate of heat loss. The data of Fig. 2 shows, for resistor SCA, the average of 24 sets of measurements at each of five power levels. Similar behavior is seen in all of the coaxial resistance standards, including R304. The resistor cases are open to the oil through 1.0 mm gaps along both sides of the cases, and stirring motion in the oil bath probably reduces loading effects at low power levels. At power levels above about 4 mW for the 1000 Ω resistors, and above 9 mW for the 100 Ω resistor, the resistance change becomes proportional to the change in current rather than the change in power. An explanation for this behavior is that induced convection of the heated oil near the wire becomes important. There is a time constant $\tau \approx \varepsilon(r/v)$ that relates the power dissipation to the temperature rise in the wire if the heated oil moves past the wire at a velocity v. If the convection velocity is also proportional to the temperature rise of the wire, then

$$\Delta T_{Wire} \propto \frac{r}{(\Delta T_{Wire})} P, \qquad (1)$$

where the r is the wire radius, and P is the power per unit length of resistance element. The temperature change of the wire, which is estimated to be on the order of 1 °C, is then proportional to the current rather than the square of the current.

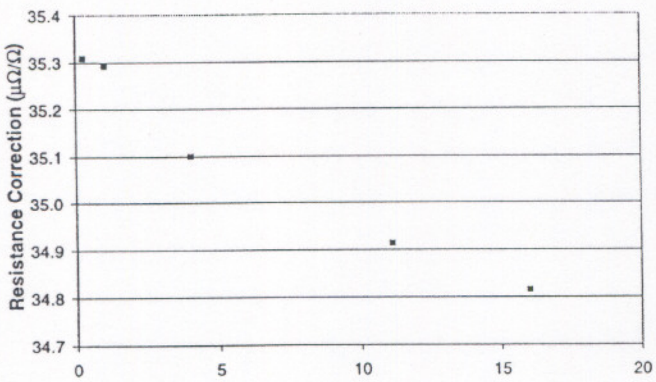

Fig. 2. Resistor SCA at various power levels

5. Ac Reference Standards

Reference standards with small, verifiable frequency dependence are needed to provide a calibration service at NIST for ac resistors. This service will support commercial thermometry bridges and other precision ac measurements. The 100 Ω and 1 kΩ coaxial references will be used for calibration of resistance values from 1 Ω to 10 kΩ over a frequency range from 20 Hz to 10 kHz. Improvements in ac resistance calibrations are also needed for the proposed NIST digital impedance bridge calibrations of standard inductors at an uncertainty of about 10 µH/H.

Ac bridge techniques that were developed for the calculable capacitor chain will be expanded to support a wider range of experiments. The resistance value of the QHR standard is a ratio of fundamental physical constants, and has no known frequency dependence. Hopefully, the calculable ac resistors will help to resolve questions about the frequency dependence of the ac QHR standard between 100 Hz and 3000 Hz. A special variable-frequency 10:1 ac bridge has been constructed, and will be used to compare the 12 906.4 Ω QHR plateau to a coaxial resistor of value 1290.6 Ω. The same bridge will also compare the QHR to a 129.06 kΩ standard resistor, which is not coaxial, but which can be compared to standard capacitors using a quadrature bridge. A coaxial 129.06 Ω standard will be used to check the frequency dependence of the 1290.6 Ω resistor, and a 100:1 ac bridge can compare the coaxial resistors to the higher-value resistance standards. The aim of this research is to base the calibration of capacitors directly on the ac QHR.

6. Conclusions

A 1000 Ω coaxial resistor has been tested as the first of a set of new calculable ac reference standards with good ac and dc properties, including low drift and temperature coefficient. Two other similar resistors of values 1000 Ω and 100 Ω have been fabricated and the construction of 1290.6 Ω and 129.06 Ω coaxial resistors is planned. Earlier works have made calculations for the frequency dependence of the coaxial resistor design. This theory can be developed with high precision due to the special design of the resistors. Experimental ac and dc measurements are used to compare the frequency dependence of various resistors to the theoretical calculations. The coaxial standards will be used to improve NIST impedance calibration services and to offer ac resistance calibrations between 1 Ω and 10 kΩ, over the frequency range from 20 Hz to 10 kHz, and to improve the chain of measurements linking the QHR to the NIST calculable capacitor experiment.

ACKNOWLEDGEMENT

The assistance of R. F. Dziuba in designing the heat treatment apparatus and A.-M. Jeffery in measuring the ac properties of resistor SCA is gratefully acknowledged.

REFERENCES

[1] Jeffery, A., Elmquist, R. E., Shields, J. Q., Lee, L. H., Cage, M. E., Shields, S. H., and Dziuba, R. F., "Determination of the von Klitzing constant and the fine-structure constant through a comparison of the quantized Hall resistance and the ohm derived from the calculable capacitor," *Metrologia*, Vol. 35, pp. 83-96, 1998.

[2] Haddad, R. J., "A Resistor calculable from DC to $\omega = 10^5$ rad/s," Master's thesis, School of Engineering and Applied Science, George Washington University, Washington, DC, (April, 1969).

[3] Cutkosky, R. D., "Four-terminal-pair networks as precision admittance and impedance standards," *IEEE Commun. Elec.*, Vol. 70, pp. 19-22, 1964.

[4] Cutkosky, R. D., "Techniques for comparing four-terminal-pair admittance standards," *NBS J. Res.*, Vol. 69C, pp. 173-179, 1965.