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SUGGESTED TRIPLE-SERIES CONNECTION MEASUREMENT TESTS OF THE AC QUANTIZED HALL RESISTANCE AND THE AC LONGITUDINAL RESISTANCE

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Abstract

Based on equivalent circuit calculations a single ac ratio bridge can be used to accurately determine the ac quantized Hall resistance and to provide an independent value of the ac longitudinal resistance in a quantum Hall device. This may be achieved by making quantized Hall resistance measurements for two different combinations of triple-series connections to a standards-quality device.

Introduction

The quantum Hall effect [1] has been successfully used as an intrinsic dc resistance standard. Many laboratories are now trying to employ this effect to realize an intrinsic ac resistance standard by using ac ratio bridges to compare the ac quantized Hall resistance R_H with reference standards; other bridges then measure the ac longitudinal resistance R_x along the device length. Because different bridges are required, only one experiment [2] has measured both R_H and R_x.

In every reported experiment to date the value of the ac quantized Hall resistance R_H has varied with the frequency of the applied current and differed from the dc value by several parts in 10^7. With one notable exception [3], the ac longitudinal resistances have increased with increasing frequency of the applied current. These frequency dependences of R_H and R_x are reported to be in the *real* (resistive) component of the ac impedance measurements, not in the *imaginary* (90° out-of-phase) component.

These puzzling results could be due to intrinsic properties of the quantum Hall devices. However, calculations [4] of the intrinsic impedances resulting from the Hall capacitance of the two-dimensional electron gas across the quantum Hall device, and the kinetic inductance and magnetic inductance along the device, have provided no plausible intrinsic impedance explanation for the observed frequency dependences of the ac quantum Hall resistance R_H and the ac longitudinal resistance R_x.

Circuit Analysis

We have therefore examined the effects of longitudinal resistances along the device on measurements of R_H and R_x by using the equivalent circuit of Ricketts and Kemeny [5], considering only real (in-phase) resistive components, and employing the multiple-series connections to the device of Delahaye [6]. To simplify the calculations, we used the diamond-shaped voltage generator arrays of Ricketts and Kemeny [5], rather than the ring-shaped arrays of Delahaye [6] and of Jeffery, Elmquist, and Cage [7]. We also assume that the device is of standards-quality, *i.e.* that the Hall resistances R_H are *all* measured on plateau regions, and that their dc values are the *same* on *all* the Hall potential probe sets at the *same* magnetic field value.

Figure 1 shows a "normal" triple-series connection to the quantum Hall effect device when the drain D and points 1, 3, and 5 are all near one potential and the source S and

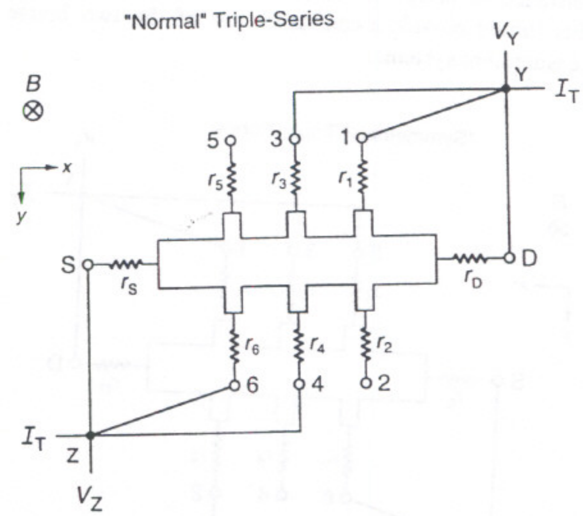

Figure 1. "Normal" triple-series connection to a quantum Hall effect device. Points Y and Z are where the four-terminal-pair definitions are realized.

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points 2, 4, and 6 are all near another potential. The magnetic flux density B points into the figure for this case. I_T is the total current injected into the device, and points Y and Z are where the four-terminal-pair definitions [8] are realized. The lead resistors r_1 through r_D include contact resistances to the two-dimensional electron gas and the inner conductor resistances of the coaxial leads. These resistances are typically 1 Ω for ac quantized Hall resistance measurements. We find from the calculations that the quantized Hall resistance $R_H(Y,Z) \equiv [V_Y - V_Z] / I_T$ is within 2 parts in 10^{11} of R_H when the lead resistances are 1 Ω, and within 1 part in 10^9 of R_H when the lead resistances are 10 Ω.

Figure 2 shows what we call the "symmetrical" triple-series connection. The quantized Hall resistance $R_H(Y,Z) \equiv [V_Y - V_Z] / I_T$ is within 3 parts in 10^{11} of $R_H - R_x(2,6)$ when the inner conductor resistances are 1 Ω, and within 1 part in 10^9 of $R_H - R_x(2,6)$ when the lead resistances are 10 Ω, where $R_x(2,6) = R_x(1,5)$ is the longitudinal ac resistance at that frequency if it had been correctly measured with a longitudinal resistance ac bridge with no triple-series connections. Therefore, the "normal" triple-series connection accurately measures R_H, and the "symmetric" triple-series connection accurately measures $R_H - R_x$ using the *same* ac ratio bridge. The value of R_x obtained with this method can be compared with that obtained from a longitudinal resistance ac bridge to assure that both results agree. This should provide a confidence test of the two bridge measurement systems.

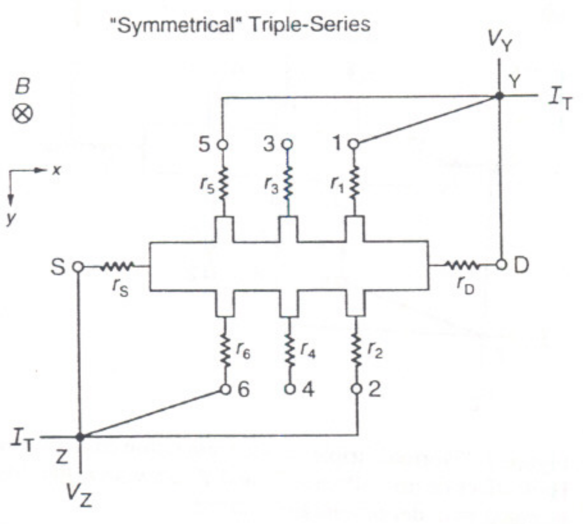

Figure 2. "Symmetrical" triple-series connection to a quantum Hall effect device. Points Y and Z are where the four-terminal-pair definitions are realized.

Conclusions

An interesting result of these calculations is that a single ac ratio bridge can be used to accurately determine the ac quantized Hall resistance R_H and to provide an independent value of the ac longitudinal resistance R_x by making four-terminal-pair measurements [8] of two different combinations of triple-series connections to the quantum Hall devices. This value of R_x can be compared with that obtained using a bridge which measures the ac longitudinal resistance to provide a test of the two bridge measurement techniques.

References

[1] R. E. Prange and S. M. Girvin, eds., The Quantum Hall Effect, Springer-Verlag, New York, pp. 1-419, 1987.

[2] F. Delahaye, "Accurate ac Measurements of the Quantized Hall Resistance from 1 Hz to 1.6 kHz," Metrologia, Vol. 31, pp. 367-373, 1995.

[3] F. Piquemal, G. Trapon, and G. Geneves, "AC Measurements of the Minimum Longitudinal Resistance of a QHE Sample from 10 Hz to 10 kHz," IEEE Trans. Instrum. Meas., Vol. 45, pp. 918-922, 1996.

[4] M. E. Cage and A. Jeffery, "Intrinsic Capacitances and Inductances of Quantum Hall Effect Devices," J. Res. Natl. Inst. Stand. Technol., Vol. 101, pp. 733-744, 1996.

[5] B. W. Ricketts and P. C. Kemeny, "Quantum Hall Effect Devices as Circuit Elements," J. Phys. D: Applied Phys., Vol. 21, pp. 483-487, 1988.

[6] F. Delahaye, "Series and Parallel Connection of Multiterminal Quantum Hall Effect Devices," J. Appl. Phys., Vol. 73, pp. 7915-7920, 1993.

[7] A. Jeffery, R. E. Elmquist, and M. E. Cage, "Precision Tests of a Quantum Hall Effect Device dc Equivalent Circuit using Double-Series and Triple-Series Connections," J. Res. Natl. Inst. Stand. Technol., Vol. 100, pp. 677-685, 1995.

[8] R. D. Cutkosky, "Techniques for Comparing Four-Terminal-Pair Admittance Standards," J. Res. Natl. Inst. Stand. Technol., Vol. 74C, pp. 63-78, 1970.