WHAT METROLOGY GAINS WITH QUANTIZED RESISTANCE STANDARDS

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Abstract. *The emergence of quantum metrology is expressed in modern electrical standards, primarily through the Josephson effect and the quantum Hall effect. The Josephson and von Klitzing constants that relate to these standards are given in quantum theory by ideal equations $K_J = 2e/h$ and $R_K = h/e^2$, respectively. While they are not definitions, these two equalities encompass invariant physical laws and are thought to be valid for certain broad types of quantum mechanical systems. It is well known that these quantum standards can provide representations of international electrical units that are equivalent and unchanging in value. This paper describes several recent experiences concerning the quantized Hall resistance (QHR) standard from a viewpoint within a National Measurement Institute (NMI). It describes the measurements of some fundamental constants that are made more accurate because the QHR standard is an unchanging representation of the ohm. The QHR standard has also helped to bring about improved international comparisons of resistance units, as well as the development of better methods of scaling from one resistance level to a higher or lower resistance. A new experiment to compare the Josephson and quantum Hall effects together with a fundamental standard of electric current is described.*
Keywords: *Measurement standards, Resistance measurement, Quantum Hall effect*

1. INTRODUCTION

As early as the 1930s there were experiments [Curtis, 1944] that provided "absolute" measurements of electrical quantities, the ampere and ohm, based in part on measurements of time, distance, and force. Now, the most fundamental metrology experiments at the National Metrology Institutes (NMIs) are based, in part, on the quantum mechanical structure of light and matter.

A structural framework must be embedded in the physical measurements, and today this consists of fundamental constants of nature, such as the speed of light in vacuum (c), the Planck constant (h), the electron charge (e), the magnetic constant (μ_0), and other basic ratios and quantities. Some of these are defined, others including h and e are constants indirectly determined by a number of experiments. The same experiments are also used to determine

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the two electromagnetic quantities, the Josephson constant (K_J) and the von Klitzing constant (R_K), which have special significance in the SystΠme International d'Unitϑs (SI).

The way in which fundamental constants are linked together mathematically in the SI is by periodic determinations based on global least-squares analysis procedures [Mohr, 1999]. Under the guidance of the Consultative Committee on Electricity and Magnetism (CCEM), calibrations and precision measurements in participating nations use standards traceable to agreed-upon values of the Josephson constant and von Klitzing constant in order to state the resulting values in consistent international units. Before 1990, resistance measurements (and voltage measurements as well, before about 1972) were traceable to independent artifacts maintained at laboratories around the world.

The National Institute of Standards and Technology (NIST) began making precise comparisons between a group of five 1 Ω wire-wound resistance standards and the QHR in 1983, after several years of testing and evaluating QHR devices. The results of these years of QHR comparisons were analyzed in 1988 [Cage, Dziuba, Vandegrift, and Yu, 1989] and showed that the U.S. Legal Ohm defined by the group of 1 Ω standards was drifting by a fractional amount of about 5.3 H 10^{-8} per year. In conjunction, values of the constant R_K and of the SI ohm were determined in 1988 by assigning a value to the QHR using the Thompson-Lampard capacitor [Thompson, 1956]. The result of this calculable capacitor experiment at NIST [Shields, 1989], with a fractional uncertainty of 2.4 H 10^{-8}, was considered along with other similar data from the NMIs in the recommendation of an exact, conventional value,

$$R_{K\text{-}90} = 25\,812.807\ \Omega \qquad (1)$$

for the plateau $i = 1$ quantized resistance of the QHR to be used starting January 1, 1990. The measured resistance values of the QHR used in metrology are then exactly $R_H(i) = R_{K\text{-}90}/i$ where i is a small integer.

The calculable capacitor determination of the SI ohm was repeated at NIST between 1992 and 1995. This time-consuming experiment was described in some detail by Jeffery, et al. [Jeffery, 1998]. In contrast to the difficulty of the SI ohm realization, the QHR standard serves as a reference for measurements that is more repeatable than the calculable capacitor (at a fractional uncertainty below 10^{-8}), and much more directly available. The next section describes how the QHR can be used directly and indirectly to determine fundamental constants, by providing a truly unchanging value as a resistance reference.

2. FUNDAMENTAL CONSTANTS

The inverse fine-structure constant α^{-1} . 137.036 is a numerical constant appearing in the equations that describe electromagnetic corrections applicable to the elementary particles, and is without units. The von Klitzing constant R_K is shown to be proportional to the inverse fine-structure constant (α^{-1}) in one of the relationships between fundamental constants,

$$R_K = \frac{\mu_0 c}{2}\alpha^{-1} \quad . \qquad (2)$$

Future improvements in the experimental realization of the SI ohm can be reflected in new "best" values of α^{-1} and R_K. Also, Eq. (2) shows why a recommended value of $R_{K\text{-}90}$ cannot be used to define the unit of resistance in the SI [Taylor and Cohen, 1991] since the constants c and μ_0 have already been given defined values ($c = 299\,792\,458$ ms^{-1} and $\mu_0 = 4\pi$ H 10^{-7} N/A^2).

For some determinations of fundamental constants, if all the electrical measurements are carried out in consistent and invariant laboratory units, then the results are valid whatever those units may be. These experiments do not depend on whether we use laboratory units based on quantum standards, or on physical artifacts. In the area of electricity, such work is carried out only at NMIs. This is because no artifact resistance standard would allow the same low level of uncertainty in determination of fundamental constants.

For example, a measurement of γ_pN (the prime indicates a spherical water sample used in nuclear magnetic resonance measurements) in laboratory units [Williams, 1989] was performed at NIST in the late 1980s. The following equation [Cage, Dziuba, Elmquist, *et al.*, 1989] for the fine-structure constant,

$$\alpha^3 = \frac{2\mu_0 R_\infty \gamma_p'}{(\mu_p'/\mu_B) R_{K\text{-}90} K_{J\text{-}90}} \quad , \qquad (3)$$

allows α to be found from the proton gyromagnetic ratio (γ_pN) measured at NIST, the Rydberg constant R_4 in SI units, the ratio of the magnetic moment of the proton to the Bohr magneton (μN$_p/\mu_B$), and the recommended values of the von Klitzing and Josephson constants. Equation (3) yields a value of the fine-structure constant largely independent of that derived from Eq. (2). Because NIST's laboratory units could be derived from the QHR and Josephson voltage standard (JVS), transfer standards (reference resistors and Zener voltage standards) were maintained consistently and were assigned values corrected for drift, reducing the uncertainty in the measurement.

More recently, the QHR has been supporting the realization of the SI-watt from moving-coil watt balance experiments, where a reference resistor that has a value with a fractional uncertainty of 10^{-8} or better is needed. The voltage reference used in the latest NIST watt balance is supplied by a programmable JVS [Hamilton, 1997] built at NIST's Boulder laboratory.

The watt balance measurement consists of two modes. In the first mode, a voltage reference value U, which is supplied by the JVS, is used to servo-control the velocity (dz/dt) of a coil moving vertically in a radial magnetic flux density. In the second mode, a current I passing through the same coil, now held stationary in the same magnetic flux density, is used to balance the force $F_z = mg$, where m is the mass of a standard mass and g is the local acceleration of gravity. In this second mode, the current I flows through both the resistor and the moving coil and the voltage across this resistor is measured in a comparison against a similar voltage from the JVS. From the product of U and I, a value of the SI-watt W is determined [Steiner, 1999]. The fractional uncertainty in W due to the influence of the 100 Ω reference resistance value is expected to be about 5 H 10^{-9}, based on $R_{K\text{-}90}$.

The following equation linking results of the watt balance to the SI mass of an atom of ^{12}C can be derived [Taylor, 1991]:

$$m(^{12}\mathrm{C}) = \left(\frac{W_{90}}{\mathrm{W}}\right)\left[8\left(\frac{m(e)}{m(^{12}\mathrm{C})}\right)^{-1}\frac{R_{\infty}}{c\alpha^2 K_{\mathrm{J-90}}^2 R_{\mathrm{K-90}}}\right] \quad . \tag{4}$$

In this equation $m(\mathrm{e})/m(^{12}\mathrm{C})$ is the electron to ^{12}C mass ratio, and W_{90} is the laboratory watt unit used in the measurement, based on 1990 recommended values. The terms inside the square brackets in Eq. (4) have a total uncertainty of less than 1 H 10^{-8}. The watt balance will first monitor the unit of mass maintained at NIST, based on the quantum electrical standards. Because it has no long-term drift, this experiment could later be considered as a practical method of permanently replacing the artifact-based definition of the kilogram in the SI.

3. INTERNATIONAL COMPARISONS

Low uncertainty in QHR determinations can be assured under the measurement conditions described in guidelines [Delahaye, 1989] sponsored by the CCEM. To complement the NMIs' programs to achieve low uncertainty, the International Bureau of Weights and Measures (BIPM) based in SΠvres, France has recently used a transportable QHR apparatus to make on-site QHR comparisons with NMIs in France, Switzerland, Germany, the United Kingdom, and the United States. Results were based on direct measurements using different QHR standards belonging to each laboratory, and help to ensure that each laboratory is successful in producing an accurate representation of the ohm. At each NMI, the unit of resistance was transferred to a 100 Ω standard provided by the BIPM. Concurrently the BIPM made its own independent QHR-based measurements of the 100 Ω resistance standard using a room-temperature 0.25 Hz current-comparator. Another QHR apparatus at the BIPM was used to calibrate the ac-dc difference of the standard resistor and provided a reference for checking the transportable measurement system. Figure 1 shows a single day's set of comparisons between the BIPM and NIST, based on the QHR standards. This type of measurement has reduced the level of uncertainty in resistance units between the NMIs listed above to about 2 H 10^{-9}, based on the typical standard combined uncertainties.

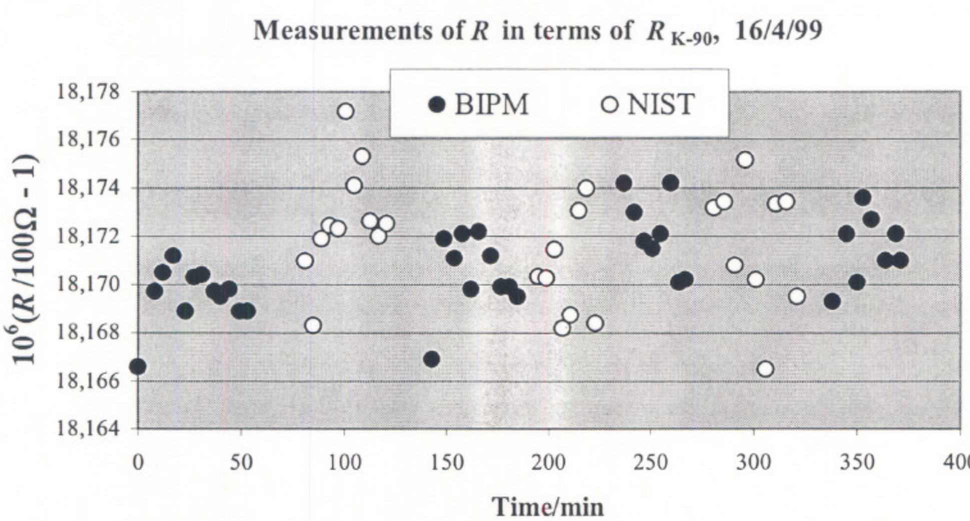

Figure 1 – One day's data comparing similar measurements of a 100 Ω resistance reference, based on QHR devices and measurement systems of the BIPM and NIST.

The most accurate commercial calibrations typically are carried out for references of value 1 Ω or 10 kΩ, using rugged transportable wire-wound resistors. Comparisons with the BIPM transportable system and working standards were used to test the ratios of cryogenic current comparator (CCC) bridges that calibrate the NIST working standards at these important levels. NIST provides these calibrations with fractional uncertainties (2σ) of 0.05 H 10^{-6} for Thomas 1 Ω standards and 0.15 H 10^{-6} for some special 10 kΩ standards.

4. ADVANCES IN TECHNOLOGY: CRYOGENIC CURRENT COMPARATORS

When K. von Klitzing first observed the quantum Hall effect in Si-MOSFETs, resistance metrology was based at most NMIs on carefully maintained groups of wire-wound resistors. In the mid 1980s, three fixed-value, 6 453.2 Ω standard resistors were measured periodically against the QHR plateau $i = 4$ and calibrated against the primary 1 Ω resistor bank at NIST based on the series-parallel network method developed by B. V. Hamon [1954]. A Hamon device with eight 800 Ω resistors, connected in series by special tetrahedral junctions and in series with a 53.2 Ω resistor, was used at NIST for these QHR comparisons. The eight 800 Ω elements could be connected in parallel and calibrated against known references with a total fractional uncertainty of about 1 H 10^{-8}, and the 53.2 Ω resistor could be similarly be measured to within about 1 H 10^{-6}. This yielded a value for the 6 453.2 Ω configuration with a total fractional uncertainty approaching 1 H 10^{-8}.

Early development of the overlapped superconducting-shield [Sullivan, 1974] type of cryogenic current comparator (CCC) at NIST helped reduce the uncertainty in QHR measurements. This device operates in liquid helium, and is very accurate because the superconducting shield allows the current comparator windings to be in nearly exact flux-linkage [Harvey, 1972] and also adds no magnetic core noise. A CCC windings-ratio of 32/4130 in one such system allows 100 Ω resistors to be calibrated against the 12 906.4 Ω plateau resistance of the QHR in direct comparisons, such as the BIPM-NIST comparison.

In the same way, NIST has developed other CCC measurement systems to scale from 100 Ω down to 1 Ω and up to 10 kΩ. These scaling measurements can be performed between resistors located in separate rooms at NIST, using shielded highly-insulated cables, eliminating the need to move transfer standards at any of these three levels. The fractional uncertainty of these scaling comparisons under typical conditions is about 3 H 10^{-9}. Commercial development of the CCC has made this technology available for use not only with the QHR, but also in routine calibrations at many NMIs.

5. QUANTUM METROLOGY AND OHM'S LAW

Recent proposals for a new type of comparison between the laboratory electrical units, sometimes called the electrical quantum metrology triangle experiment, are made possible by the single-electron tunneling (SET) pump. The SET pump is a cryogenic device made from a series array of gated, weakly isolated islands of conducting material. The small size of each island results in a very small capacitance, so that one electron can self-bias the region and produce a large change in the potential barrier for the tunneling of a second electron. With careful tuning, the external gate bias levels can be controlled to transfer one electron per period through the array at frequency f_s, and under this condition the pump provides a current of exactly ef_s where e is the electronic charge.

The first proposal to measure a SET pump current was made in 1992 [Hartland, 1992], based on amplification of the low level of SET current. A CCC could be designed to amplify the SET current by an integer factor of up to about 10^5, producing a larger current in an isolated secondary circuit. This current from the CCC device would generate a small but measurable voltage across a QHR device. Comparing this to a voltage based on the Josephson effect voltage, Ohm's law then yields the SET current by the equation,

$$ef_s = \frac{aK_J^{-1} f_J}{R_K} \quad , \qquad (5)$$

where a is an experimentally-determined coefficient related to the values of resistance, current, and voltage, and f_J is the frequency of microwave radiation applied to the Josephson junction device. The SET pump thus links the values of QHR and JVS standards and tests the self-consistency of the SI-realization experiments such as the calculable capacitor and Watt balance.

Several different types of SET pumps have been proposed and tested, and some are now operating at a pumping frequency of up to 10 MHz and producing a dc current of ef_s – 1 pA with a fractional uncertainty of about 15 H 10^{-9}. So far, only one type of experiment has succeeded in the attempt to measure this current with low uncertainty. Scientists at NIST recently developed a technique [Keller, 1999] in which they collected current from a 7-island SET pump with a cryogenic capacitor and then measured the capacitor voltage after a certain number of electrons had passed through the pump. The value of the capacitor in SI units was obtained to within a fractional uncertainty of 10^{-6}, and the SET current was then calculated to the same level of uncertainty. Several national laboratories have built similar low-loss cryogenic capacitors. Some of these are designed with adjustable value, to allow the use of more accurate ac-bridge calibration techniques.

5.1 New experimental design

In order to make a better determination of the consistency of our system of laboratory units, NIST is considering a third method, using a cryogenic resistor of value $10^7\ \Omega$ to $10^9\ \Omega$, placed in the same circuit with the SET pump (Fig. 2). This experiment combines several advanced technologies.

A SET pump supplies current i_S with almost infinite source resistance, but can operate only if the voltage across the pump is small. Likewise, a voltage source and high-value resistor act as a current source that supplies a constant current i_R, as long as the voltage across the resistor is constant. In this version of the triangle experiment, a high-value cryogenic resistor will carry a current nearly equal to the SET pump current. The resistor current could be generated by a small, precise dc-voltage output from a pulse-driven Josephson voltage standard [Benz, 1999].

The relative magnitudes of the currents can be adjusted using the two frequencies f_J and f_s in Eq. (5). The experiment will aim to determine those frequencies for which the mean current difference $+i_\Delta, = (+i_R, - +i_S,)$ is zero.

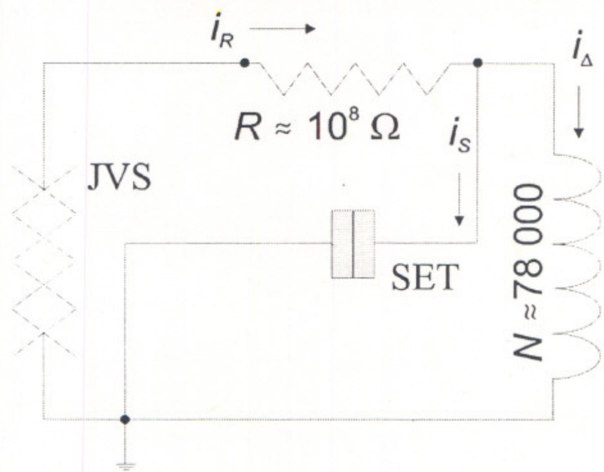

Figure 2 – Comparison of the JVS, QHR, and SET standards of voltage, resistance, and current using an experiment that detects the current difference $i_\Delta = i_R - i_S$, where i_S is the SET current and i_R is a calibrated current generated in a high-value cryogenic resistor.

5.2 Balance detection

This experimental design shown in Fig. 2 could achieve low noise current-detection and zero voltage across the SET current terminals using a superconducting winding of N – 78 000 turns. This input winding allows a superconducting quantum interference device or SQUID to detect the small current difference with a noise level about equal to the Johnson noise in the cryogenic resistor.

The analog output signal of the SQUID would be synchronously measured with a DVM, with some time delay after each reversal. With improved SET pumps being developed at NIST in Boulder, it should be possible to measure a SET pump current of 10 pA with a fractional measurement uncertainty of 10^{-7} (10^{-18} A) in about six hours. Over several weeks or months, it may be possible to achieve a fractional uncertainty of 10^{-8}.

5.3 Cryogenic Resistor Calibration

In order to determine the exact current flowing through the cryogenic resistor, the resistor would be compared to the QHR. The resistor calibration would be based on a special CCC installed in the same cryostat with the SET device and cryogenic resistor, with a current of 50 µA or less passing through a QHR device on the $i = 2$ plateau in a separate cryostat.

5.4 Expected Results of the Experiment

At a level of uncertainty approaching 10^{-8}, results from this type of experiment could help to improve the self-consistency of R_K and $K_J$values that are assigned periodically, and also to lower the uncertainties of a number of fundamental constants and test theoretical models from quantum electrodynamics.

6. CONCLUSION

The QHR device is both an ideal and a real-world standard, through which both practical metrology and physics are advanced in the laboratory. For the practical unit of resistance in international metrology, the CCEM has assigned a value, $R_{K\text{-}90}$, that is exact. This reference value can be used when the QHR is measured in a manner consistent with international guidelines. On this basis, a number of NMIs today maintain the QHR standard at a level of reproducibility near 2×10^{-9}. To determine several fundamental constants, various experiments use the QHR in several ways. The experimental designs up to now either assume the value of $R_K = h/e^2$ from quantum theory, or use repeated comparisons with the QHR standard to eliminate drift and therefore to provide a consistent unit of resistance over time. In the near future, comparisons between quantum electrical standards (the JVS, QHR, and SET) may provide a test of consistency among the equations $K_J = 2e/h$, $R_K = h/e^2$, and $i_S = ef_s$.

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