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Transient Errors in a Precision Resistive Divider

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Abstract: **Resistive dividers have the advantages of dc response and stability. However unlike capacitive dividers, they inevitably involve power dissipation and also generally involve an appreciable inductance. These aspects of a resistive divider result in transient errors, i.e., errors which are a function of the applied waveform. This paper discusses transient measurement errors of precision high voltage resistive dividers such as the one recently developed by NIST.**

INTRODUCTION

High voltage dividers are essential to electrotechnology as many commercial transactions require tests to be performed at specified transient voltage levels and waveshapes. Impulse voltage dividers have typically been calibrated either (1) through measurements at low voltage together with some verification of linearity such as comparison of the divider output with impulse generator charging voltage or (2) through comparison with another divider calibrated in the manner of (1). Until recently, international standards required such devices to measure impulse voltage peaks with uncertainties of ±3%. New versions of these standards [1,2] require peak voltage measurements with ±1% relative standard uncertainties for reference dividers used to check other dividers. NIST continues to work at reducing measurement uncertainties in high voltage impulse measurements. Kerr cells are useful laboratory standards that complement voltage dividers, but they are limited at low frequencies and are too complex for routine industrial laboratory usage. Precision resistive voltage dividers offer the possibility of easily manufactured laboratory standards.

Resistive high voltage dividers typically have high voltage arms that are wound from uniformly resistive wire having a very low temperature coefficient of resistance. The high voltage arm generally comprises two concentric coils, counter-wound to minimize the residual inductance. Such a divider can have transient errors produced by residual inductance in both the high and low voltage arms, thermally-induced errors caused by unequal changes in overall resistance ratio due to heating of the divider, and by stray capacitances. The focus of this paper is an analysis of possible thermal effects and the effects of residual inductances on the accurate measurement of high voltage impulses.

THE NIST HIGH VOLTAGE DIVIDER

Table I gives basic divider characteristics [3]. The divider consists of high voltage and low voltage arms, each formed of two, counter-wound coils of the same turns density wound with wire of the same diameter but with differing resistance per unit length on mandrels of differing diameters. The high voltage arm is about 26 cm long and is rated at 300 kV for a standard lightning impulse waveform (1.2 x 50 µs).

THERMAL MODELING

The divider was modeled thermally using a finite element program which solves Poisson's equation and the (thermal) diffusion equation with time-dependent boundary conditions and field (electric or thermal) dependent material properties. In the present application, short sections of the top and bottom arms of the divider were each modeled as a layer of homogeneous resistive metal over a cylindrical substrate with the thermal properties of Macor* on one side of the resistance layer and with air on the other side. This divider in question normally operates in oil; however, many such dividers operate in air, and

Table I

NIST High Voltage Divider Construction

High Voltage Arm	
Wire Resistance	128.9 Ω/m (#37 Evanohm*)
Wire heat capacity, density	455 J/(kg K), 6562 kg/m^3
Turns density	0.1344 mm/turn
Total turns per winding	1944
Mandrel diameter	25.4 mm, Macor* ceramic
Macor* heat capacity, density	755 J/(kg K), 2520 kg/m^3
Total resistance of HV arm	10,000 Ω (20 kΩ/winding)
Wire conductor diameter	114.3 µm
Wire insulation thickness	6.4 µm polyesteramide
Insulation heat cap., density	2170 J/(kg K), 920 kg/m3 (est)
Wire temperature coefficient	-11 ppm/°C
Low Voltage Arm (where different from HV arm)	
Wire Resistance	25.82 Ω/m (#10 Evanohm*)
Mandrel diameter	6.35 mm Macor* ceramic
Total turns per winding	7
Total resistance of LV arm	1.80 Ω

*Certain commercial materials are identified for completeness. In no case does this identification imply a recommendation by the National Institute of Standards and Technology, nor does it imply that the materials are the best available.

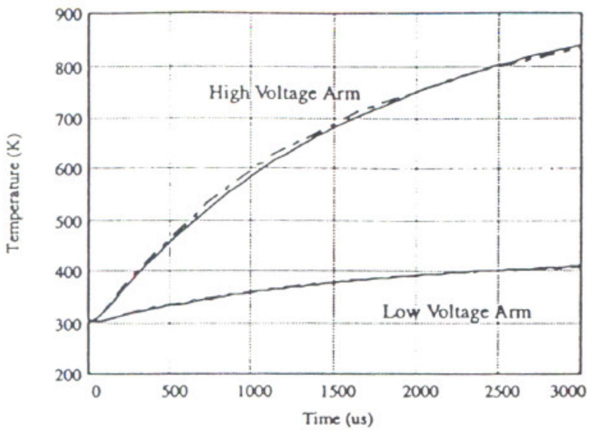

Figure 1. Finite element (FEM) (dashed lines) and adiabatic (solid lines) computations of the temperature vs. time during application of a 200 kV switching impulse. The temperature rise is nearly adiabatic; however, the effect of thermal diffusion can be seen in the crossing of the FEM and adiabatic temperatures at 2300 µs. The adiabatic and finite element computations were undertaken using identical material parameters and are compared with no adjustable parameters.

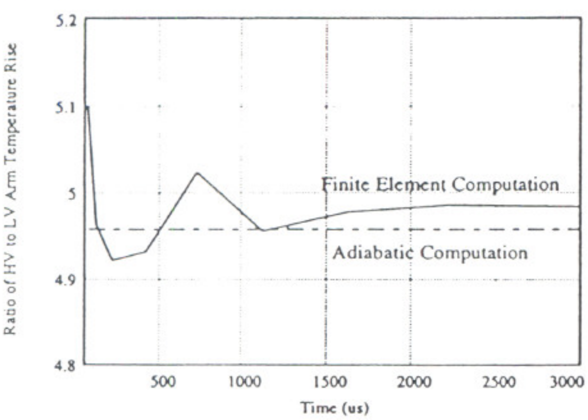

Figure 2. Ratio of temperature rise in the HV arm to that in the LV arm for finite element (solid line) and adiabatic (dashed line) computations. Since the power dissipation goes as I^2R with I common to the two arms, in the adiabatic approximation, we expect the ratio of temperature rise to be given by ratio of the wire resistance per unit length, which is 4.99. The roughness at short times is caused by the small temperature rise.

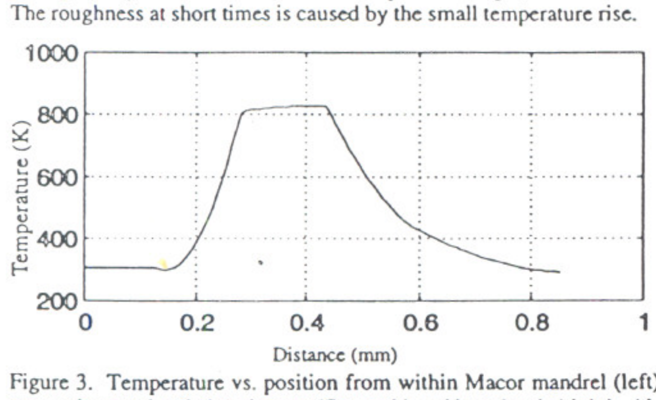

Figure 3. Temperature vs. position from within Macor mandrel (left) across the metal resistive element (flat peak) and into the air (right). Although the air temperature has increased appreciably, little heat has been transferred as the air has a very low heat capacity.

air is the worst case condition. The thickness of the metal layer was adjusted to give the same volume, mass, etc. per unit length of divider arm as provided by the two parallel, concentric windings of the high voltage divider being modeled. Short sections of the high voltage and low voltage windings were so modeled in series, connected by a material of high electrical conductivity but very low thermal conductivity. For comparison, the temperature rise of the winding was computed analytically in an adiabatic approximation, i.e., assuming that all power dissipated in the winding heats the wire. Figure 1 shows the temperature rise of the upper and lower arms for a switching surge (50 x 2500 µs) with an amplitude of 200 kV, as the switching surge rating of power apparatus is generally about two-thirds of the lightning impulse rating.

THERMALLY-INDUCED ERROR

The thermally-induced transient error is caused by the unequal change in temperature of the two arms of the divider during application of the voltage waveform, as shown in Figure 2. Figure 3 shows the radial temperature distribution in the high voltage arm of the divider. Assuming that the temperature coefficient of resistance remains constant at its room temperature value of -11 ppm/K, Figure 4 shows the error in the divider ratio as a function of time during the switching surge. As the high voltage arm heats up more than the low voltage arm and the arms have a negative temperature coefficient of resistance, the division ratio decreases as a function of time and the divider reads high. The error is small at the peak of the switching surge (about 100 µs) and increases during the tail of the waveform as the divider heats up. Similar computations have been undertaken for a 300 kV lightning impulse, for which the temperature rise in the high voltage arm is less than 30 K, which makes the divider error roughly a factor of 20 less than for the switching surge. The divider will take substantial time to return to thermal equilibrium. Although the NIST divider was designed for lightning impulse measurement and is never used with switching surge waveforms, this analysis demonstrates the errors that can arise when a precision resistive divider is used for switching surge measurements. Obviously care must be exercised in using such a divider for repeated lightning impulse measurements, as high repetition rates could cause substantial heating.

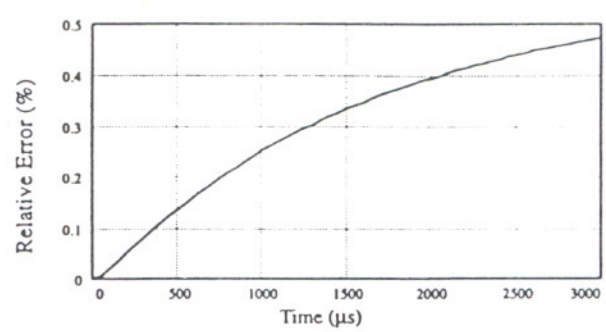

Figure 4. Thermally-induced divider error vs. time during application of a 200 kV switching surge. Positive error indicates that the divider reads high.

INDUCTIVE ERROR

Inductance is caused by energy stored in the magnetic field by current flow through a circuit. Inductance can be computed from the energy condition that

$$\frac{1}{2} L I^2 = E = \frac{\mu_o}{2} \int H^2\, dV \text{ or } L = \frac{2E}{I^2} = \frac{\mu_o}{I^2} \int H^2\, dV \tag{1}$$

where L is the inductance, E is the energy stored in the magnetic field, I is the current through the circuit, H is the magnetic field intensity and μ_o is the magnetic permeability of free space which is assumed constant throughout the volume. The volume integral covers all regions with current-induced magnetic field.

For the divider in question, a magnetic field exists in the region between and within the wires of the two windings of each coil. The inductive voltage drop in each arm of the divider is proportional to the inductance of that arm and the rate of change of the current through the arm. The high voltage arm is much longer, has a much larger number of turns, and has a larger diameter than the low voltage arm, so that the volume integral of magnetic field is much greater with the result that the inductance is much greater. Thus the inductive voltage drop across the high voltage arm is much greater than that across the low voltage arm, which results in an error proportional to dI/dt through the divider.

We compute the inductance of the high voltage arm by assuming that the current is always distributed evenly through the cross section of the wire. As this is resistance wire, skin effect should be negligible. The resistivity of the high voltage arm wire is $1.32 \times 10^{-6}\ \Omega$ m which results in a skin depth of about 0.5 mm at 1 MHz. The skin depth becomes equal to the wire radius at about 100 MHz, which justifies our assumption. Since the two windings are counter-wound, the outer winding lies on top of the inner winding. Going in from the outside surface of the outer winding, the magnetic field increases with the cross section of the outer wire until the magnetic field generated by the outer coil is achieved in the insulation and air space between the two wires. The magnetic field drops as the cross section of the inner wire is crossed and is zero inside the inner winding.

The cross section, A(x) of the wire as a function of distance, x, into the wire is given by (a = the wire diameter)

$$A(x) = \frac{a^2}{8}\left[2 \operatorname{acos}\left(\frac{a-2x}{a}\right) - 4\left(\frac{ax - x^2}{a^2}\right)\frac{a-2x}{a}\right] \tag{2}$$

The current vs. position is therefore given by

$$A(r) = \frac{a^2}{8}\left[2 \operatorname{acos}\left(\frac{a-2R_o-2r}{a}\right) - 4\,\frac{a(R_o-r)-(R_o-r)^2}{a^3}(a-2R_o+2r)\right] \tag{3}$$

$$I(r) = \frac{4\,A(r)}{\pi a^2}\, 0.5 \text{ amp} \tag{4}$$

$$E_1 = 2\,\frac{1}{2}\int_{R_1}^{R_o} \mu_o\, (I(r)\, n)^2\, (2\pi r)\, dr \tag{5}$$

where A(r) is the area of the wire as a function of distance r inward from the outer coil surface at R_o, I(r) is the current outside radius R_o-r, and the integral is taken from the inner radius of the outer coil to the outer radius of the outer coil. As we are computing the inductance of the high voltage arm and not the coil, we assume that 1 A flows through the high voltage arm, which means that only 0.5 A runs through the outer coil. The total energy stored in the magnetic field is simply twice this integral, since the same magnetic field exists within the wire of each coil. Thus the total energy stored in the magnetic field of the wire is found to be 50.6 μJ/m for a high voltage arm current of 1 A. This implies an inductance of 101 μH/m length of high voltage arm or about 26.5 μH for the 26.2 cm length of the high voltage arm. In addition, we have the magnetic field within the insulation space between the windings which results in a magnetic field energy of 8.9×10^{-6} J/m which results in an inductance of 4.66 μH so that the total high voltage arm inductance is 31.1 μH. Similar computations for the low voltage arm result in an inductance of 25 nH, which is negligible.

We are now in a position to predict the transient error caused by the residual inductance of the high voltage arm of the divider. As skin effect should not cause significant errors in the resistance of the divider below 100 MHz, we might reasonably expect the divider to be good for risetimes as short as 3.5 ns and certainly for 10 ns risetimes. The current through the divider caused by an applied step voltage is

$$I(t) = \frac{V_p}{2\,R_d}\left[1 + \tanh\left(\frac{t \cdot 10^9}{t_r\, 0.45 \cdot 10^9} - 3\right)\right] \tag{6}$$

where t_r is the 10% to 90% risetime in seconds, V_p is the peak voltage, and R_d is the divider resistance. The derivative of this waveform is

$$\frac{dI(t)}{dt} = 1.111\,\frac{V_p}{R_d}\,\frac{1 - \tanh\left(\frac{2.222\,t}{t_r} - 3\right)^2}{t_r} \tag{7}$$

We can take the derivative of (7), set it equal to zero, and find the time at which the maximum in the derivative takes place. We can substitute this back into (7) to find the maximum dI/dt as a function of the waveform risetime, which is given by (8). The peak inductive voltage drop across the high voltage arm is then simply $L_{arm}\,(dI/dt)_{max}$ or

$$V_L(t_r) = 31.1 \cdot 10^{-6} \text{ henry}\left[1.111\,\frac{V_p}{R_d\, t_r}\right] \tag{8}$$

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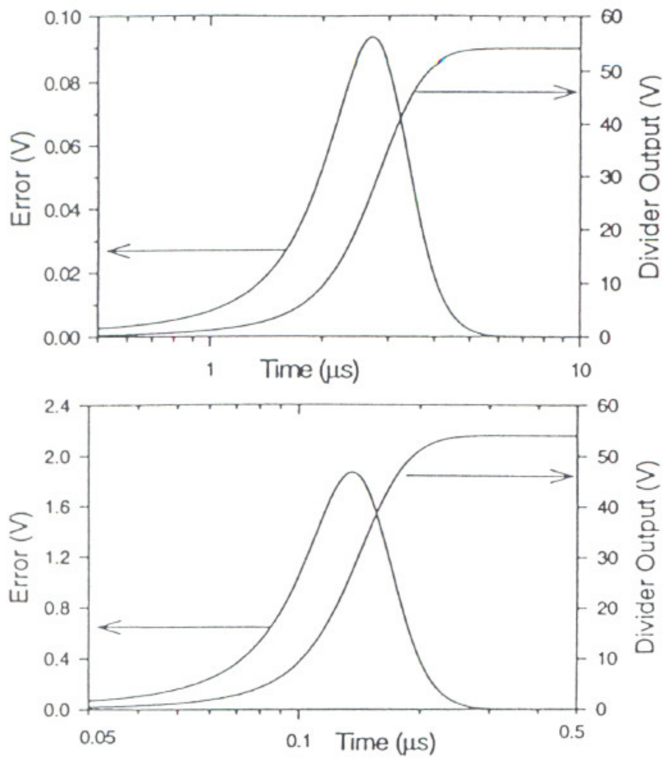

Figure 5. Top, divider output and inductive error in divider output for a 300 kV, 2 µs (10 to 90%) step wave. Below, similar data for a 100 ns risetime (10 to 90%) waveform of 300 kV amplitude. In both cases, the error peaks at the inflection point of the waveform, as it should.

Figure 5 plots divider output and the inductive error in the divider output for 300 kV, 2 µs and 100 ns (10 to 90%) risetime waveforms. For the latter waveform, the relative error (error voltage over output voltage) is roughly constant at 12% early in the waveform, as dI/dt and the voltage are increasing in proportion. The percent error drops later in the waveform and is about 6% at the waveform inflection point, where the error voltage is maximum. In principle, the inductive error should be zero at the peak of a surge waveform, as dV/dt and, therefore dI/dt should be zero. The substantial resistance of this divider should damp any tendency toward oscillation. For example, a 100 ns risetime corresponds to a -3 dB high frequency cutoff of about 3.5 MHz. At that frequency, the inductive impedance of the divider is about 600 Ω (resulting in a 6% error). Given the resistive impedance of 10,000 Ω, the Q is about 0.06, so that the divider should be very well damped. A Q of 1 will be reached at a risetime of about 6 ns corresponding to a bandwidth of 60 MHz. For the 2 µs risetime waveform, which is more typical of a lightning impulse, the relative error is about 20 times less at about 0.6% early in the waveform dropping to about 0.3% at the inflection point

ERROR CORRECTION

Given that transient errors are inevitable in a resistive divider, the obvious question is whether such errors can be corrected easily with digital post processing. In the case of inductive errors, this would be relatively simple as long as the divider remains well damped, as it should. The high voltage arm inductance simply causes a high frequency rolloff and phase shift which can be inverted with an appropriate inverse digital or analog filter.

Correction of the thermally-induced error would be much more difficult, as the thermal effects are long-term and to some degree cumulative. One method of correction would be to insert a miniature thermocouple or thermistor in contact with the winding at the bottom end of the high voltage arm. Such a temperature measurement would allow correction for average, long-term shifts in temperature resulting from application of multiple impulses. Given knowledge of the starting temperatures of the top and bottom windings, a measured waveform could be corrected to first order by assuming an adiabatic temperature rise of the winding during the impulse waveform. As seen above, the adiabatic assumption appears to be good even for switching impulse (50 x 2500 µs) waveforms.

CONCLUSION

We have identified two sources of transient error in a high voltage resistive divider. Thermal errors become significant for long-duration waveforms such as switching surges and affect the tail of the waveform. Inductive errors affect the rise of a waveform (or any region with high dV/dt) and become significant for risetimes less than about 1 µs. Inductive errors will affect measurement of waveform risetime but have little effect on measurement of the peak voltage amplitude.

The present analysis indicates that the range and magnitude of transient errors that can occur in even very well constructed resistive dividers. Interestingly, measurement of peak amplitudes for standard power engineering waveforms are not likely to be much affected as thermal errors early in the waveform are small and inductive errors are theoretically zero at the peak, where dI/dt is zero.

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