IMEKO TC-4 SYMPOSIUM AND ADC WORKSHOP

AN UNCERTAINTY ANALYSIS OF THE HIGH-SPEED PULSE PARAMETER MEASUREMENTS AT NIST

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Abstract - A detailed uncertainty analysis for NIST's pulse parameter measurement service is presented that represents the new pulse parameter measurement and extraction processes. Uncertainties for pulse amplitude, transition duration, overshoot, and under(pre)shoot are given. Effects of temperature, temperature variation, impulse response estimate, pulse parameter extraction algorithms, time-base distortion, calibration procedures, and the waveform reconstruction process are included.

Keywords - uncertainty analysis, pulse parameters, high-speed samplers, transition duration, pulse amplitude, overshoot, under(pre)shoot

1. INTRODUCTION

The National Institute of Standards and Technology (NIST) provides a measurement service for estimating the pulse parameters of amplitude and transition duration[1] for high-speed (transition durations < 20 ps) pulse generators and samplers. NIST previously provided overshoot and undershoot (preshoot) parameters as well but support for these parameters had been discontinued because of the lack of a viable uncertainty analysis[2]. The NIST measurement service presently uses commercially-available high-bandwidth sampling oscilloscopes (-3 dB attenuation bandwidths of approximately 50 GHz) and pulse generators (-3 dB attenuation bandwidths of approximately 20 GHz). NIST is one of two national laboratories that provide a pulse parameter measurement service; the other national laboratory is the National Physical Laboratory in the United Kingdom.

The pulse parameter computations are based on histogram methods. The first step in the calculations is to compute the histogram of the waveform. Next the topline (V_{S2}) and bottomline (V_{S1}) values are obtained from the histogram. From the waveform, V_{S2}, and V_{S1}, the pulse parameters are obtained.

2. UNCERTAINTY ANALYSIS

2.1 Measurement process

Several sets of data are acquired for the customer's pulse generator or sampler (device under test, DUT). A set of data consists of M_1 sampler-acquired DUT waveforms and one measurement of the time-base errors. Measurements of the time-base errors[3-6] are done routinely as part of the DUT measurement procedure. Measurements of the system jitter and the dynamic gain of the sampler are also done routinely, but not necessarily as part of the DUT measurement sequence, and a control chart is maintained from which the appropriate parameters and their uncertainties are obtained. A diagram of the NIST pulse parameter measurement system is shown in Fig. 1. The DUT measurement sequence is as follows:

- Measure time-base error (one independent measurement)
- Acquire waveforms (independent measurements of DUT output)

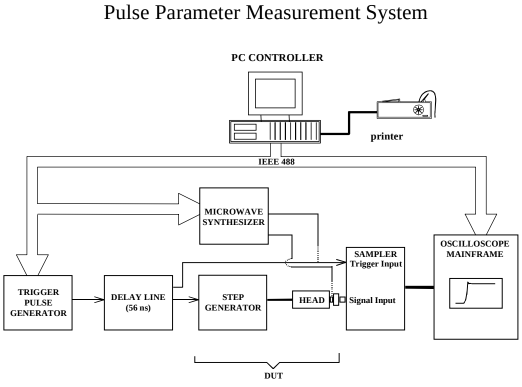

Fig. 1 - Diagram of NIST pulse measurement system. The dotted lines indicate insertion of instruments used in time-base calibration.

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The DUT waveforms are subsequently corrected for gain and time-base errors only if these errors are large relative to the reported uncertainties. The (un)corrected waveforms are then used in a reconstruction process to obtain a waveform that is an accurate estimate of the pulse measured by the sampler. The accuracy of this estimate (the reconstructed waveform) is dependent on the reconstruction process and the accuracy of the estimate of the sampler impulse response. From each reconstructed waveform, pulse parameter values are extracted. The set of pulse parameter values thus extracted is used to determine the mean value and standard deviation for the given pulse parameter.

An estimate of the impulse response of the NIST 50 GHz samplers is presently obtained using the "nose-to-nose" method[7,8] which has been compared to results using swept frequency and optoelectronic methods[9]. We have examined the "nose-to-nose" method and its limitations in sampler calibration[10,11].

2.2 Measurement results

The reported pulse parameters are obtained using a histogram-based algorithm and are an average of the particular pulse parameters obtained from a set of M_1 pulse waveforms. The uncertainty for this average pulse parameter, $\langle W \rangle$ for example, is given by:

$$\begin{aligned} u_{\langle W \rangle} &= k_{eff} \sqrt{\sum_{i=1}^{M_1} \left[\left(\frac{\partial \langle W \rangle}{\partial W_i} \right)^2 \sum_j \left(\frac{\partial W_i(\alpha_j)}{\partial \alpha_j} \right)^2 u_j^2 \right]} \\ &= k_{eff} \sqrt{\sum_{i=1}^{M_1} \left[\frac{1}{M_1^2} \sum_j \left(\frac{\partial W_i(\alpha_j)}{\partial \alpha_j} \right)^2 u_j^2 \right]} && \text{(a)} \quad \mathbf{(1)} \\ &= k_{eff} \sqrt{\frac{1}{M_1} \sum_j \left(\frac{\partial W_i(\alpha_j)}{\partial \alpha_j} \right)^2 u_j^2} && \text{(b)} \end{aligned}$$

where M_1 is the number of values for the parameter W, one value for each waveform, W is dependent on a number, j, of variables, the α_j, and u_j is the uncertainty of the j^{th} variable. It is assumed in (1a) that the α_j are uncorrelated, which is the reason there are no cross terms in the partial derivatives with respect to the α_j. In (1b) it is further assumed that the u_j are the same for each W_i; that is, the uncertainties in the variables for a given parameter are the same for each waveform. The k_{eff} is the statistical weight[12] applied to the uncertainties of variables obtained from a limited number of trials.

2.3 Pulse amplitude uncertainty

Calculating the uncertainty in the pulse amplitude requires having an equation that describes the reported pulse amplitude, $V_{A,}$, and for our system is:

$$V_A = \frac{\overline{V_{A,c} + V_{\Delta T}}}{\overline{g}} = \left(\overline{V}_{A,c} + \overline{V}_{\Delta T} \right) \Big/ \overline{g}, \qquad \mathbf{(2)}$$

where the horizontal bars represent the arithmetic mean, $\overline{V}_{A,c}$ is the average of the set of M_1 pulse amplitudes corrected for sampler offset errors, $\overline{V}_{\Delta T}$ is the average of the amplitude corrections required for a change in measurement temperature, and $\overline{g}$ is the transient gain of the sampler. Ideally, $\overline{g} = 1$ if the sampler exhibits no pulse gain or attenuation and the sampler has settled within the waveform epoch. The $\overline{V}_{A,c}$ is given by:

$$\begin{aligned} \overline{V}_{A,c} &= \overline{V}_{S2,c} - \overline{V}_{S1,c} \\ &= \overline{V}_{S2,m} - \overline{V}_{S2,off,m} - \left(\overline{V}_{S1,m} - \overline{V}_{S1,off,m} \right), \end{aligned} \qquad \mathbf{(3)}$$

where $\overline{V}_{off}$ is the voltage offset. The "c" and "m" in the subscripts refer to corrected and measured voltage values. We have observed that for the presently-available, high-bandwidth samplers, the voltage offset error is the same for both the top line (S2) and bottom line (S1) voltage levels, therefore, the offset voltage contribution can be ignored.

The temperature correction term is obtained by measuring the change in the observed pulse amplitude with temperature (see Fig. 2)[13] and is given by:

$$\overline{V}_{\Delta T} = S_{\Delta V/\Delta T} \left(\overline{T}_{meas} - \overline{T}_{ref} \right), \qquad \mathbf{(4)}$$

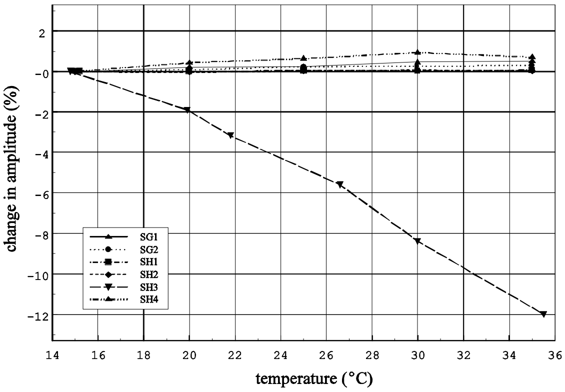

Fig. 2 - The percent change in pulse amplitude with temperature relative to 15 °C.

where $\overline{T}_{meas}$ is the average of M_T temperature values taken during the pulse measurement process, S is the slope of a straight line fit through a set of previously-acquired amplitude versus temperature data, and $\overline{T}_{ref}$ is the average temperature of the sampling head taken when the sampler impulse response was determined.

The transient gain term, $\overline{g}$, is obtained by taking the ratio of the amplitude of the reference pulse as measured using the sampler and the amplitude of the reference pulse as measured using a reference instrument. As mentioned earlier, the gain term is obtained from a control chart and is given by:

$$\overline{g} = \frac{1}{M_g}\sum_{i=1}^{M_g} g_i = \frac{1}{M_g}\sum_{i=1}^{M_g}\frac{\overline{V}_{S2,m,r,i} - \overline{V}_{S1,m,r,i}}{\overline{V}_{S2,r,r,i} - \overline{V}_{S1,r,r,i}}, \quad \textbf{(5)}$$

where the "r,r" in the subscript denotes the reference measurement instrument with reference pulse generator, "m,r" denotes test instrument and reference pulse generator, and there are M_g independent gain measurements.

In addition to measurement-related uncertainties, the reported amplitude values are also subject to uncertainties from the method used to calculate these values, which in this case, is a histogram method. The histogram-derived amplitude values, for example for V_{S2}, are given by:

$$V_{S2} = N_{S2}\,\frac{V_{max} - V_{min}}{N_{bins}} + V_{min}, \quad \textbf{(6)}$$

where N_{bins} is the number of histogram bins and N_{S2} is the bin number of the topline mode bin in the histogram.

2.4 Transition duration uncertainty

The reported (and therefore, reconstructed) waveform transition duration, t_d, is the average transition duration extracted from M_1 reconstructed pulse waveforms. The t_d is related to the transition duration of the measured waveform, $t_{d,m}$, and the transition duration of the sampler step response, $t_{d,r}$:

$$\begin{aligned} t_d &= \frac{\sum_{i=1}^{M_1} t_{d,R,i}}{} \\ &= f_{dec}\left(t_{d,m}, t_{d,r}\right) + \Delta t_{d,\Delta T}, \end{aligned} \quad \textbf{(7)}$$

where T is temperature and $t_{d,R}$, $t_{d,m}$, and $t_{d,r}$ are the transition durations of the reconstructed, measured, and sampler step response waveforms. The specific deconvolution functional relationship, f_{dec}, between t_d, $t_{d,m}$, and $t_{d,r}$ is dependent on the type of waveforms used. For example, for a Gaussian waveform, t_d is equal to the square root of the difference of the squares of transition durations of the measured and step response waveforms. The $\Delta t_{d,\Delta T}$ is the temperature-induced incremental change in transition duration[13].

Since we do not know, a priori, the functional relationship between t_d, $t_{d,m}$, and $t_{d,r}$, we obtain an empirical relationship for the three parameters. We obtain this relationship by fitting a curve (such as a polynomial) to $t_{d,R}$ versus $t_{d,m}$ data and separately to $t_{d,R}$ versus $t_{d,r}$ data where both $t_{d,m}$ and $t_{d,r}$ are varied within expected values and the t_d is obtained from the reconstructed waveforms.

The $t_{d,m}$ and $t_{d,r}$ can be put in terms of the sampling intervals:

$$\begin{aligned} t_{d,m} &= X_m \delta t, \\ t_{d,r} &= X_{rj} \delta t, \end{aligned} \quad \textbf{(8)}$$

where X_m and X_{rj} are the real-valued (non-integer) number of sampling intervals describing the transition duration for the measured sampler step response waveforms (which includes jitter) and δt is the duration of the equispaced sampling interval. The δt is the average duration of the sampling intervals that span either the transition region of the waveform or the entire waveform[15] and is measured using sin-fit techniques[5] during the time-base error measurement process. Fig. 3 shows the time-base errors taken from two different sampling oscilloscopes. X_{rj} is the result of the convolution of the measurement jitter and the sampler step response:

$$X_{rj} = \sqrt{X_r^2 + X_j^2}, \quad \textbf{(9)}$$

where X_r and X_j are the number of sampling intervals in the sampler step response and equivalent jitter step response transition durations. Although X_r may not accurately be described by a Gaussian waveform, X_r and X_j are added in quadrature (Gaussian approximation) to get X_{rj}. Errors are associated with this approximation[14] and the uncertainty bounds are adjusted accordingly. The X_j includes drift of the

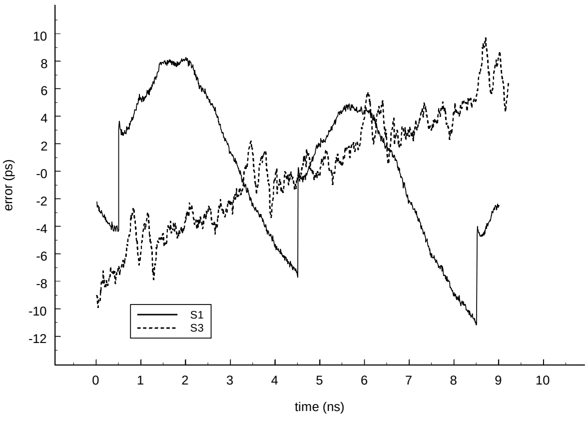

Fig. 3 - Time base errors.

sampling aperture with respect to its trigger.

The temperature dependent change in transition duration can be expanded:

$$\Delta t_{d,\Delta T} = S_{\Delta t/\Delta T}\left(\overline{T}_{meas} - \overline{T}_{ref}\right). \qquad \textbf{(10)}$$

The curve, $S_{\Delta t/\Delta T}$, is a straight line fit to the M_S measured transition-duration versus temperature data pairs that is measured independently of the M_1 acquired waveforms. Using (8) and (10) in (7) gives:

$$t_d = \overline{f_{dec}(X_m, X_{rj})}\delta t + S_{\Delta t/\Delta T}\left(\overline{T}_{meas} - \overline{T}_{ref}\right). \qquad \textbf{(11)}$$

For samplers and pulse generators presently in use at NIST, the value of $\Delta t_{d,\Delta T}$ is approximately zero or is much less than the reported uncertainties, however, it will be maintained here for completeness.

The values of X_m, X_r, and X_j are determined by linear interpolation to obtain the instant in time (the reference level instant) corresponding to the given reference level. The value of X_m (and analogously for X_r and X_j), is:

$$X_m = \frac{t_{L2} - t_{L1}}{\delta t}, \qquad \textbf{(12)}$$

where t_{L1} and t_{L2} are the time instances corresponding to the first (L1) and second (L2) reference levels of the transition duration. For example, in the 10 % to 90 % transition duration, L1 is the 10 % reference level instant and L2 is the 90 % reference level instant. The t_{L1} is given by:

$$t_{L1} = t_{L1-} + \frac{t_{L1+} - t_{L1-}}{L1_+ - L1_-}\left(L1 - L1_-\right), \qquad \textbf{(13)}$$

where the "+" and "-" subscripts denote the actual data values found immediately above and below the reference level and, for the time variables, the subscripts correspond to sampling instances of the those data. The t_{L2} can be expanded similarly to that done for t_{L2}. The values of L1 and L2 can be expanded:

$$\begin{aligned} L1 &= V_{S1,m} + P1(V_{S2,m} - V_{S1,m}), \\ L2 &= V_{S1,m} + P2(V_{S2,m} - V_{S1,m}), \end{aligned} \qquad \textbf{(14)}$$

where P1 and P2 are the percent reference values, such as 10 % and 90 % or 20 % and 80 %.

2.5 Overshoot uncertainty

Voltage offset errors will not be considered here because they will cancel as they did for the uncertainty calculation of $V_{A,c}$. The equation describing the calculation for the overshoot is:

$$OS = \overline{\left(\frac{V_{max,R} - V_{S2,R}}{V_{A,R}}\right)}. \qquad \textbf{(15)}$$

Overshoot is usually presented as a percentage. $V_{max,R}$ can be written as:

$$V_{max,R} = V_{S2,m} + V_{OS,R}, \qquad \textbf{(16)}$$

where

$$V_{OS,R} = \frac{\beta_{OS} V_{OS,m} t_{d,m}}{t_{d,R}}, \qquad \textbf{(17)}$$

and β_{OS} is a correction factor that is determined experimentally. Equation (17) describes an empirical relationship between the overshoot and transition duration of the reconstructed (reported) waveform and that of the measured waveform. In (17), we assume that the product of the overshoot voltage and transition duration is not affected by an all-pass filter, which is how the sampler impulse response is expected to behave for an input signal that has a bandwidth lower than that of the sampler. $V_{OS,m}$ can be expanded:

$$V_{OS,m} = V_{max,m} - V_{S2,m}. \qquad \textbf{(18)}$$

The $t_{d,R}$ can be expanded similar to that of $t_{d,m}$, and using this expansion and (8), (16), (17), and (18) in (15) yields for OS:

$$OS = \overline{\left(\frac{1}{V_{S2,R} - V_{S1,R}}\right)\left(V_{S2,m} + \beta_{OS}\left[V_{max,m} - V_{S2,m}\right]\frac{X_m}{X_R} - V_{S2,R}\right)}, \qquad \textbf{(19)}$$

where X_R is the real-valued number of sampling intervals describing the transition duration of the reconstructed waveform. The correction factor β_{OS} is determined by fitting a curve to a set of M_9 $t_{d,m}V_{OS,m}$ versus $t_{d,R}V_{OS,R}$ data. The uncertainty in β_{OS} is the standard deviation in the fitted curve relative to the set of corresponding $t_{d,m}V_{OS,m}$ versus $t_{d,R}V_{OS,R}$ data, and the coverage factor is determined by the number of β_{OS}s.

2.6 Undershoot (preshoot) uncertainty

The undershoot uncertainty calculation is performed similarly to the overshoot uncertainty calculation (see sec. 2.5) with the appropriate change in variables. This uncertainty estimate yields:

$$\mathrm{US} = \overline{\left(\frac{1}{V_{S2,R} - V_{S1,R}}\right)\left(V_{S1,m} + \beta_{US}\left[V_{min,m} - V_{S1,m}\right]\frac{X_m}{X_R} - V_{S1,R}\right)}, \qquad \mathbf{(20)}$$

where β_{US} is a correction factor that is determined experimentally as is done for β_{OS}. Equation (20) provides an empirical relationship between the undershoot and transition duration of the reconstructed (reported) waveform and that of the measured waveform. The correction factor β_{US} is determined by fitting a curve to a set of M_9 $t_{d,R}V_{US,R}$ versus $t_{d,m}V_{US,m}$ data. The uncertainty in β_{US} is the standard deviation in the fitted curve relative to the set of corresponding $t_{d,m}V_{US,m}$ versus $t_{d,R}V_{US,R}$ data, and the coverage factor is determined by the number of β_{US}s.

ACKNOWLEDGMENTS

We would like to thank W.F. Guthrie and B.C. Waltrip of the National Institute of Standards and Technology (NIST), Gaithersburg, MD for technical comments and G.N. Stenbakken of NIST for technical comments and administrative support.

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