Investigating the Use of Multimeters to Measure Quantized Hall Resistance Standards

Marvin E. Cage, Dingyi Yu, Beat M. Jeckelmann, Richard L. Steiner, and Robert V. Duncan

Abstract—**A new generation of digital multimeters was used to compare directly the ratios of the resistances of several wire-wound resistors and a quantized Hall resistor. The accuracies are better than 0.1 ppm for ratios as large as 4 : 1 if the multimeters are calibrated with a Josephson array.**

I. Introduction

THE INTEGRAL quantized Hall resistance $R_H(i)$ of a two-dimensional electron gas [1] became the new worldwide resistance standard on January 1, 1990, where

$$R_H(i) = \frac{V_H(i)}{I} = \frac{R_K}{i}. \tag{1}$$

$V_H(i)$ is the Hall voltage of the ith resistance plateau, I is the current through the sample, and R_K is the von Klitzing constant, which for the purposes of practical electrical metrology has been assigned the value 25 812.807 Ω exactly [2].

The national standards laboratories require the highest possible accuracies for realizing the new representation of the ohm. They can achieve total one-standard-deviation uncertainties of 0.015–0.030 ppm using 1 : 1 potentiometric measurement systems [3], [4] and Hamon resistance scaling networks [5], or 0.015–0.037 ppm using cryogenic current comparators [6], [7].

In the future, other types of laboratories may find it desirable to have quantized Hall resistance (QHR) standards rather than depend solely on resistor calibrations at the national standards laboratories or on measurement assurance programs (MAPs). Those laboratories will require accuracies of at least 0.1–0.2 ppm in order to compete with the MAPs method and justify the time and expense of building and maintaining a QHR measurement system.

II. Choice of Measurement Technique

There are at least seven questions to consider when deciding to build a QHR measurement system of at least 0.1-ppm accuracy. These might be: 1) What is the quantum Hall resistance sample availability? There is no point in building a QHR system unless an adequate sample supply is assured. 2) What temperatures are required for the samples? There is a big difference in cost and complexity if one has to use a ^{3}He refrigerator instead of a pumped ^{4}He system. Most QHR samples still require the temperatures achieved with a ^{3}He refrigerator. 3) What technique should be used to measure the QHR? 4) Will that technique allow one to make all the measurements recommended by the Consultative Committee on Electricity [8] to test the reliability of the sample? 5) What resistance scaling method should be used? 6) What is the system cost and complexity? 7) What are the personnel training requirements? This paper begins to address questions 3)–7).

We begin by considering possible measurement systems. Potentiometric measurement systems and Hamon resistance scaling networks are not reasonable choices for this type of QHR measurement system because they are too labor-intensive to build. Cryogenic current comparator systems are rather complex, and it can be difficult to achieve a satisfactory performance initially. They do, however, solve the resistance scaling problem. Josephson array measurement systems are another possibility, as demonstrated by an experiment using a Josephson potentiometer consisting of series connections of individual Josephson junctions [9]. However, this would again be a rather complex technique, and one would have to increase substantially the leakage resistance of the RF filters when comparing resistors whose ratio was not 1 : 1 because the quantized Hall resistances are quite large.

One practical possibility is to use direct current comparator (DCC) potentiometers to compare quantized Hall voltages with the voltage drops across series-connected 10-kΩ reference resistors. These comparators already exist in most of the prospective laboratories and 10-kΩ reference resistors are already used in MAP calibrations. In fact, it has been demonstrated [10] that improved versions of a commercial DCC potentiometer can achieve uncertainties less than 0.1 ppm if the DCC potentiometer is calibrated by a Josephson potentiometer consisting of series connections of Josephson junctions [11].

We investigate here the feasibility of an even simpler QHR measurement system, namely, the use of a digital multimeter to compare the dc voltage of a QHR sample with that of a resistor connected in series with the sample. The multimeter is calibrated with a Josephson array voltage measurement system since a Josephson array would almost certainly be an integral part of any laboratory wanting an accurate QHR measurement system.

III. The DVM-Method

In the DVM-method, the same current is passed through a resistor R' and a resistor R, and the resistance ratio is R'/R. R' and R have nominal values R'_{nom} and R_{nom}, so their nominal

Manuscript received June 14, 1990; revised September 28, 1990. This work was supported in part by the Calibration Coordination Group of the Department of Defense, the Naval Strategic Systems Program Office, and Sandia National Laboratories.

M. E. Cage and R. L. Steiner are with the National Institute of Science and Technology (NIST), Gaithersburg, MD 20899.

D. Y. Yu is with NIST, Gaithersburg, MD 20899, on leave from Shanghai University of Science and Technology, Shanghai, China.

B. M. Jeckelmann was with NIST, Gaithersburg, MD, on leave from the Federal Office of Metrology, Wabern/Bern, Switzerland.

R. V. Duncan was with NIST, Gaithersburg, MD, on leave from Sandia National Laboratories, Albuquerque, NM.

IEEE Log Number 9041852.

0018-9456/91/0400-0262$01.00 © 1991 IEEE

resistance ratio r is $R'_{\text{nom}}/R_{\text{nom}}$. For example, if $R' \approx R'_{\text{nom}} = 25\,812.80\ \Omega$ and $R \approx R_{\text{nom}} = 6\,453.20\ \Omega$, then $r \equiv 4$.

The resistance ratio R'/R should equal the voltage ratio if the voltages were measured by a "perfect" DVM. A real DVM requires corrections. We obtained these corrections by calibrating the DVM with a Josephson array. To simplify the method, we used uncorrected DVM voltages and then corrected the voltage *ratios* rather than correcting the *voltages* individually.

The direction of the current through the resistors must be reversed in order to reduce the effects of thermoelectric voltages. Therefore, the uncorrected voltage ratio is $\bar{V}_{R'}/\bar{V}_R$, where

$$\bar{V}_{R'} = \tfrac{1}{2}\left[V_{R'}^{+} - V_{R'}^{-}\right] \tag{2}$$

and

$$\bar{V}_{R} = \tfrac{1}{2}\left[V_{R}^{+} - V_{R}^{-}\right] \tag{3}$$

where the + and − superscripts indicate the direction of current. Let

$$d_{\text{DVM}}(\text{unc}) = \frac{\bar{V}_{R'}}{r\bar{V}_R} - 1 \tag{4}$$

where $d_{\text{DVM}}(\text{unc})$ is the difference of the uncorrected ratio $\bar{V}_{R'}/r\bar{V}_R$ from unity. For this measurement resistor R' is initially in position 1) of the measurement system and resistor R is initially in position 2), as shown in Fig. 1. If the resistors are interchanged then

$$d_{\text{DVM}}^{\text{int}}(\text{unc}) = \frac{r\bar{V}_R(\text{Pos1})}{\bar{V}_{R'}(\text{Pos2})} - 1$$

and

$$\bar{d}_{\text{DVM}}(\text{unc}) \equiv \tfrac{1}{2}\left[d_{\text{DVM}}(\text{unc}) - d_{\text{DVM}}^{\text{int}}(\text{unc})\right]$$

and

$$\bar{d}_{\text{DVM}}(\text{unc}) = \frac{1}{2}\left[\frac{\bar{V}_{R'}(\text{Pos1})}{r\bar{V}_R(\text{Pos2})} - \frac{r\bar{V}_R(\text{Pos1})}{\bar{V}_{R'}(\text{Pos2})}\right]. \tag{5}$$

We must next correct the voltage ratios in (5). The DVM voltage V can be calibrated as a function of the applied voltage V_{arr} of a Josephson array. V can be expressed as a linear function of V_{arr} with a small nonlinearity term $N(V_{\text{arr}})$:

$$V = A + (1 + S)V_{\text{arr}} + N(V_{\text{arr}}) \tag{6}$$

where $(1 + S)$ is the gain, which may deviate from unity by the small quantity S. A is the intercept of the straight line at $V_{\text{arr}} = 0$ and, if $N(0)$ is defined to be zero, then A is the offset voltage V_{off} of the DVM. It then follows from (6) that the DVM correction voltage $C(V)$ is

$$C(V) \equiv V - V_{\text{off}} - V_{\text{arr}} = SV_{\text{arr}} + N(V_{\text{arr}}) \approx SV + N(V) \tag{7}$$

where V and V_{arr} are either both positive or both negative. Straight lines can be fitted to $C(V)$ versus V data using the least-squares method. This can either be done by fitting the positive and negative polarity values of V separately, yielding the coefficients S^+ and S^- and the deviations $N(V^+)$ and $N(V^-)$, or by fitting the combined values of V for both polarities, yielding an average slope S and another set of deviations $N(V^+)$ and $N(V^-)$. The two approaches provide the same final results.

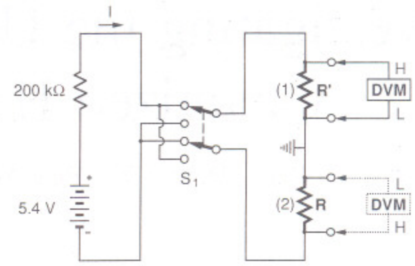

Fig. 1. Schematic diagram of the automated measurement system used in the DVM-method. All of the system to the left of the DVM is identical to the corresponding part of the automated potentiometric system described in [13]. The DVM replaces the potentiometer and detector used in [13]. This system uses thermally insulated mercury batteries for the current source and a stepping motor to rotate the current reversal switch S_1. The current I is 25.5 μA when the wire-wound resistors R' and R are both 6 453.20-Ω ($i = 4$) resistors. A separate stepping motor and rotating switch moves the DVM from position (1) to position (2). The low input side L of the DVM is always near the circuit ground.

It can be shown that the voltage corrections given by (7) lead to a correction term $\Delta\bar{d}_{\text{DVM}}$ that must be subtracted from the expression given in (5) for $\bar{d}_{\text{DVM}}(\text{unc})$. This correction term is

$$\Delta\bar{d}_{\text{DVM}} = \frac{1}{2}\left[\frac{C(V_{R'}^{+}) - C(V_{R'}^{-})}{\bar{V}_{R'}} - \frac{C(V_{R}^{+}) - C(V_{R}^{-})}{\bar{V}_{R}}\right]. \tag{8}$$

Using (7) in (8), and noting that the terms $S^{\pm}(V_{R'}^{\pm} - rV_R^{\pm})$ are negligible because $S^{\pm}$ and $(V_{R'}^{\pm} - rV_R^{\pm})$ are all very small, one obtains the result

$$\Delta\bar{d}_{\text{DVM}} = \frac{1}{2\bar{V}_{R'}}\left[\left[N(V_{R'}^{+}) - N(V_{R'}^{-})\right] - r\left[N(V_{R}^{+}) - N(V_{R}^{-})\right]\right]. \tag{9}$$

The final expression for the deviation of the ratio R'/rR from unity is

$$\bar{d}_{\text{DVM}}(\text{cor}) \equiv \frac{1}{2}\left[\frac{R'}{rR} - \frac{rR}{R'}\right] = \bar{d}_{\text{DVM}}(\text{unc}) - \Delta\bar{d}_{\text{DVM}}. \tag{10}$$

The quantities in expression (10) are very small, and can be expressed in parts per million (ppm). In general, one expects $\bar{d}_{\text{DVM}}(\text{cor})$ to be nonzero because the values of resistors R' and R are rarely exactly equal to their nominal values.

IV. Measurements and Results

We tested the accuracy of this method by using two different Hewlett-Packard HP 3458A Multimeters[1] operated in the dc voltage mode to obtain the voltage ratios. The multimeters were designated as DVM1 and DVM2. They were calibrated with the NIST Josephson array voltage calibration system [12]. The voltage ratios obtained by this method were compared with those obtained from very accurate measurements using an automated QHR potentiometric comparator system [13].

An actual QHR measurement system would most likely use a 10-kΩ resistor for either resistor R or R' of Fig. 1 and a quan-

[1]Brand names are used only for purposes of identification. Such use implies neither endorsement by the National Institute of Standards and Technology nor assurance that the equipment is the best available.

tum Hall sample in place of the other resistor. We wanted, however, to make an accurate assessment of this method by comparing it with results obtained from the automated potentiometric comparator system. Therefore, we used instead six wire-wound QHR resistors. Four of the resistors were constructed [3] to have values within several ppm of the nominal 6 453.20-Ω ($i = 4$) quantized Hall resistance plateau. They are designated $R_1(4)$, $R_2(4)$, $R_3(4)$, and $R_4(4)$. $R_1(4)$ and $R_2(4)$ are in individual temperature-regulated air-bath enclosures. $R_3(4)$ and $R_4(4)$ are in a third air-bath enclosure and can be used individually or connected in series to form a nominal 12 906.40-Ω ($i = 2$) resistor. The last two resistors were constructed to have values within several ppm of the nominal 12 906.40-Ω ($i = 2$) quantized Hall resistance plateau. They are designated $R_5(2)$ and $R_6(2)$, and are in a fourth air-bath enclosure. They can also be used individually or connected in series to form a nominal 25 812.80-Ω ($i = 1$) resistor. These six wire-wound resistors can therefore be used to obtain 1:1, 2:1, and 4:1 resistance ratios, i.e., $r = 1$, 2, or 4. The four air-bath enclosures have all been continuously controlled to within $\pm$ 0.002°C at a nominal temperature of 27.4°C for at least three years. The drift rates of the six resistors, relative to the quantized Hall resistances, are all less than 0.15 ppm/year.

We used an automated quantized Hall resistance potentiometric measurement system [13] to intercompare accurately combinations of 1:1 resistance ratios of these six wire-wound resistors, and thereby determined the deviations $\overline{d}_{POT}$ from unity, where $\overline{d}_{POT}$ is defined by (5) with $r = 1$. No voltage ratio corrections are required, as can be seen in (9), because the voltages are nominally equal. A potentiometer canceled most of the voltage drops across the wire-wound resistors. Voltage differences were amplified by a Leeds and Northrup 9829 Linear Amplifier[1] detector. A digital voltmeter measured the output of the detector. The total one-standard-deviation random or type A uncertainty was typically ± 0.004 ppm for each resistor intercomparison. This uncertainty was achieved with a 25.5 μA current after 1.5 h of multiple measurements in one configuration and then another 1.5 h with the resistors interchanged. There was a 15-s wait time and then a 30-s or a 60-s integration period after each current polarity reversal.

We then used DVM1 or DVM2 to measure total voltage drops directly across the QHR wire-wound resistors, as shown in Fig. 1. The standard deviations of the DVM-method results were only about twice as large as those using the Leeds and Northrup detector, and that particular detector is unusually quiet. One would have to measure four times longer with the DVM-method to obtain the same random uncertainty as with the potentiometer-method. This would take about 6 h for each resistor configuration. Typically we measured between 4.5–5.5 h. No DVM calibrations were necessary for 1:1 ratios because the voltages were nominally equal. Therefore $\Delta \overline{d}_{DVM} = 0$ for this situation, as can be seen in (9). The results of the $\overline{d}_{DVM}(\text{cor})$ and $\overline{d}_{POT}$ values obtained for the DVM and potentiometer methods were identical within the ± 0.007 ppm experimental random uncertainties for the fourteen measurements.

We next verified that the HP 3458A multimeters could be used with quantum Hall samples by comparing the nominal 6 453.20-Ω ($i = 4$) quantized Hall resistance of the GaAs/AlGaAs heterostructure sample that now maintains the U.S. ohm with the wire-wound resistor $R_1(4)$. Once again the measured values of $\overline{d}_{DVM}(\text{cor})$ and $\overline{d}_{POT}$ were in agreement for 1:1 ratios. Also, the standard deviations of the DVM-method data were the same whether using the quantum Hall sample or wire-wound resistors. Therefore, the digital multimeters do not appear to disturb the quantum Hall sample significantly at this level of accuracy.

Finally, we measured various 2:1 and 4:1 resistance ratios by the DVM-method, using DVM1 and DVM2, and obtained $\overline{d}_{DVM}(\text{unc})$ values via (5). The digital voltmeters were then calibrated so that the corrected results, $\overline{d}_{DVM}(\text{cor})$, could be compared with those of $\overline{d}_{POT}$ obtained from appropriate combinations of 1:1 ratios of resistors R_1 through R_6 measured by the potentiometer-method.

An example of a 4:1 ratio is $[R_5 + R_6]:R_1$, where $R' \equiv [R_5(2) + R_6(2)] \approx 4R \equiv 4R_1(4)$. It can be shown that this DVM-method ratio can be expressed, in the form $d = \{[R_5(2) + R_6(2)]/4R_1(4) - 1\}$, as combinations of the measurable 1:1 potentiometric-method ratios $R_5:[R_3 + R_4]$, $R_6:[R_3 + R_4]$, $R_3:R_1$, and $R_4:R_1$. The result is

$$1/2\left\{R_5(2)/[R_3(4) + R_4(4)] - 1\right\} + 1/2\left\{R_6(2)/[R_3(4) + R_4(4)] - 1\right\} + 1/2\left\{R_3(4)/R_1(4) - 1\right\} + 1/2\left\{R_4(4)/R_1(4) - 1\right\}.$$

Four 2:1 and four 4:1 ratios were measured for DVM1, and four 2:1 and six 4:1 ratios were measured for DVM2. All eighteen measurements were made using the 1–V DVM range. The data were always reproducible within the ± 0.007 ppm random uncertainties.

The digital multimeters must be calibrated against a Josephson array for ratios that are not 1:1. Correction voltages $C(V)$, expressed in the form of (7), were determined for DVM1 and DVM2 using the least-squares fitting method for the $C(V)$ versus V data. Fig. 2 shows the resulting $\overline{N}(V)$ versus V curves for DVM1, where the $\overline{N}(V)$ data have been averaged for six different calibration runs. The error bars represent the standard deviations of the mean of the individual $N(V)$ data. Values of $\Delta \overline{d}_{DVM}$ and $\overline{d}_{DVM}(\text{cor})$ were then obtained using (9), (5), and (10).

The differences in the results between the DVM and potentiometric methods are $\delta(\text{unc})$, where the DVM voltage ratios have not been corrected, and $\delta(\text{cor})$, where they have. These differences are

$$\delta(\text{unc}) = \overline{d}_{DVM}(\text{unc}) - \overline{d}_{POT} \tag{11}$$

and

$$\delta(\text{cor}) = \overline{d}_{DVM}(\text{cor}) - \overline{d}_{POT}. \tag{12}$$

Table I lists these differences for DVM1 and DVM2, along with the random uncertainties for $\delta(\text{cor})$. The largest discrepancies between the DVM and potentiometric methods were for DVM1. Fig. 3 displays the corrected $\delta(\text{cor})$ results for DVM1 and DVM2. The corrected results for the DVM-method and for the potentiometric-method are in agreement to within ± 0.1 ppm for these two digital voltmeters.

If the input impedance Z of the HP 3458A digital voltmeter is not large enough, then significant current will be shunted around the wire-wound resistors. The resulting shunting error of the ratio R'/rR is approximately $(r - 1)R/Z$ if one neglects the small changes in current that arise when the DVM is moved from position 1) to position 2) of Fig. 1.

The HP 3458A input impedance is specified to be greater than 10^{10} Ω over a wide range of temperature and relative humidity

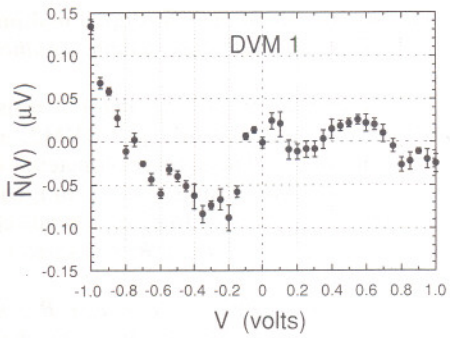

Fig. 2. $\overline{N}(V)$ versus V curve for DVM1. The $N(V)$ are the deviations in μV from the linear least-squares fit to the $C(V)$ versus V data, as given by (7). The $\overline{N}(V)$ are the averages obtained from six different Josephson array calibration runs.

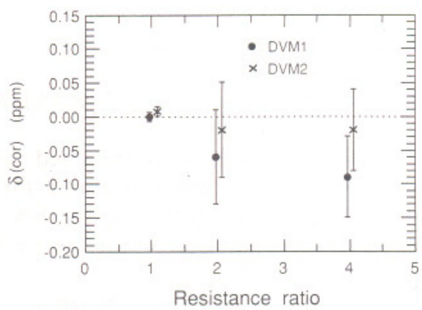

Fig. 3. $\delta(\text{cor}) = \bar{d}_{\text{DVM}}(\text{cor}) - \bar{d}_{\text{POT}}$ results for 1 : 1, 2 : 1, and 4 : 1 resistance ratios using DVM1 and DVM2. $\delta(\text{cor})$ would be zero for exact agreement between the DVM and potentiometric methods. The data for DVM1 and DVM2 are slightly displaced for clarity.

TABLE I
DIFFERENCES IN ppm BETWEEN THE DVM AND POTENTIOMETRIC METHOD RESULTS. THE UNCORRECTED AND CORRECTED DIFFERENCES, $\delta(\text{unc})$ AND $\delta(\text{cor})$, ARE DEFINED IN (11) AND (12)

DVM	r	$\delta(\text{unc})$ (ppm)	$\delta(\text{cor})$ (ppm)
1	2	−0.11	−0.06 ± 0.07
1	4	−0.19	−0.09 ± 0.06
2	2	0.04	−0.02 ± 0.07
2	4	0.04	−0.02 ± 0.06

and to be about 10^{12} Ω at 23°C and 40% relative humidity. If Z was only 10^{10} Ω, then this would give systematic shunting errors of approximately 0.64 ppm and 1.94 ppm, respectively, for $r = 2$ and $r = 4$ ratios when $R \approx 6\,453.2$ Ω. We measured the input impedance by placing a 1.35-V mercury battery in series with a 10^8 Ω resistor R_0 across the input of the HP 3458A multimeter to obtain the DVM voltage V', and then shorting the resistor R_0 to obtain the DVM voltage V. Z is then equal to $R_0/(V/V' - 1)$. We found that Z was 1.4×10^{12} Ω for DVM1 and 7.6×10^{11} Ω for DVM2. This led to 0.005-ppm, 0.014-ppm, 0.008-ppm, and 0.025-ppm corrections for the $r = 2$ and $r = 4$ ratios, respectively, for DVM1 and DVM2. The small input impedance effects of these particular DVMs are therefore masked by the ±0.06–0.07-ppm random uncertainties of our data.

V. DISCUSSION

The DVM calibration data, because of Josephson array operational constraints, were collected for one polarity at a time within about a half hour time period to minimize DVM voltage offset drifts. Therefore, the calibration measurement time is much shorter than the QHR measurement time. This results in larger uncertainties in the DVM calibration measurements than in the QHR measurements. We reduced the calibration uncertainties by averaging several calibration runs. This was a time-consuming task, but it is practical if the DVM calibrations can be shown to be constant over many months. Our preliminary data indicate that this may indeed be the case.

The difference between the DVM and potentiometric method results is $\delta(\text{cor}) = \bar{d}_{\text{DVM}}(\text{unc}) - \Delta\bar{d}_{\text{DVM}} - \bar{d}_{\text{POT}}$. The random uncertainty is dominated by the uncertainty of the $\Delta\bar{d}_{\text{DVM}}$ term given by (9). Our data indicate that, even though the random uncertainties of the $\overline{N}(V)$ values of (9) are about ±0.01 ppm of 1 V, the resulting uncertainty of the $\Delta\bar{d}_{\text{DVM}}$ term is typically ±0.06–0.07 ppm, and that it is not significantly reduced by averaging more than four or five calibration runs. This appears to be the fundamental limitation to this method.

We have used least-squares fitting to obtain the calibration corrections because it provides corrections for all voltages on the 1–V range of the DVM. Another approach would be to make calibrations only for the four specific voltages used in each ratio measurement. One would then use (5), (8), and (10) to determine $\bar{d}_{\text{DVM}}(\text{cor})$, rather than (5), (9), and (10). This would be slightly more difficult because the voltage polarity of the Josephson array would have to be reversed quickly and nearly exactly.

The DVM-method achieves the desired 0.1 ppm accuracy. Therefore this method shows great promise as a secondary quantized Hall resistance standard measurement technique. It could provide a simple and inexpensive method for calibrating 10-kΩ resistors via QHR standards.

ACKNOWLEDGMENT

The authors thank A. C. Gossard of the AT&T Bell Laboratories (now with the University of California at Santa Barbara) who made the MBE-grown GaAs / AlGaAs heterostructure. D. C. Tsui of Princeton University, NJ did the photolithography and made ohmic contacts to the heterostructure sample. G. Marullo Reedtz, a Guest Researcher from the Galileo Ferraris National Electrotechnical Institute, Turin, Italy, designed and built the automated potentiometric comparator and made some very useful comments. C. T. Van Degrift designed and built the new quantum Hall effect laboratory in which many of the measurements were made, and K. C. Lee assisted in writing the computer program for the DVM method.

REFERENCES

[1] K. v. Klitzing, G. Dorda, and M. Pepper, "New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance," *Phys. Rev. Lett.*, vol. 45, pp. 494–497, Aug. 1980.
[2] *BIPM Proc.—Verb. Com. Int. Poids et Mesures*, vol. 56, p. 20, 1988.
[3] M. E. Cage, R. F. Dziuba, C. T. Van Degrift, and D. Y. Yu, "Determination of the time-dependence of Ω_{NBS} using the quantized Hall resistance," *IEEE Trans. Instrum. Meas.*, vol. 38, pp. 263–269, Apr. 1989.
[4] G. W. Small,, B. W. Ricketts, and P. C. Coogan, "A reeval-

uation of the NML absolute ohm and quantized Hall resistance determinations," *IEEE Trans. Instrum. Meas.*, vol. 38, pp. 245–248, Apr. 1989.

[5.] B. V. Hamon, "A 1–100 ohm build-up resistor for the calibration of standard resistors," *J. Sci. Instrum.*, vol. 31, pp. 450–453, Dec. 1954.

[6] F. Delahaye, A. Satrapinsky, and T. J. Witt, "Recent determinations of R_H in terms of $\Omega_{69\text{-BI}}$," *IEEE Trans. Instrum. Meas.*, vol. 38, pp. 256–259, Apr. 1989.

[7] A. Hartland, R. G. Jones, B. P. Kibble, and D. J. Legg, "The relationship between the SI ohm, the ohm at NPL, and the quantized Hall resistance," *IEEE Trans. Instrum. Meas.*, vol. IM-36, pp. 208–213, June 1987.

[8] F. Delahaye, "Technical guidelines for reliable measurements of the quantized Hall resistance," *Metrologia*, vol. 26, pp. 63–68, 1989.

[9] J. Kinoshita, K. Inagaki, Y. Murayama, T. Endo, C. Yamanouchi, K. Yoshihiro, J. Wakabayashi, and S. Kawaji, "An improved Josephson potentiometer system for the measurement of the quantum Hall effect," *IEEE Trans. Instrum. Meas.*, vol. IM-36, pp. 230–233, June 1987.

[10] T. J. Witt, T. Endo, and D. Reymann, "The realization of the quantum Hall resistance at the BIPM," *IEEE Trans. Instrum. Meas.*, vol. IM-36, pp. 234–239, June 1987.

[11] K. Inagaki, Y. Sakamoto, and T. Endo, "Accuracy of measurements of the quantized Hall resistivity by a direct current comparator type potentiometer: calibration using a Josephson potentiometer," *IEEE Trans. Instrum. Meas.*, vol. 38, pp. 276–278, Apr. 1989.

[12] R. L. Steiner and B. F. Field, "Josephson array voltage calibration system: operational use and verification," *IEEE Trans. Instrum. Meas.*, vol. 38, pp. 296–301, Apr. 1989.

[13] G. Marullo Reedtz and M. E. Cage, "An automated potentiometric system for precision measurement of the quantized Hall resistance," *J. Res. Natl. Bur. Stand.*, vol. 92, pp. 303–310, Sept. 1987.