MULTIPORT NOISE CHARACTERIZATION AND DIFFERENTIAL AMPLIFIERS*

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Abstract: I address the issue of the definition and measurement of noise figure and parameters to characterize multiport devices, particularly differential amplifiers. A parameterization in terms of the noise matrix appears to be the most practical. The noise figure for a given output port is defined and related to the noise matrix and scattering parameters of the device. As an example, the case of a differential amplifier with reflectionless terminations is considered in detail. The noise figure, effective input temperature, and gains are related to the results of a series of hot-cold measurements, as in the familiar two-port case.

1. INTRODUCTION

There are several equivalent parameterizations for the noise characteristics of two-port amplifiers and transistors, including the well known IEEE noise parameters [1,2] and their variants, the noise matrix in either its voltage-current [3] or its wave amplitude [4,5] incarnation, and the terminal-invariant set of Engen [6]. The noise figure or effective input noise temperature of a two-port amplifier as a function of source impedance or reflection coefficient can be expressed in terms of any of these sets. For more than two ports, or for more than one mode in a port, the situation is not so well developed. The basic multiport noise-matrix formalism was introduced long ago [3], but the expression of multiport noise figures in terms of a common set of parameters has not been developed. Even the definition of multiport noise figures is not well established. The IEEE definitions [7] allow for multiple input ports as well as different input and output frequencies (since they were developed with receivers in mind), but they are restricted to one output port, and even for that case they stop short of defining a noise figure. Differential amplifiers present two complications not included in the two-port noise figure definition and parameterization: multiple input ports and a signal input channel which is a linear combination of the two physical input channels. Other multiports, such as mixed-mode two ports [8] introduce the additional complication of multiple output ports. The widespread use of such components in cell phones and other applications makes it desirable to have a convenient standard description of their noise characteristics. Such a description should be simple, have a physical basis or interpretation, and reduce to a familiar form for the two-port case.

This paper suggests a description of noise in differential amplifiers and other multiports based on a wave description of the noise matrix [4,5]. A more complete account of the work will be published elsewhere [9]. Our interest is in multiple (especially two) input and output ports, at a single frequency, with all ports at the same frequency. A definition of the noise figure for a given port is suggested, and that noise figure is expressed in terms of the noise matrix and the

S-parameters of the amplifier. As an example, the simple case of a three-port differential amplifier with reflectionless terminations is considered in detail. For this case, the noise figure, effective input temperature, and gains are related to the results of a series of hot-cold measurements, as in the familiar two-port case. The next section reviews the noise matrix formalism as applied to multiports and goes on to develop a definition of the noise figure for each output port of a linear multiport device. Section 3 treats the special case of a three-port differential amplifier with reflectionless terminations, and Section 4 is devoted to a summary.

2. FORMALISM

2.1 Noise Matrix

For purposes of this paper, a port will refer to a single mode in a single physical port. Multiple modes in a single physical port are treated as multiple ports. Thus a four-port may refer to an amplifier with two input and two output ports or to an amplifier with two different modes in a single input port and two in an output port (or to some combination of these). Our primary interest is in three-port and four-port amplifiers, but in principle the formalism applies to any N greater than one. All the work in this paper is in terms of wave amplitudes. They may be defined in terms of voltages and currents [4,8,10], or they may be introduced and used with no reference to voltage and current [5,11]. Details of the modes and waves are not of concern. What are important are two general properties. In order for the formalism of this paper to be valid, the modes must be power-orthogonal, *i.e.*, the total power across a reference plane must be the sum of the powers in each of the individual modes or ports. If this is not the case, and the total power contains cross terms between the modes, it is possible to regain power orthogonality by a linear transformation, at least for lossless or low-loss lines [12]. The second general property that we assume about the waves is that they can be physically generated in practical applications, otherwise the discussion of measurements based on these waves is purely academic.

A linear N-port amplifier can be represented by its N×N scattering matrix ($\boldsymbol{S}$) and an N-vector of internal (noise) sources ($\hat{\boldsymbol{b}}$) ,

$$\boldsymbol{b} = \boldsymbol{S}\,\boldsymbol{a} + \hat{\boldsymbol{b}}, \tag{1}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are N-vectors of the usual incident and outgoing wave amplitudes. The *i*th element of $\hat{\boldsymbol{b}}$, $\hat{b}_i$, is the amplitude of the generator wave at port *i*, which would be the output noise amplitude for reflectionless terminations and no input noise. We work in a normalization such that the spectral power density is given by the square of the absolute value of the wave amplitude. We assume that the noise amplitudes are approximately constant in a small bandwidth (1 Hz, for example) around the frequency of interest, and we have divided out that bandwidth. We also use bold characters to indicate vectors or matrices. The incident wave vector can be written as

$$\boldsymbol{a} = \boldsymbol{\Gamma}\,\boldsymbol{b} + \hat{\boldsymbol{a}}. \tag{2}$$

where $\hat{\boldsymbol{a}}$ is the vector of generator waves of the sources connected to the N ports, and $\boldsymbol{\Gamma}$ is the N×N matrix of reflection coefficients. Combining eqs. (1) and (2) in the usual manner yields the expression for the outgoing wave vector in terms of the generator waves,

$$\boldsymbol{b} = [\mathbf{1} - \boldsymbol{S}\boldsymbol{\Gamma}]^{-1}[\boldsymbol{S}\hat{\boldsymbol{a}} + \hat{\boldsymbol{b}}]\,. \tag{3}$$

We can then define the noise matrix (or noise correlation matrix) and write it in terms of the intrinsic parameters of the N-port and the properties of the N terminations on its ports. We can define two distinct noise matrices: an intrinsic noise matrix, which depends only on the properties of the amplifier itself, and an *in situ* noise matrix, which depends on the characteristics of the circuit in which the amplifier is embedded. The full, *in situ* noise matrix $\boldsymbol{N}$ is defined as

$$\boldsymbol{N} \equiv \overline{\boldsymbol{b}\,\boldsymbol{b}^{\dagger}}\,, \tag{4}$$

or

$$N_{ij} = \overline{b_i b_j^{\,*}}\,, \tag{5}$$

where the bar indicates either an ensemble or time average (assumed equal), and † indicates hermitian conjugate. Diagonal elements of the noise matrix give the power spectral density of the output noise in the respective port, while off-diagonal elements are the correlation between the output noise in different ports. We can use eq. (3) to write the noise matrix in terms of the generator waves,

$$\boldsymbol{N} = \overline{\boldsymbol{b}\,\boldsymbol{b}^{\dagger}} = [\mathbf{1} - \boldsymbol{S}\boldsymbol{\Gamma}]^{-1}\boldsymbol{S}\,\overline{\hat{\boldsymbol{a}}\hat{\boldsymbol{a}}^{\dagger}}\boldsymbol{S}^{\dagger}[\mathbf{1} - \boldsymbol{S}\boldsymbol{\Gamma}]^{-1\dagger} + [\mathbf{1} - \boldsymbol{S}\boldsymbol{\Gamma}]^{-1}\overline{\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}^{\dagger}}[\mathbf{1} - \boldsymbol{S}\boldsymbol{\Gamma}]^{-1\dagger}\,, \tag{6}$$

where we have used the fact that the generator waves from the amplifier are uncorrelated with those from the terminations. The first term is due to the noise (and signal) from the sources and terminations connected to the ports, and the second term is due to the noise generated by the amplifier itself, suitably modified by reflections from the terminations. In the absence of any external noise ($\hat{\boldsymbol{a}} = \mathbf{0}$) the first term vanishes, and we are left with only the amplifier noise represented by the second term. Conversely, for a noiseless amplifier $\hat{\boldsymbol{b}} = \mathbf{0}$, and only the first term is present.

The *intrinsic* noise matrix is defined by

$$\hat{\boldsymbol{N}} = \overline{\hat{\boldsymbol{b}}\,\hat{\boldsymbol{b}}^{\dagger}}\,. \tag{7}$$

It is the noise matrix which would occur if all terminations of the amplifier were reflectionless and noiseless. Because the intrinsic noise matrix (supplemented by the scattering matrix) contains full information on the intrinsic noise parameters of the amplifier, we find it useful to introduce a more physical parameterization of it. It will also be convenient to have a more compact notation. For the diagonal elements, we associate a characteristic noise temperature with each port,

$$\overline{|\hat{b}_i|^2} = k_B\,\hat{T}_i\,, \tag{8}$$

where k_B is Boltzmann's constant. Although $\hat{T}_i$ is called a characteristic noise temperature, it does not actually correspond to the noise temperature of port i for any simple physical case. The quantity $k_B\hat{T}_i$ is the noise power which would be *delivered* to a noiseless, reflectionless load attached to port i if all other ports were also terminated in noiseless, reflectionless loads. The noise temperature of port i is related to the *available* noise power, and for the case of all noiseless, reflectionless terminations the noise temperature is given by $T_{i,0} = \hat{T}_i/(1-|S_{ii}|^2)$.

The off-diagonal elements of the intrinsic noise matrix are appropriately scaled correlation functions,

$$\overline{\hat{b}_i \hat{b}_j^*} = k_B \sqrt{\hat{T}_i \hat{T}_j}\ \rho_{ij}\,. \qquad \textbf{(9)}$$

The absolute values of the ρ_{ij} can range from 0 to 1, as befits a correlation function.

2.2 Noise Figure Definition

We are now prepared to consider the definition of the noise figure and its expression in terms of the intrinsic parameters of the amplifier and its terminations. We deal only with the noise figure at a single frequency, the spot noise figure. The IEEE definition of operating noise temperature and effective noise temperature for multiple input ports [7] stops short of defining a noise figure for multiple inputs. For a two-port, the IEEE definition [1] is that the noise figure (or noise factor) at a given frequency is the ratio of total output noise power per unit bandwidth to the portion of the output noise power which is due to the input noise, evaluated for the case where the input noise power is $k_B T_0$ (T_0 = 290 K). Equivalently, it is one plus the ratio of the output noise due to the amplifier to the output noise due to T_0 input noise. This definition can be generalized to the N-port case in a fairly straightforward manner. In principle, we can define a noise figure F_i for each port i, but in practice we will consider noise figures only for output ports. As in the two-port case, we define the noise figure of a given port to be the ratio of the total output noise in that port to the output noise power which is due to the input noise for the case when the input noise is T_0. Ambiguities arise in the details of how the input ports are terminated. Is T_0 input to all the input ports, and if so, is the input noise to different ports correlated? How are the other output ports terminated; is T_0 input to them as well? And for differential amplifiers, is the noise input to the physical ports 1 and 2 or to the differential and common modes?

The definition which is likely to bear the most resemblance to actual conditions is for T_0 to be input to each of the input ports and for this reference input noise at each physically separate port to be uncorrelated with the input noise at each other port. To require that the noise input to separate ports be correlated could require difficult measurement techniques, and in most practical situations the input noise to different ports will not be correlated. Even in the case of a differential amplifier, where one might think that the noise in the differential mode might be correlated with that in the common mode (since they receive contributions from common sources), the correlation between the two vanishes approximately. To see this, let a_1 and a_2 be the wave amplitudes in the two wires relative to ground, and define the differential mode $a_- = (a_1 - a_2)/\sqrt{2}$ and common mode $a_+ = (a_1 + a_2)/\sqrt{2}$. Then the correlation between the two is $\overline{a_+ a_-^*} = (\overline{|a_1|^2} - \overline{|a_2|^2})/2$, which is the difference between the noise powers in the two wires. This vanishes if the two wires are at the same temperature. (The cross terms vanish because fluctuations in the two wires are uncorrelated.) Consequently, in our definition of noise figure, we specify that the reference case be T_0 applied to each input port and that they be uncorrelated. This will be discussed further in the example in the following section.

For the terminations of the output ports, we follow the spirit of the two-port definition, namely that the noise figure measures the noise added by the amplifier for a given choice of reflection coefficients for the input terminations, but it should not include noise contributions from the various output loads. (Note, however, that the IEEE definition of the operating temperature does include such contributions [7].) We will (tentatively) adopt the convention that no noise source is connected to the output ports. In practice, it should make little difference, since the isolation between the different output ports should be great enough that the output of a given port would be insensitive to whether T_0 is applied to some other output port, especially considering that T_0 is applied to the input channels, which are being amplified.

The definition of the noise figure for a given output channel is then complete. In terms of the notation introduced in the preceding section, it takes the form

$$F_i = 1 + \frac{\{[\mathbf{1} - \boldsymbol{S\Gamma}]^{-1}\,\overline{\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}^{\dagger}}\,[\mathbf{1} - \boldsymbol{S\Gamma}]^{-1\dagger}\}_{ii}}{\{[\mathbf{1} - \boldsymbol{S\Gamma}]^{-1}\,\boldsymbol{S}\,\overline{\hat{\boldsymbol{a}}\hat{\boldsymbol{a}}^{\dagger}(\boldsymbol{T_0})}\,\boldsymbol{S}^{\dagger}\,[\mathbf{1} - \boldsymbol{S\Gamma}]^{-1\dagger}\}_{ii}}\,, \tag{10}$$

$$\overline{\hat{\boldsymbol{a}}\hat{\boldsymbol{a}}^{\dagger}(\boldsymbol{T_0})} = (k_B\,T_0)\times\mathbf{1}\,,$$

where the subscript i's indicate the element of the matrix within the braces.

The definition of eq. (10) reduces to the usual definition for the two-port case and embodies the intuitive idea that the noise figure measures how much noise the amplifier adds to a 290 K reference signal. If it seems a bit formal at this point, it should become clearer in the next section, when we work through the $\boldsymbol{\Gamma} = 0$ case in detail. An alternate definition, which may be more appropriate for signal-to-noise considerations, will be discussed in [9].

Besides defining the noise figure, eq. (10) constitutes a parameterization of its dependence on the reflection coefficients of the sources and loads. The noise parameters are the independent elements of the intrinsic noise matrix. For a three-port amplifier, there would be nine real parameters: three characteristic noise temperatures and three complex correlation functions. In principle, one could develop a parameterization analogous to the IEEE parameterization for two-port noise figure or effective noise temperature. This would also require nine real parameters: a minimum noise figure, optimal complex values for the reflection coefficients of the two sources, and four parameters describing the rate of variation of the noise figure as the reflection coefficients deviated from their optimal values. This set of nine parameters could be expressed in terms of the elements of the noise matrix and the S-parameters of the amplifier, but that is well beyond the scope of this paper.

3. EXAMPLE: DIFFERENTIAL AMPLIFIER, $\boldsymbol{\Gamma} = 0$

A differential amplifier is a three-port device with a single output port whose signal (ideally) is proportional to the difference between the signals at the two input ports. Let the output port be port 3, and define input waves and S-parameters to describe the common (+) and differential (−) modes,

$$a_{\pm} \equiv (a_1 \pm a_2)/\sqrt{2}\,,$$
$$S_{3\pm} = (S_{31} \pm S_{32})/\sqrt{2}\,. \qquad \textbf{(11)}$$

We can then write the output amplitude at port 3 as

$$b_3 = S_{3-}\,a_{-} + S_{3+}\,a_{+} + \hat{b}_3\,, \qquad \textbf{(12)}$$

where ideally $S_{3+} = 0$. One immediate, important consequence of the definitions of eq. (11) is that if the noise waves represented by a_1 and a_2 are uncorrelated, then the noise temperatures input to the common and differential modes are equal, $\hat{T}_{+} = |a_{+}|^2/k_B = |a_{-}|^2/k_B = \hat{T}_{-}$. Therefore, to obtain different input noise temperatures for the common and differential modes requires correlated noise sources for ports 1 and 2.

We consider the simple case of all ports terminated with matched (reflectionless) loads or sources. Because there are no reflections from the terminations, the off-diagonal elements of the noise matrix do not contribute to the output noise at port 3, nor do the characteristic noise temperatures of the input ports, $\hat{T}_1$ and $\hat{T}_2$. Only $\hat{T}_3$, the characteristic noise temperature of port 3, contributes to the output noise, just like the case of a two-port amplifier with reflectionless terminations.

The average noise power per unit bandwidth emerging from port 3 is given by

$$N_3 = \overline{|S_{31}\,a_1 + S_{32}\,a_2 + \hat{b}_3|^2}\,. \qquad \textbf{(13)}$$

If two uncorrelated noise sources with noise temperatures $T_1^{(i)}$ and $T_2^{(i)}$ are input to ports 1 and 2, eq. (13) becomes

$$N_3 = G_{31}\,T_1^{(i)} + G_{32}\,T_2^{(i)} + \hat{T}_3\,, \qquad \textbf{(14)}$$

where $G_{31} = |S_{31}|^2$, and k_B is understood to multiply the noise temperatures.

The unknown parameters in eq. (14) can be determined from a series of hot/cold measurements similar to the two-port case. Let T_{h1} denote the noise temperature of the hot source connected to port 1, etc. In principle T_{h1} and T_{h2} could be equal, and T_{c1} and T_{c2} probably will be equal to the ambient temperature and therefore to each other, but we begin with the general case. There are then four different measurements that can be performed. Let $N_{3,hc}$ be the output noise power per unit frequency measured at port 3, for a hot source on port 1 and a cold source on port 2. $N_{3,hh}$, $N_{3,ch}$, and $N_{3,cc}$ are defined in a similar manner. The results of the four measurements are then given by

$$
\begin{aligned}
N_{3,hh} &= G_{31} T_{h1} + G_{32} T_{h2} + \hat{T}_3 , \\
N_{3,hc} &= G_{31} T_{h1} + G_{32} T_{c2} + \hat{T}_3 , \\
N_{3,ch} &= G_{31} T_{c1} + G_{32} T_{h2} + \hat{T}_3 , \\
N_{3,cc} &= G_{31} T_{c1} + G_{32} T_{c2} + \hat{T}_3 .
\end{aligned}
\tag{15}
$$

where Boltzmann's constant is again understood to multiply all the noise temperatures. There are only three unknowns in the four equations of (15), G_{31} , G_{32} , and $\hat{T}_3$; and consequently the equations are not all independent. Indeed, one notes that

$$
N_{3,hh} + N_{3,cc} = N_{3,hc} + N_{3,ch} . \tag{16}
$$

Therefore, it is sufficient to measure only three of the four hot/cold combinations to determine the gains and $\hat{T}_3$. The set of hc, ch, and cc may give slightly better accuracy, and it requires only one hot noise source, so we begin with that set. The measured values for the gains are then

$$
\begin{aligned}
G_{31} &= \frac{N_{3,hc} - N_{3,cc}}{T_{h1} - T_{c1}} , \\
G_{32} &= \frac{N_{3,ch} - N_{3,cc}}{T_{h2} - T_{c2}} ,
\end{aligned}
\tag{17}
$$

and the intrinsic output noise temperature for port 3 is given by

$$
\hat{T}_3 = \frac{(T_{h1} T_{h2} - T_{c1} T_{c2})}{(T_{h1} - T_{c1})(T_{h2} - T_{c2})} N_{3,cc} - \frac{T_{c1}}{(T_{h1} - T_{c1})} N_{3,hc} - \frac{T_{c2}}{(T_{h2} - T_{c2})} N_{3,ch} . \tag{18}
$$

The equivalent input temperature, which is equal for the two input ports [7], is given by

$$
T_e = \frac{\hat{T}_3}{G_{31} + G_{32}} . \tag{19}
$$

Assuming the two cold temperatures are equal, this can be written as

$$
T_e = \frac{T_{h1} T_{h2} - (T_{h1} Y_{ch} + T_{h2} Y_{hc}) T_c + Y_{hh} T_c^2}{(Y_{ch} - 1) T_{h1} + (Y_{hc} - 1) T_{h2} - (Y_{hh} - 1) T_c} , \tag{20}
$$

where $Y_{hc} = N_{3,hc}/N_{3,cc}$, etc. Although $N_{3,hh}$ may not have been measured, Y_{hh} can be determined from eq. (16), $Y_{hh} = Y_{hc} + Y_{ch} - 1$. If we further assume that only one hot noise source is used, so that $T_{h1} = T_{h2} = T_h$, eq. (20) reduces to

$$
T_e = \frac{T_h - Y_{hh} T_c}{Y_{hh} - 1} , \tag{21}
$$

which is the familiar two-port result, with Y_{hh} playing the role of the two-port Y.

Equation (21) indicates that T_e can be determined either from the set of three measurements (hc, ch, and cc) or from just two measurements (hh and cc) if $T_{h1} = T_{h2}$. If only hh and cc are measured, we can still determine the sum of the gains and $\hat{T}_3$,

$$G_{31} + G_{32} = \frac{N_{hh} - N_{cc}}{T_h - T_c} ,$$
$$\hat{T}_3 = \frac{(T_h - Y_{hh} T_c)(T_h - T_c)}{(Y_{hh} - 1)(N_{hh} - N_{cc})} , \tag{22}$$

but we cannot determine either gain separately as in eq. (17).

The discussion in this section has not yet treated the differential or common mode, nor has it mentioned noise figure. From $G_{3\pm} = |S_{3\pm}|^2$ and eqs. (11) and (12), it follows that

$$G_{3+} + G_{3-} = G_{31} + G_{32} . \tag{23}$$

Since $\hat{T}_3$ is the same no matter how we describe the input ports and since the sum of the gains is the same, the effective input noise temperature in the differential and common modes is the same as for ports 1 and 2. The hot-cold measurements with uncorrelated sources, described above, are therefore sufficient to determine $\hat{T}_3$, T_e , and $G_{3+} + G_{3-}$ for the differential and common modes, but not G_{3+} or G_{3-} individually. Since G_{3-} is designed to be much larger than G_{3+} , we might use the approximation $G_{3-} \approx G_{31} + G_{32}$, but it would be useful to measure G_{3+} or G_{3-} independently. Using noise to measure G_{3+} or G_{3-} requires correlated noise input to ports 1 and 2. If $a_1 = a_2$, then $a_- = 0$ and $a_+ = \sqrt{2}\, a_1$, which in turn leads to $T_- = 0$ and $T_+ = 2\, T_1$. If the measured noise power out of port 3 in such a measurement is called $N_{3,+}$, then

$$N_{3,+} = 2 G_{3+} T_1 + \hat{T}_3 , \tag{24}$$

from which it follows that

$$G_{3+} = \frac{(G_{3+} + G_{3-})}{2 T_1} [T_c Y_+ + T_e (Y_+ - 1)] , \tag{25}$$

where $Y_+ = N_{3,+} / N_{3,cc}$. All the quantities on the right side except Y_+ can be determined from the uncorrelated measurements described above.

Once all the relevant parameters have been measured, as in the preceding subsection, we can compute the noise figure of the differential amplifier. For two input ports and one output port, all with reflectionless terminations, eq. (10) reduces to

$$F_3 = 1 + \frac{\overline{|\hat{b}_3|^2}}{|S_{31}|^2\, \overline{|\hat{a}_1|^2} + |S_{32}|^2\, \overline{|\hat{a}_2|^2}}$$
$$= 1 + \frac{\hat{T}_3}{(G_{31} + G_{32})\, T_0} = 1 + \frac{T_e}{T_0} . \tag{26}$$

Note that this definition for the noise factor has the advantage that it does not require separate measurement of G_{3+} or G_{3-} , and thus does not require any measurements with correlated noise input.

To summarize the matched case, with just one hot source and two equal-temperature cold sources, a set of three measurements (hc, ch, and cc) with uncorrelated input noise will determine $\hat{T}_3$, T_e , G_{31} , G_{32} , and $G_{3+} + G_{3-}$. (Obviously, if a second hot source is available, hh could be done as a consistency check or to reduce the uncertainty.) If two equal-temperature hot sources and two equal-temperature cold sources are available, then just two measurements (hh and cc) suffice to determine $\hat{T}_3$, T_e , $G_{31} + G_{32}$, and $G_{3+} + G_{3-}$, but not any individual gain. To determine G_{3-} or G_{3+} individually (in a noise measurement) requires the noise input to ports 1 and 2 to be correlated. To determine the noise factor, as defined by eq. (10), it is sufficient to measure T_e .

4. SUMMARY

A formalism based on the wave-amplitude form of the noise matrix was presented for multiport amplifiers, particularly differential amplifiers. The noise figure for an output channel was defined and written in terms of the noise matrix and scattering parameters of the amplifier and the reflection coefficients of the terminations. Two special cases were considered, a three-port differential amplifier with reflectionless terminations and a four-port mixed-mode amplifier, also with reflectionless terminations. For each case, the noise figures, effective input temperatures, and gains were related to the results of a series of hot-cold measurements, as in the familiar two-port case. In both examples, the off-diagonal elements of the intrinsic noise matrix, the correlation coefficients ρ_{ij} , were not determined, since they do not affect the noise figure in the $\Gamma = 0$ case. To characterize a multiport amplifier for nonzero reflection coefficients, additional measurements would be required to measure the ρ_{ij} . That more general case is left for future work.

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