TH3E-5

SIMULATIONS OF NOISE-PARAMETER UNCERTAINTIES*

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Abstract — **This paper reports results for uncertainties obtained from a Monte Carlo simulation of noise-parameter measurements. The simulator permits the computation of the dependence of the uncertainty in the noise parameters on uncertainties in the underlying quantities. Results have been obtained for the effect due to uncertainties in the reflection coefficients of the input terminations, the noise temperature of the hot noise source, connector variability, the ambient temperature, and the measurement of the output noise. Representative results are presented for both uncorrelated and correlated uncertainties in the underlying quantities.**

I. INTRODUCTION

Propagation of uncertainty in measurements of amplifier or device noise parameters can be a complicated task that does not admit an analytical solution. The dependence of the noise parameters on the measured quantities is generally nonlinear, and the noise parameters are typically determined by a least-squares fit to an overdetermined system of equations. Monte Carlo methods are well suited to such problems. They have been used to compare different choices of input terminations [1]-[3] in noise-parameter measurements, and recently they have been used to study the dependence on the uncertainties in the underlying quantities [4], [5].

The present paper extends the work of [4] and [5] in several respects. The possibility of correlations among uncertainties in the underlying quantities has been added to the simulator, as has the choice of either a Gaussian or a rectangular distribution for uncertainties in the ambient temperature. The presence of correlations in particular can lead to important effects in the final uncertainties. Also, a different analysis program has been used. The analysis program used in the earlier work lumped together the device under test (DUT) and the receiver used in the measurement. The uncertainties in the noise parameters were obtained by assuming that the DUT and the receiver could be disentangled without the introduction of any additional uncertainty. Equivalently, the uncertainties arising from the power measurement were all contained in one power uncertainty, assuming a perfectly matched, noiseless power meter. The present work uses a different analysis program, which includes a full and realistic estimate for the uncertainty in measurement of the output of the DUT for the different input sources.

The following section briefly reviews the simulator and the measurement process to be simulated. Section III presents representative results obtained for the noise-parameter uncertainties, and Section IV summarizes the results and discusses possible future directions.

II. MODEL AND PROCEDURES

The measurement process to be simulated is a variation of the one originally proposed by Adamian and Uhlir [6]. A number of different terminations of known reflection coefficient Γ_i and noise temperature T_i are connected to the input of the DUT, and the output noise is measured for each. An equation relates the output noise to what we call the underlying quantities (Γ_i, T_i, and the amplifier's S-parameters) and to the parameters to be determined (the four noise parameters and the gain). More than five different input terminations are used, resulting in an overdetermined system of equations, which is solved for the five unknown parameters by a least-squares fit. In this paper, the output noise *temperature* is measured, and the radiometer used to measure it has already been calibrated. This constitutes a minor departure from [6], as well as from [4] and [5], where the output *power* was measured. The noise temperature is chosen in the present case because it corresponds to the quantity measured by the radiometers at NIST, which is the application of most interest to the author. Most of the results obtained should also apply to the case in which the output noise power is measured.

There are several different parameterizations of the dependence of an amplifier's effective input noise temperature T_e on the impedance or reflection coefficient of the input termination. The particular parameterization used in this paper is one of the common variants of the IEEE set of parameters [7],

$$T_e = T_{e,\min} + t\frac{\left|\Gamma_{opt} - \Gamma_G\right|^2}{\left|1+\Gamma_{opt}\right|^2 (1-\left|\Gamma_G\right|^2)}\,, \qquad (1)$$

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where Γ_G is the reflection coefficient of the input termination, and the four noise parameters are $T_{e,\min}$, t, and the real and imaginary parts of the optimal reflection coefficient Γ_{opt}.

In studies of noise-parameter measurements, there are a myriad of variables whose interdependent effects can be studied. The current paper focuses on the dependence of the noise-parameter and gain uncertainties on the uncertainties in the underlying quantities, for both correlated and uncorrelated uncertainties. For the other variables entering the problem, typical or representative values are chosen. Thus, for the set of input terminations we chose 13 terminations, one of them hot, the rest at ambient temperature. Their reflection coefficients were distributed in the complex plane as shown in Fig. 1, where point 1 is the hot termination. Similarly, we are not studying the manner in which the uncertainties depend on the actual noise parameters themselves, so we consider just one particular set of noise parameters, measured for a low-noise amplifier at a single frequency. The values used for the "true" values are $G = 2399$ (33.80 dB), $T_{e,min} = 109.6$ K ($F_{min} = 1.392$ dB), $\Gamma_{opt} = 0.050 + 0.142$ j, and $t = 176.3$ K.

A good description of the use of Monte Carlo simulation for uncertainty analysis is given in reference [8]. For the simulation, we first chose "true" values for the underlying quantities. These comprise the noise and scattering parameters of the amplifier and the noise temperature and reflection coefficient of each termination. We then chose uncertainties for the S_{ij}, $T_{G,i}$, $\Gamma_{G,i}$, and $T_{out,i}$. All measurement distributions were taken to be Gaussian except for the ambient temperature, whose distribution could be either rectangular or Gaussian. We also chose a value for the connector variability.

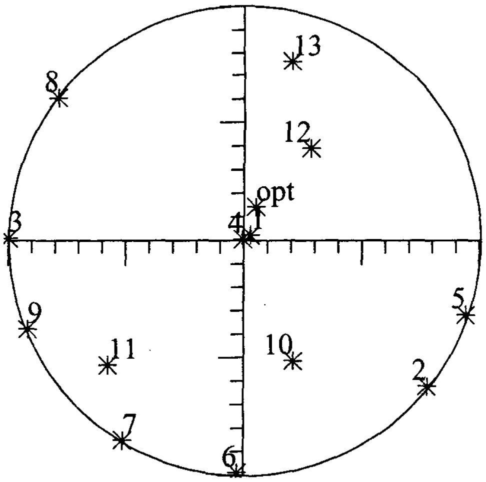

Fig. 1 Distribution of reflection coefficients of terminations in and on the unit circle.

We generated simulated measured values for the S_{ij}, $T_{G,i}$, and $\Gamma_{G,i}$ in the standard manner, randomly choosing a value from a Gaussian (or rectangular, for ambient temperatures) distribution centered at the true value. For the complex quantities, real and imaginary parts were generated independently. To generate the simulated noise-temperature measurement, we first calculated the true output noise temperature from the equation for output temperature, using the true values for the noise parameters and the termination noise temperatures, and using the true values for the S-parameters and the reflection coefficient *for that connection*. Once the true output temperature for the given connection was calculated, the measured value was generated with the uncertainty in the noise-temperature measurement used as the standard deviation. A complete simulated measurement set then consisted of the measured values for S_{ij} and the measured $T_{G,i}$, $\Gamma_{G,i}$, and $T_{out,i}$ for each of the $N_{meas} = 13$ terminations.

The complete simulated measurement set was analyzed and the noise parameters and gain determined in the same way as for a real data set. A weighted least-squares fitting routine was used. To assess the uncertainties in the noise parameters, we generated a large number N_{sim} of simulated measurement sets with the given uncertainties in the underlying quantities. Each simulated measurement set was analyzed to produce a set of "measured" noise parameters, yielding N_{sim} measured values for each parameter. The average and standard deviation of the measured values were computed. The uncertainty in a single measurement of a parameter was then computed by combining the standard deviation in quadrature with the difference between the average and the true value. (Statistics for Γ_{opt} were computed on real and imaginary parts, not on magnitude and phase.) For all the results in this paper, $N_{sim} = 1000$ was used.

III. RESULTS

There are five "measured" parameters whose uncertainties we wish to determine (four real noise parameters plus the gain), and there are four underlying variables whose correlated and uncorrelated uncertainties can be varied. That leads to far more different combinations than can be treated in this relatively short paper. Only a few of the more interesting or representative results will be shown. Some approximate general features can be summarized without resorting to figures: the uncertainties in G and T_{min} are dominated by the uncertainty in T_{hot}; the uncertainties in Γ_{opt} are dominated by the uncertainties in the reflection coefficients of the input terminations; t is sensitive to just about everything; and the uncertainty in T_{amb} has very little effect on any of the measured parameters (though it may, of course, affect the actual properties of the device itself).

Selected results are shown in Figs. 2 – 4. To isolate the effect of a single underlying uncertainty, these figures show the dependence on one underlying uncertainty, with all other underlying uncertainties set to zero. Figure 2 shows the dependence of the uncertainty in the gain on the fractional uncertainty in the measurement of hot noise temperatures for both the case with the errors in all hot noise temperatures completely uncorrelated, and the case with the errors in the hot noise temperatures perfectly correlated. The fractional uncertainty in the hot noise temperature applies both to the hot source used as one of the input terminations and to the measurements of the output noise temperatures. Figure 2 indicates that the uncertainty in measuring the noise temperature has a major effect on the uncertainty in the gain, as would be expected. What may be rather surprising is that if the uncertainties in the noise-temperature measurements are all perfectly correlated, the resulting uncertainty in the gain is very small. This can be understood by recalling that the gain is determined primarily by a ratio of differences, and correlated errors cancel in taking the difference of two noise-temperature measurements. A similar, but less

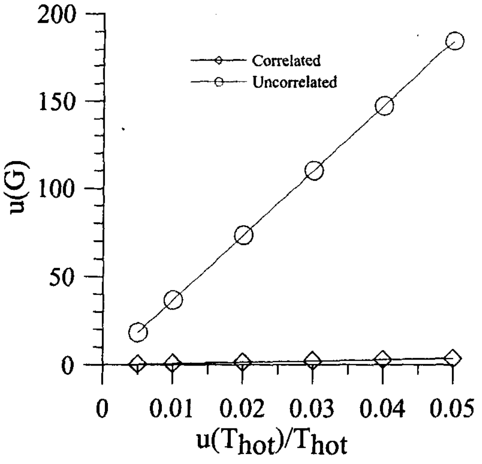

Fig. 2 Dependence of the uncertainty in the gain on the uncertainty in measurement of hot noise temperatures, for correlated and uncorrelated uncertainties in the hot noise-temperature measurements.

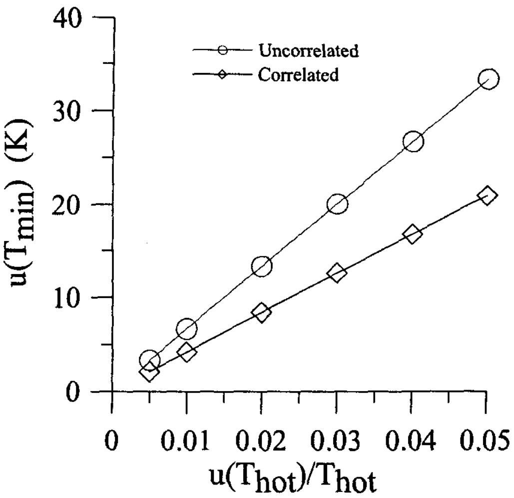

Fig. 3 Dependence of the uncertainty in T_{min} on the uncertainty in measurements of hot noise temperatures.

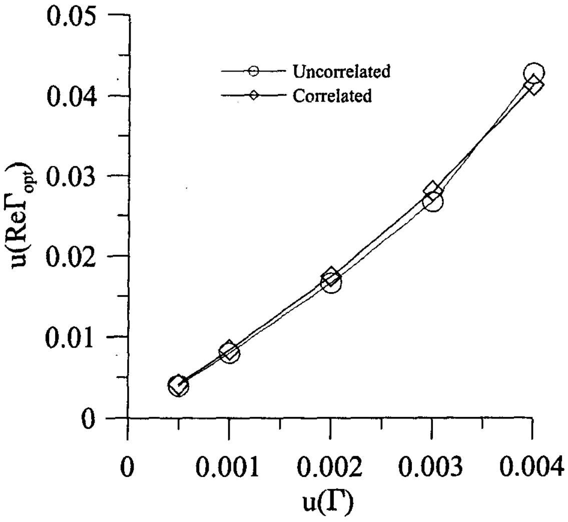

Fig. 4 Dependence of the uncertainty in $\mathrm{Re}\Gamma_{opt}$ on the uncertainty in the reflection coefficients of the input terminations.

pronounced, effect occurs for the uncertainty in T_{min}, Fig. 3. For those accustomed to measuring the characteristics in decibels, an uncertainty in G of 100 (for $G = 2400$) corresponds to about 0.18 dB, and an uncertainty of 20 K in T_{min} (for $T_{min} = 110$ K) corresponds to an uncertainty of approximately 0.2 dB in the minimum noise figure. The uncertainty in the real part of Γ_{opt} is shown in Fig. 4 as a function of the uncertainty in the real or imaginary part of the reflection coefficients of the input terminations. (Uncertainties in the real and imaginary parts of the input reflection coefficients were taken to be equal and uncorrelated.)

The Monte Carlo program can also be used to compare different measurement strategies. As a practical example, the effect of using a cold noise source instead of the hot noise source was computed. For a cold noise source (78 K) with a fractional uncertainty somewhat larger than that of the hot noise source, the cold noise source led to smaller uncertainty for T_{min}, but larger uncertainty for G. Use of both the cold and the hot noise source resulted in significantly smaller uncertainties for both T_{min} and G.

IV. SUMMARY AND PLANS

A Monte Carlo program was used to study the dependence of uncertainties in noise parameters on the uncertainties in the underlying measured quantities, including reflection coefficients and noise temperatures of the sources, output noise temperature, and connector repeatability. The effect of correlations among the underlying uncertainties was included, and some general features of the results were presented. A more complete description of the work and results will be presented elsewhere.

The present program assumes measurement of the output noise temperature, rather than noise power, from the amplifier. It would be surprising if the results were radically different if the output noise power were measured, but it should be straightforward to modify the present program to accommodate measurements of output noise power, rather than noise temperature. This would make the results more directly applicable to the most common methods for noise-parameter measurements, and so the extension to power measurements is planned for the near future. Other possible extensions include the option of different measurement strategies and, if there is sufficient demand, development of a user-friendly version of the program for general distribution.

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