APPLICATION OF THE NIST TESTING STRATEGIES TO A MULTIRANGE INSTRUMENT

A. D. Koffman, T. M. Souders
National Institute of Standards and Technology[1]
Gaithersburg, MD

Abstract: A new modeling and test point reduction technique for analog and mixed-signal devices has been developed at the National Institute of Standards and Technology (NIST). This technique has been applied as a case study to a manufacturer's thermal transfer standard for potential use in testing and calibration. An empirical model is formulated using complete test data from many devices collected from several production runs. The model is then algebraically reduced using singular value decomposition and QR decomposition. Once the final reduced model is obtained, it is used to test devices which are measured only at a reduced set of test points. The model allows accurate prediction of device behavior at all other test points. Techniques for optimal model size selection are discussed. Device modeling results are presented and compared to complete test data.

I. INTRODUCTION

Testing is an integral part of maintaining the functionality of electronic instruments. At production time and periodically during the life of an instrument, the device is tested over a large set of test states, specified by the manufacturer. This can be very costly and time-consuming for both the manufacturer and the user. A savings of time and money would result if the instrument could be tested at a limited number of test points or test states with subsequent accurate estimation of the performance at all remaining test points. The work presented here is a case study of the application of the analog and mixed-signal device testing strategies work carried out at the National Institute of Standards and Technology over the last several years [1-6].

The NIST testing strategy is based on building a linear coefficient matrix model that relates the error response of a device, at all candidate test conditions, to a set of underlying device parameters. The model can be generated using physical device information, a priori information, or empirical data. Empirical modeling is especially attractive for performance testing applications such as production-line testing. Detailed design knowledge is not needed to produce the model. Instead, the model is built using complete measurement data from a representative sample of devices [3,4].

[1]U. S. Department of Commerce, Technology Administration

Once an accurate model is obtained, algebraic operations can be performed on the model to select an optimum set of test points which will minimize the cost/effort involved in the testing and maximize the test confidence. An estimate of the parameter vector of the model can be calculated from measurements made at the selected set of test points. The device response can be predicted at all candidate test points from the estimated parameters. The accuracy of the parameter estimates and response predictions can be calculated based on knowledge of the random error in the measurement process [3].

The subject instrument of this case study is a multirange thermal transfer standard, the Fluke 792A[2]. A thermal transfer standard converts rms ac voltages to dc values over a given input voltage range and frequency range. A large set of thermal transfer standard device test data was obtained from which empirical models were derived. Each data vector consists of a set of ac-dc differences measured at 255 input test conditions relative to a standard device of known accuracy. Data from a total of 139 devices was used. This large data set was divided into a modeling set of 100 devices used to build models and a validation set of 39 devices used to test the models developed. The model was formulated by algebraically reducing the number of columns of the modeling set to approximately equal the number of parameters governing the behavior of the device. The number of rows was similarly reduced to form a square matrix model. Additional rows (test points) were then added via an algorithm involving the prediction variance ratio to produce an overdetermined system suitable for solution using a least-squares error approach.

[2]For completeness, the instrument used in this case study is identified by manufacturer's name and model number. In no case does such identification imply recommendation or endorsement by the National Institute of Standards and Technology.

II. MODEL FORMULATION

The formulation of the model is begun by obtaining a modeling set of data vectors, in this case study, 100 vectors of length 255. A typical data set for the thermal transfer standard is shown in Figure 1. The manufacturer also specified the uncertainties for each test point. A test point refers to a specific input voltage range setting, input voltage, and input frequency. The data used to create the model are normalized by the uncertainty. This is done by dividing each data point by the uncertainty specified for that data point. This allows the analysis to be performed relative to the manufacturer's uncertainty. Thus, the modeling set is initially a matrix of size 255×100. It is desired to minimize the model size in a manner such that all of the useful device information is maintained. Minimizing the model size ultimately reduces the number of test points required to calibrate the device. This reduction in model size occurs in two dimensions, column and row. Two matrix decomposition methods are used within the modeling algorithm to reduce the model size. The singular value decomposition (SVD) is first used to reduce the column size and then the QR decomposition (QRD) is used to reduce the row size. An algorithm that minimizes the prediction variance ratio is next used to add test points to the model to reduce the effects of measurement noise and to provide a means of detecting model error [2].

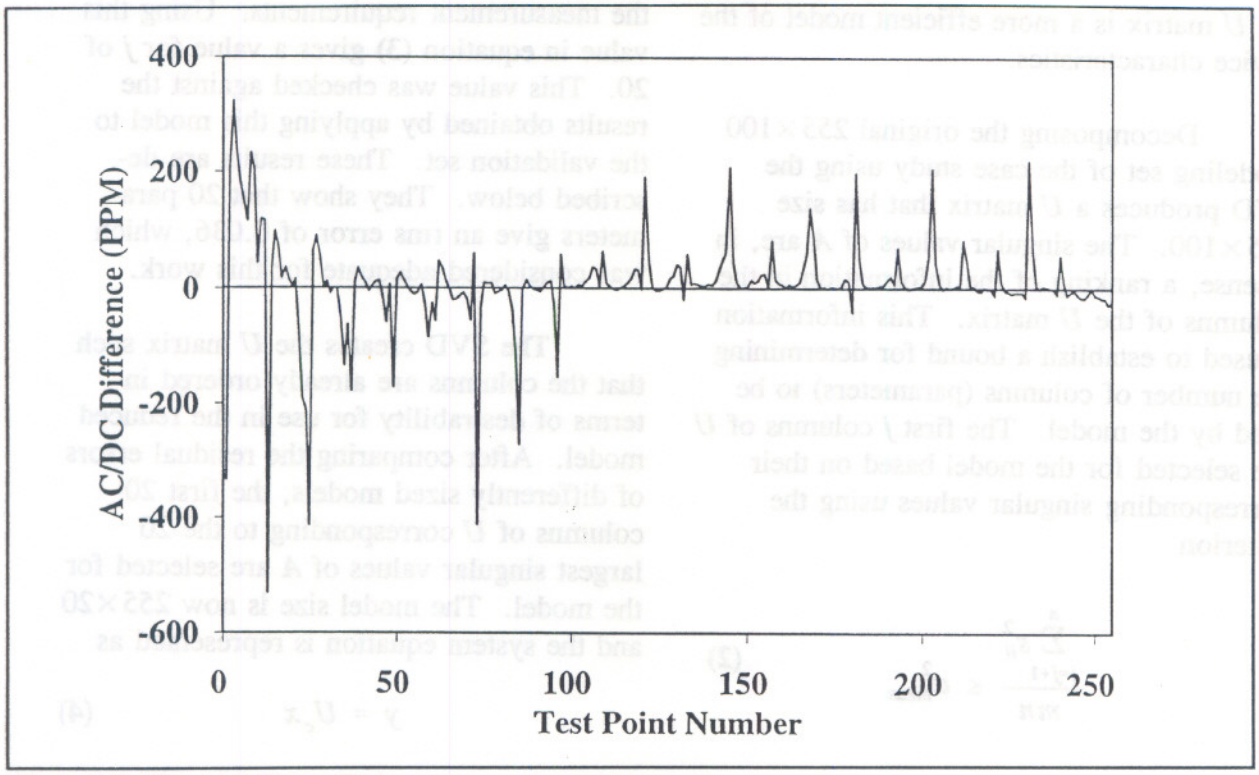

Figure 1 Typical Error Data

A. COLUMN REDUCTION OF MODEL

The column dimension of the system model represents the number of parameters used by the model to characterize the system. Ideally, this dimension should equal the number of significant underlying variables governing the manufacturing process. If the dimension is too small, the model will not adequately describe the system performance. If it is too large, then it will not be efficient because the data must estimate insignificant parameters, and too many test points will be required. Unfortunately, there is no straightforward *a priori* method to determine the exact number of process variables in a typical manufacturing process, or their relative importance. However, the number can be deduced by algebraic analysis of suitable measurement data (e.g., the modeling set) to produce an efficient yet complete column size for the model.

A method found to be very effective in performing the column reduction of the modeling set, A, is the SVD. The SVD factors a matrix into a product of three matrices,

$$A = USV^T. \qquad (1)$$

where U and V are orthonormal matrices, $()^T$ signifies the transpose of a matrix, and S is a diagonal matrix whose elements are the singular values associated with the A matrix [7]. The singular values occur along the diagonal of S in descending order. The U matrix is created by orthogonalizing linear combinations of the vectors in A and therefore, has the property that it spans the same space as the modeling set, A. However, the columns are orthogonal and thus, fewer columns of U can contain information that is contained in more columns of A. Therefore,

the U matrix is a more efficient model of the device characteristics.

Decomposing the original 255×100 modeling set of the case study using the SVD produces a U matrix that has size 255×100. The singular values of A are, in a sense, a ranking of the information in the columns of the U matrix. This information is used to establish a bound for determining the number of columns (parameters) to be used by the model. The first j columns of U are selected for the model based on their corresponding singular values using the criterion

$$\frac{\sum_{i=j+1}^{n} s_{ii}^2}{mn} \leq \sigma_{meas}^2 \qquad (2)$$

where s_{ii} is the i^{th} element of the main diagonal of S, i.e., the i^{th} singular value, n is the original column dimension and m is the row dimension of A, and σ_{meas} is the standard deviation of the measurement noise. If too many vectors are required to satisfy (2), then an alternative bound can be used, for example

$$\frac{\sum_{i=j+1}^{n} s_{ii}^2}{mn} \leq t^2 \quad , \quad t \geq \sigma_{meas} \qquad (3)$$

where t is determined based on specific accuracy needs. The bound t corresponds to the rms accuracy needed for the specific test or calibration. In this example, the measurement noise, σ_{meas}, was significantly smaller than the accuracy requirements. In addition, due to normalization with respect to the manufacturer's specified uncertainty, σ_{meas} is not constant for all test points in normalized units. Instead, a reasonable bound of 0.027 was selected for t to meet the measurement requirements. Using this value in equation (3) gives a value for j of 20. This value was checked against the results obtained by applying this model to the validation set. These results are described below. They show that 20 parameters give an rms error of 0.036, which was considered adequate for this work.

The SVD creates the U matrix such that the columns are already ordered in terms of desirability for use in the reduced model. After comparing the residual errors of differently sized models, the first 20 columns of U corresponding to the 20 largest singular values of A are selected for the model. The model size is now 255×20 and the system equation is represented as

$$y = U_c x \qquad (4)$$

where the matrix U_c is a column-reduced version of the U matrix, y is the vector of measurement data taken at the 255 test points, and x is the vector of 20 unknown parameter values specific to the device being tested.

B. ROW REDUCTION OF MODEL AND TEST POINT SELECTION

The SVD cannot be used to perform the row reduction of the model because the original test point identity would be lost. The QRD is used because it maintains the identity of the columns or test points. In the QRD, a matrix A is factored into two other matrices,

$$PA = QR. \qquad (5)$$

where P is a pivot matrix which reorders the columns of A such that the diagonal of R is monotonically decreasing, the Q matrix is a

square matrix with orthonormal columns, and the R matrix is an upper triangular invertible matrix the same size as matrix A [8]. The main diagonal of the R matrix contains values corresponding to how much independent information a column of A contributes relative to the previous columns in A. The QRD is applied to the transpose of the model in order to operate on the rows of A, which correspond to test points. The QRD method is used to select the j rows (j is the column dimension) which provide the most independent information about the row space of the model. In our example case, 20 rows (test points) are selected reducing the model size from 255×20 to 20×20. A model row size of 20 corresponds to using only 20 test points to predict the behavior of the device at all 255 of the test states specified by the manufacturer. To minimize errors caused by measurement noise, it is good practice to use more than the minimum number of test points (j). Therefore, more rows (test points) should be added to reduce the effects of measurement noise (lower prediction variance) and to provide redundancy so that model errors can be detected. Adding rows will create an overdetermined model that can be solved using a least-squares procedure. An overdetermined system is one in which there are more equations than unknowns. An algorithm which minimizes the prediction variance ratio is used to determine which test points to add to increase the row size. The ratio of the prediction variance, σ_p^2, to the measurement variance, σ^2, is calculated using the equation

$$\frac{\sigma_p^2}{\sigma^2} = diag[U_c(\tilde{U}_c^T\tilde{U}_c)^{-1}U_c^T] \qquad (6)$$

where σ_p^2 is a vector of the prediction variance at each of the 255 test points, U_c is the full model (length 255), and $\tilde{U}_c$ is the row reduced model (length 20). To get additional test points, the test point with the largest prediction variance is selected, and the row of U_c corresponding to that test point is added to the row-reduced model to produce a model of length 21. The prediction variance ratio **(6)** is recalculated using the length 21 model and the point of largest prediction variance is selected. This procedure is repeated until the desired model size is achieved. To keep track of which test points or test states are used by the model, there must be a test point vector containing the rows of the length-255 model that are included in the reduced model. As each new test point is added to the reduced model, the test point vector is updated with the index pointing back to the corresponding row of the length-255 model. As more test points are added to the model, the overall prediction variance will decrease. Since more test points ultimately implies longer test time, the decision to stop adding test points is an economic one. It is generally appropriate to have 2-4 times as many test points as parameters. In this case study, 50 test points were chosen because that number gave a suitably low prediction variance and still afforded substantial savings in test time, i.e., 50 vs. 255 measurements to be made. Therefore, the resulting model chosen for the Fluke 792A has size 50×20. This reduced model gives the reduced system equation

$$\tilde{y} = \tilde{U}_c x \qquad (7)$$

where $\tilde{y}$ represents the measurement data at only the reduced set of test points, in this case, 50. Using only the data at the selected test points, a least-squares estimate of the parameter vector, x, can be calculated as

$$\hat{x} = (\tilde{U}_c^T\tilde{U}_c)^{-1}\tilde{U}_c^T\tilde{y}. \qquad (8)$$

Then the complete set of test point data, **y**, can be predicted from $\hat{x}$ and the length-255 model via the system equation

$$\hat{y} = U_c \hat{x}. \quad (9)$$

The reduced model determines at which reduced set of test points to take measurements and is used in the calculation of the parameter estimates and the length-255 model is used to predict the response at all specified test points.

III. RESULTS

Along with the modeling set of 100 measurement vectors, a validation set of 39 measurement vectors was obtained from devices coming off the production line. This set was used to perform a check on the validity and accuracy of the model developed. Validation measurements are taken at all test points. Thus, a residue can be determined at these points using

$$\begin{aligned} \varepsilon_v &= y_m - \hat{y} \\ &= y_m - U_c \hat{x}. \end{aligned} \quad (10)$$

During production, test measurements are taken only at the reduced set of test points. Because more than the minimum number of test points are measured, the residue at these points can be calculated using

$$\varepsilon_p = \tilde{y} - \tilde{U}_c \hat{x}. \quad (11)$$

This residue can be monitored during production testing to assure that the derived model continues to reflect the production process.

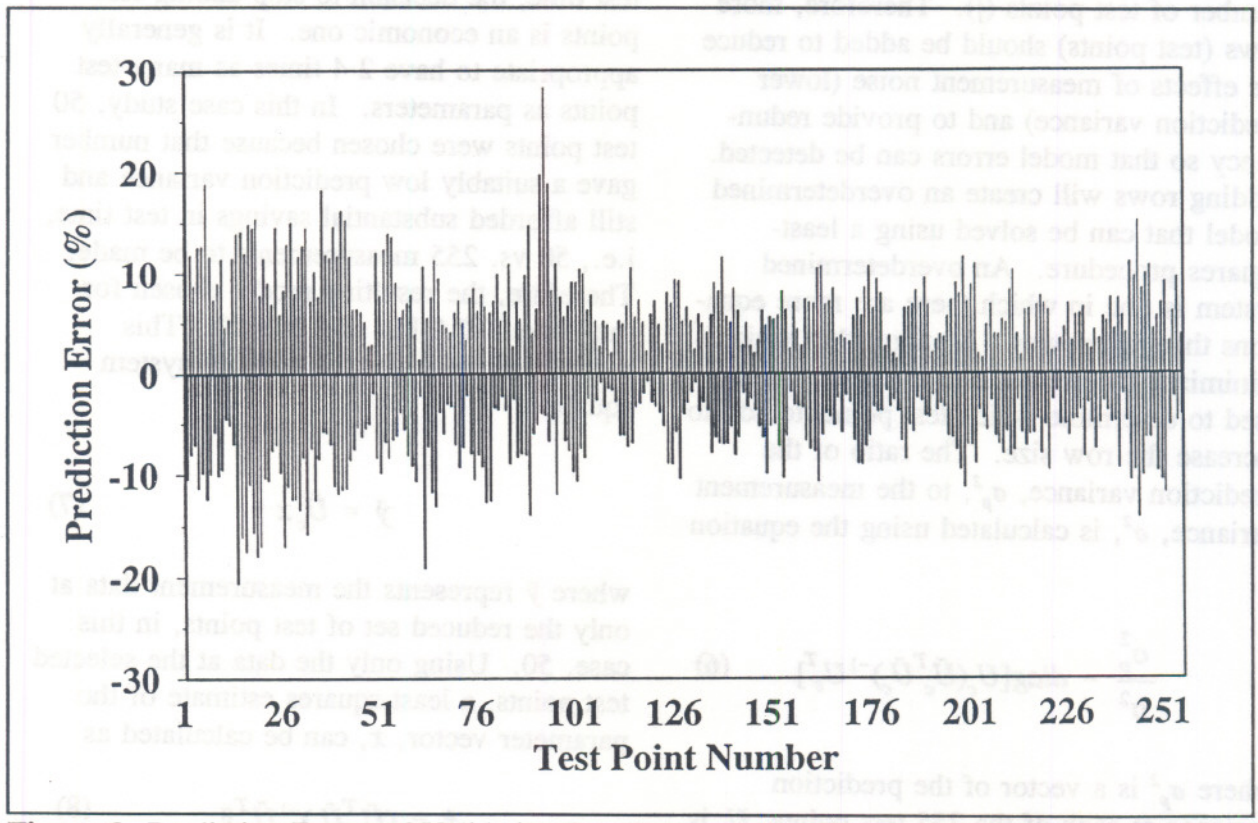

Figure 2 Prediction Errors of Validation Set (Errors in Percentage of Specified Uncertainty)

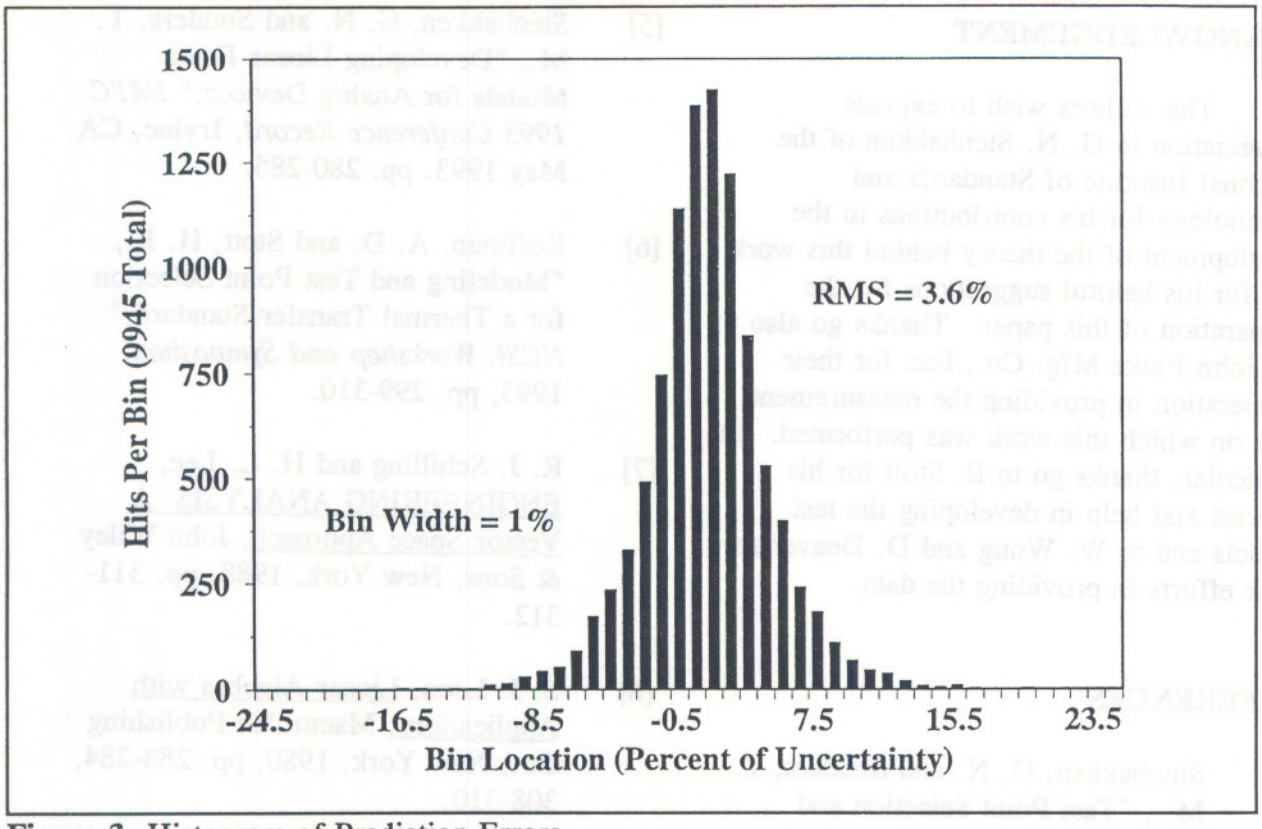

Figure 3 Histogram of Prediction Errors

When tested with the validation set, the 50×20 model produced an rms error of 0.036 or 3.6 percent of the manufacturer's specified uncertainty. Figure 2 shows the residue error vectors (calculated from **(10)**) produced from applying the 50×20 model to all 39 of the validation vectors. All 39 error vectors are overlaid. Figure 3 displays the same data in histogram form. More than 98 percent of the errors were less than 10 percent of the uncertainty values. All but two of the 9945 residues were less than 20 percent and the remaining two were less that 30 percent of the manufacturer's specified uncertainty. Weighing the small loss in accuracy against the reduction in test effort, this addition to the uncertainty of the testing process is acceptable.

IV. CONCLUSIONS

The technique illustrated in this paper allows for fewer measurements to be taken to fully characterize a multirange instrument. Fewer test points will reduce test time and calibration cost. This technique can generally be applied to other types of instruments for which enough exhaustive measurement sets are available to construct and test a model. Since the development of the test model is usually only done once (with periodic checks and updates), the greatest cost of testing will likely be incurred with taking the measurements needed to build the model. Therefore, this method could prove very useful in production line testing and in calibration.

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