Calculable Coaxial Resistors for Precision Measurements

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Abstract—**Coaxial straight-wire resistors have been constructed with the purpose of improving comparisons between resistors, capacitors, and inductors in the audio-frequency range. The design is based on the principle that a coaxial line with a cylindrical shield can be described by relatively simple equations for the real and imaginary parts of the impedance. The resistors, with values at and above 100 Ω will be used as transfer standards for characterization of the frequency dependence of standard resistors and of the quantum Hall resistance in the audio frequency range.**

Index Terms—**Impedance measurement, measurement standards, resistance measurement, skin effect, thermoelectricity.**

I. Introduction

THE National Institute of Standards and Technology (NIST) realizes the units of resistance and capacitance based on cross-capacitance measurements made with a Thompson-Lampard calculable capacitor [1]. Calculable cross-capacitors were constructed and used at several national laboratories, beginning in the 1960's, with four or more symmetrically mounted cylindrical bars which were partially shielded from one another by a central, moveable guard electrode. This design allows the farad to be related to the International System of Units (SI) through measurements of length using laser interferometry. SI resistance values can then be determined from the calculable capacitor through a comparison of a resistor and a capacitor using a quadrature bridge that relies on a frequency standard. Historically, the calculable capacitor and quadrature bridge measurements have been made at a frequency of $\omega = 10^4$ rad/s (1592 Hz).

In order to link the NIST calculable capacitor to dc resistance standards, it was necessary to first calibrate a bank of 10 pF fused-silica capacitors. The SI values of the 10 pF capacitors were transferred to an ac resistor by a series of bridges. The measurements that comprise these experiments are known collectively as the calculable capacitor chain, because of the many linked comparisons that are required to obtain a resistance value from the calculable capacitor.

The quantized Hall resistance (QHR) was adopted as the SI representation of the ohm on January 1, 1990 [2]. NIST has reported on experiments [3], [4] that determined the SI value of the QHR from the calculable capacitor and also tied the SI ohm to certain fundamental constants of nature. As a fixed reference, the QHR allows the ohm to be maintained independently of the farad, while providing a conventional value of resistance based in SI units. This representation of the ohm can be established at national and industrial laboratories in order to meet commercial and scientific requirements for international consistency of units, and accurate measurements of the QHR are readily obtainable using commercial equipment. Several groups have applied audio-frequency measurement techniques to study the frequency dependence of the QHR standard. So far, the results of alternating current (ac) QHR measurements [5]–[7] have not reached the level of accuracy that can be obtained from direct current (dc) QHR measurements. Since few laboratories have a calculable capacitor, it is reasonable to base impedance as well as resistance measurements on the dc QHR representation, i.e., the calculable capacitor chain could be reversed to obtain ac resistance and capacitance values based on dc QHR measurements.

Special coaxial resistors with calculable frequency response were built in 1969 by Haddad [8] and Cutkosky to transfer values between ac and dc resistance. The resistance elements of these resistors consist of single straight lengths of fine Evanohm[1] wire about 23 cm long. With values of 100 Ω and 1000 Ω, the frequency dependence of these standards is calculable up to at least $\omega = 10^5$ rad/s (15920 Hz). This property allows dc measurements of the 1000 Ω standard resistor to be related to its ac resistance at 1592 Hz with an uncertainty of about 0.01 $\mu\Omega/\Omega$. The Haddad 1000 Ω coaxial resistor was used to transfer the SI ohm to a primary group of 1 Ω Thomas-type resistors, which were the U.S. representation of the ohm before 1990. The 100 Ω resistor was compared to the 1000 Ω resistor as an experimental test of the relative frequency dependence. This paper describes the construction and characterization of new coaxial resistors of a similar design. The resistors will be used as transfer standards with known frequency dependence.

II. Frequency Dependence of Calculable Coaxial Resistors

The NIST coaxial resistors are often used in bridge measurements based on the four-terminal-pair definition of admittance [9]. (A terminal-pair consists of inner and outer conductors of a connector to the four-terminal-pair standard.) Since resistors can be precisely defined in such a bridge without introducing

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[1]Evanohm is a registered trademark of the Wilbur B. Driver Company. Certain commercial equipment and materials are identified in this paper in order to specify the experimental procedure. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment are necessarily the best available for the purpose.

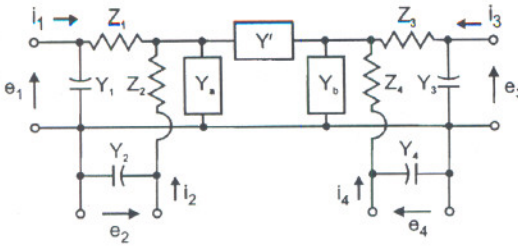

Fig. 1. Four-terminal-pair coaxial resistor represented by an equivalent circuit, with Y' as the series admittance of the main resistance element, Y_a and Y_b as the shunt admittances of the coaxial elements, and other impedances and admittances of the leads and connections as shown.

errors due to leads and connections, the same value can be obtained when the standard is placed in many different bridges. The four-terminal-pair equivalent circuit diagram of a coaxial resistor is shown in Fig. 1. Ideally, the current into the inner conductor of each coaxial connector is equal to the current returning by way of its outer shield, and only terminals 1 and 3 carry such currents. In this configuration the resistor can be described as a coaxial transmission line and the effects of capacitance, inductance, non-ideal current distributions, and eddy currents on the frequency dependence of the coaxial resistor can be calculated [8].

Frequency dependence cannot be measured by comparing a resistor against a capacitor, since the balance condition of the bridges requires a single fixed frequency for such a comparison. Comparisons of two calculable coaxial resistors against one another at several frequencies can provide tests of certain theoretical results. For example, inductance of the resistance wire is not as significant for a 1000 Ω resistor as for a similar 100 Ω resistor, because the effect is smaller relative to the main impedance. Likewise, eddy current losses in the resistor shell are smaller relative to the 1000 Ω value than to the 100 Ω value. The total relative theoretical in-phase frequency dependence, $\{1-R(\omega)/R(0)\}$, at $\omega = 10^5$ rad/s sum to about –0.03 μΩ/Ωat 1000 Ω and about –0.24 μΩ/Ω at 100 Ω for the Haddad resistors. The calculated out-of-phase part of the resistor admittance yields the phase angle, and the resistors can be measured against standards of phase angle to determine the accuracy of that part of the calculation.

III. Construction of the Resistors

The resistors consist of a straight Evanohm resistance wire surrounded by a 51 mm diameter coaxial brass case. Each case is made from thin brass, assembled with silver solder and then split lengthwise into two equal halves by the electrical discharge method. The cases have chlorotriflouroethylene insulators at each end. Evanohm rods are clamped in the insulators and attached via Pt wires to two shielded connectors, which make voltage and current terminations at each end of the resistor. A single length of heat-treated resistance wire is attached to the rods by spot-welding. Once the wire is mounted and suspended near the axis of the cylindrical case, it is necessary to adjust the position of the insulators and bring the wire as close as possible to the axis without applying tension.

TABLE I
Physical Characteristics of Coaxial Resistors

Coaxial resistor	SCA	SFA	SGA
Nominal value (Ω)	1000	1000	100
Wire diameter (mm)	0.020	0.019	0.064
Wire length (mm)	233	210	237
Case resistance (mΩ)	0.192	0.192	0.196

The design of the ac four-terminal-pair bridge measurement ensures that ac current enters through one current terminal and returns through the shield of the same terminal. Thus the case forms the return current conductor in the ac bridges, and the case resistance must be well defined. The cases were sealed by soldering to the shields of the four electrical terminations, leaving a 1 mm gap along the sides of the case. Sets of copper rings serve as clamps for the shields at the terminations and help to equalize the current distribution between the two halves of the case. Table I gives several characteristics of these new resistors.

For the most part the design of these resistors is the same as that developed by Haddad [8]. The following changes were made to improve the ac and dc performance.

1) In the original Haddad resistors, the mounting rods were 1 mm diameter copper rods. Alternating current changes the resistance of the mounting rods due to the skin effect, and this source of resistance change is especially important in copper at higher frequencies. Instead, 2.1 mm diameter Evanohm rods are used in the new resistors. The higher resistivity reduces the ac skin effect in the rods, and the 100 Ω resistor's relative theoretical in-phase frequency dependence $\{1 - R(\omega)/R(0)\}$ improves at $\omega = 10^5$ rad/s from about –0.24 μΩ/Ω to about –0.18 μΩ/Ω.
2) Haddad's Evanohm resistance wire was attached by spot-welding to small platinum rectangles, and the platinum was spot-welded to the copper mounting rods. The new resistors have the resistance wire attached to Evanohm rods. The resistor wire is attached by spot welding using an intermediate ribbon made from flattened Evanohm wire as shown in the diagram of Fig. 2. A micrometer was used to position the resistor case for low-power spot-welding between the wire and the ribbon, in order to adjust the four-terminal resistance to within about 50 μΩ/Ω of the nominal value. Spot-welding of similar alloys at the junctions should minimize the Peltier effect in the low frequency and dc measurements.
3) Techniques of heat-treating the resistance wire (as described below) allow greater control over the temperature coefficient of the wire, resulting in lower uncertainty due to self-heating in some of the resistors.

The dc measurement properties of the straight-wire resistors depend on the composition of the alloy, annealing and heat treatment of the wire, and on the mounting process. There are two types of Evanohm alloy, nominally composed of about 75% Ni, 20% Cr, 2.75% Al, and 2% Cu. The Evanohm-R and -S alloys differ in composition by the addition of a small amount of silicon to the -S alloy. In order that similar cases could be used to make resistors with values of 100 Ω, 1 kΩ, 129.06 Ω, and

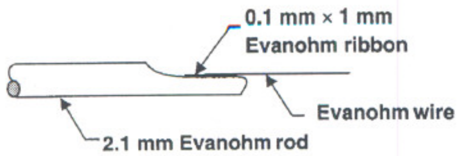

Fig. 2. Spot-welded Evanohm wire as used in the new coaxial resistors.

1290.6 Ω, four spools of wire of each type were obtained, with resistance values ranging from about 4 Ω/cm to about 50 Ω/cm.

The Evanohm-S wire was annealed and heat-treated by the manufacturer. The resulting temperature coefficient of resistance (T_C) was about +7 ($\mu\Omega/\Omega$)/°C. In order to produce lengths of wire with a T_C within ±0.5 ($\mu\Omega/\Omega$)/°C, pieces were spot-welded to temporary mounts and further heat-treated in air using a regulated oven. The variations in T_C of adjacent sections of wire from a single spool were considerable for both the alloys, presumably because of inhomogeneous composition. Thus each resistance element was heat-treated and monitored individually, with the length of the wire adjusted to be only a few percent longer than required. Four-terminal resistance was used to monitor the heat treatment process. Samples of Evanohm-S alloy, treated at 390 °C to 405 °C, resulted in the most reproducible changes in T_C and the three resistors constructed so far were made using this alloy.

IV. Measurements

A. AC Bridge Comparisons

AC techniques have been used to characterize one of the new 1000 Ω coaxial resistance standards. Using a four-terminal-pair bridge, two ac resistors of different nominal value can be compared with a total relative uncertainty of about 0.01 $\mu\Omega/\Omega$ at 1592 Hz. A 100:1 ratio equal-power resistance bridge was used to compare the 1000 Ω coaxial resistor made by Haddad (R304) with one new 1000 Ω coaxial resistor (SCA). Each was compared against the same 100 kΩ resistor at frequencies of 1592 Hz and 1000 Hz. The direct bridge readings include the large frequency dependence of the 100 kΩ resistor, the bridge frequency dependence, and the frequency dependence of the coaxial standard. Although the bridge has not recently been characterized at 1000 Hz, only the differences in these bridge readings are needed in order to determine a ratio of values for the two coaxial resistors at two frequencies. These measurements were taken at two frequencies and the difference in the bridge readings (α) for similar measurements is given below for each coaxial resistor

$$\alpha_{304}(1592\text{ Hz}) - \alpha_{304}\,(1000\text{ Hz}) = 5.894\ \mu\Omega/\Omega$$
$$\alpha_{\text{SCA}}(1592\text{ Hz}) - \alpha_{\text{SCA}}\,(1000\text{ Hz}) = 5.899\ \mu\Omega/\Omega.$$

The resulting values differ by only 0.005 $\mu\Omega/\Omega$, indicating that the two coaxial resistors have nearly the same frequency dependence. The measurements were repeated with the sets of leads interchanged, switching the high potential to the opposite end of the resistor, and no change in α_{SCA} larger than 0.005 $\mu\Omega/\Omega$ were seen at 1592 Hz.

Bridge measurements can determine relative variations in phase angle as a function of frequency [10], and the calculated phase angle (δ) at a frequency of 1592 Hz was compared to the measured phase angle of two coaxial resistors. This was measured using the 100:1 bridge by comparing SCA against two resistors for which the phase angles are known [11]. The measured value $\delta_{\text{SCA}} = -10.0\ \mu\text{rad} \pm 1.0\ \mu\text{rad}$ agrees with the value $\delta_{304} = -9.9\ \mu\text{rad} \pm 1.0\ \mu\text{rad}$ obtained for the Haddad resistor, and with the calculated theoretical result, $\delta_{1000} = -9.4\ \mu\text{rad} \pm 0.6\ \mu\text{rad}$.

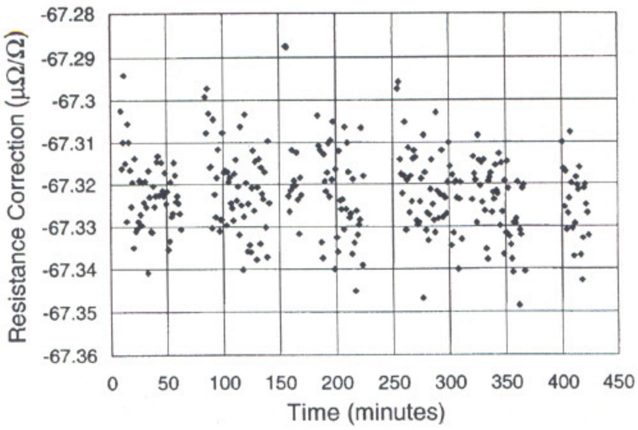

Fig. 3. Sequence of data showing the resistance correction from the nominal value of 100 Ωfor coaxial resistor SCA. The random uncertainty of each data point is about 0.01 $\mu\Omega/\Omega$.

B. Comparison of ac and dc Resistance

DC comparisons generally involve either 1:1 bridge ratios or current comparator bridge measurements at ratios of 10:1 or higher. A 1:1 dc bridge was operated to compare R304 and SCA with 2 mA measurement current, producing a 4 mW load on each coaxial resistor. Measurements using the 100:1 ac bridge were made with the same load before and after these dc measurements. With the resistance of each coaxial case taken into account the ac-dc difference of SCA relative to R304 was 0.02 $\mu\Omega/\Omega \pm 0.03\ \mu\Omega/\Omega$. These last measurements are tedious; however, they need to be repeated to improve the precision. The calculated ac-dc resistance change at 1592 Hz for the 1000 Ω resistors is less than 1 part in 10^9.

C. Stability

Initial dc comparisons show that the three new coaxial resistors have drift rates within about ±1 ($\mu\Omega/\Omega$)/year. The short-term stability has been measured for two resistors, SGA (100 Ω) and SFA (1000 Ω), using dc cryogenic current comparator (CCC) bridges. Fig. 3 shows data comparing SGA to a stable commercial resistor over a period of about 7 h. Over this period, and under good conditions, the resistance value of SGA is stable to ±0.005 ($\mu\Omega/\Omega$)/h. The value of resistor SFA is slightly less stable, possibly due to motion of the very fine diameter wire.

Instability in the coaxial resistors can usually be traced to the effect on the wire resistance of changes in strain. According to Starr, *et al.* [12], strain in Evanohm causes resistivity to decrease. Tension in the wire should however cause a net increase in resistance, because the dimensional changes more than offset the increase in strain. In pure bending, a decrease in resistance

TABLE II
ELECTRICAL CHARACTERISTICS OF RESISTIVE ELEMENTS IN FOUR COAXIAL RESISTORS

Coaxial resistor	R304	SCA	SFA	SGA
Nominal value (Ω)	1000	1000	1000	100
Resistance correction at 25 °C (μΩ/Ω)	–91	+35	–76	–67
First order T_c at 25 °C (μΩ/Ω)/°C	–1.0	–0.44	–0.18	+0.20
Loading at 4 mW power level (μΩ/Ω)	–0.30	–0.22	–0.41	–0.03
L_c at 4 mW power level (μΩ/Ω)/mW	–0.07	–0.03	–0.08	–0.01
Convection coefficient (μΩ/Ω)/mA	–0.38	–0.14	–0.37	–0.07

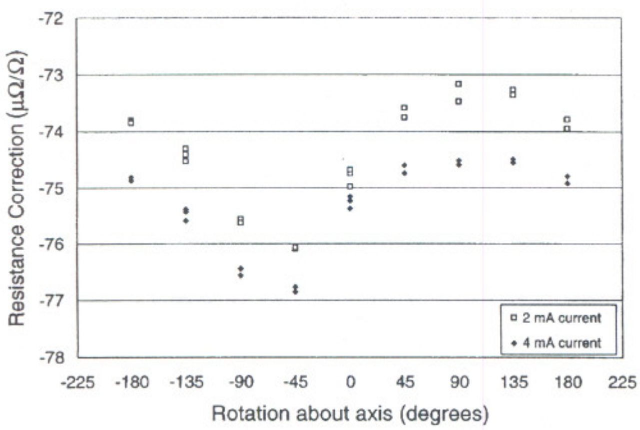

Fig. 4. Resistance correction from the nominal value of 1000 Ω for resistor SFA. The graph shows changes in resistance for two current levels as a function of the placement of the resistor, for rotation of the resistor case about the long axis. The changes are related to reversible strain caused by bending of the resistance element.

is expected. The coaxial resistors were constructed with no tension, and some bending naturally occurs whenever the resistor is moved. Fig. 4 shows values of resistor SFA versus position of the case, for various angles of rotation of the case about its own axis. The resistance changes smoothly with the angle by 2 $\mu\Omega/\Omega$ per 180° of rotation. The data show little or no hysteresis, in that the positions were changed randomly and the values were repeatable. Resistor SGA is affected in very much the same way by rotation. To reduce the changes in strain, stands have been constructed. The stands hold the resistor cases at the angle for which the maximum value of resistance is obtained.

D. Temperature and Load Dependence

The approximate resistance offset from the nominal value for each resistor is given in Table II. The overall T_C of each of the three new resistors has been characterized at dc, by repeated measurements in a controlled oil bath at temperatures between 23 °C and 27 °C. Both 1000 Ω resistors have a purely first-order temperature dependence, and the second-order T_C of the 100 Ω resistor is about ±0.015 $(\mu\Omega/\Omega)/(°C)^2$ at 25 °C.

The three new coaxial resistors were measured at several of the above oil bath temperatures using power levels of between 0.25 mW and 16 mW for the 1000 Ωresistors, and between 1.6 mW and 30 mW for the 100 Ω resistor. Joule heating produces a significant and immediate rise in the temperature of the wire, and a resulting change in resistance (loading effect) occurs which is summarized in Table II. In many standard resistors the load coefficient of resistance (L_C) is proportional to the T_C value.

The coaxial resistor design provides little direct conductive cooling for the wire element. The change in resistance due to the loading effect is also complicated by the fact that the T_C of the element varies over short lengths of wire. In the simple static approximation, power dissipation from the wire takes place by conduction into the mounting rods and outward into the surrounding oil. Near the mounting rods there is a temperature gradient in the resistance wire when power is applied. Thus, the change in resistance of the wire is more closely related to the T_C of the central part than to that of the ends. This may explain why, in Table II, L_C and T_C values of some of the resistors show only weak correspondence. The values in the table are relatively small compared to the variations in the untreated wire T_C, and have remained constant over several weeks or longer.

In the data of Fig. 3, one can observe a positive offset in the data in segments starting near minutes 10, 80, 155, and 250. These segments were preceded by at least 10 min with no current through the resistor. The decrease in the resistance with load over a period of several minutes indicates that heating of the oil inside the resistor case may cause this effect. The dc measurement current was on continuously before the segments beginning at minutes 180, 320, and 400.

Any bulk motion of the oil will increase the rate of heat loss. The data of Fig. 5(a) show, for resistor SCA, the average of 24 sets of measurements at each of five power levels. The same data is plotted versus the measurement current in Fig. 5(b). A similar trend is seen in all of the coaxial resistance standards, including R304. At power levels above about 4 mW for the three 1000 Ω resistors, and above 9 mW for the 100 Ω resistor, the resistance change becomes proportional to the change in current rather than the change in power. An explanation for this behavior is that induced convection of the heated oil near the wire becomes important. It is necessary to assume that oil temperature very near the wire is in equilibrium with the wire temperature. Convection causes this heated oil to rise and be displaced by unheated oil at a constant rate. The time required for the heated oil to be displaced is $\tau \approx (T/v)$ where v is the velocity of the heated oil moving past a wire of radius r. The time constant τ

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