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EVALUATION OF HIGH-VOLTAGE IMPULSE WAVEFORMS USING MODEL-BASED DECONVOLUTION

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Abstract

Accurate measurement of high voltage (hv) impulse waveforms is of critical importance in the testing of hv apparatus and for fundamental insulation studies. Quantifying the measurement system errors can be performed by following procedures recommended by the international standards for high voltage testing. Application of deconvolution techniques to reduce measurement errors by calculating the input waveform using the measured step response have been explored, but they are particularly sensitive to noise in digitized data; small random errors in the measured waveforms can result in large oscillations in the deconvolved waveforms. This paper presents a technique for effectively performing a deconvolution of measured high voltage impulse waveforms using the step response of the measurement system and input waveform model based upon the impedance parameters of the test circuit. Data are presented that demonstrates a significant advantage of this approach to estimating the deconvolved input, its insensitivity to random noise in the measured data. Results for different analytic models for the input waveforms are also included.

Introduction

The use of wide-bandwidth digital recorders has made possible improved precision in the measurement of high-voltage impulses. Signal processing techniques can be readily applied to the measured data to perform waveform smoothing to increase the signal-to-noise ratio, extract the characteristic waveform time parameters, and correct for systematic errors in the measurements. Several approaches can be taken to evaluate the waveform parameters specified by the international standards on high voltage testing [1, 2]. Smoothing routines can be applied to the waveforms or alternatively, if the waveform can be modeled with an analytic function, the parameters of the function can be adjusted using a least-squares fitting routine to find the best-fit model [3]. Although smoothing may result in more reproducible results, it does not correct for errors of a systematic nature introduced by the voltage divider, cables, terminators, and digital recorder. These errors can be estimated, if the impulse response $h(t)$ of the system is known, through the convolution integral, which relates the waveform at the input $x(t)$ of the measurement system to the output $y(t)$:

$$y(t) = \int_0^t h(t-s)\ x(s)\ ds\ . \qquad (1)$$

Deconvolution techniques use the measured output $y(t)$ and impulse response $h(t)$ to calculate the input $x(t)$. It can be done numerically in time domain, or by transforming the waveforms to frequency domain, performing the deconvolution, and then inverse-transforming to obtain the time domain input waveform [5]. The great difficulty in performing the numerical deconvolution directly is that due to the presence of noise in the digitized data, the deconvolution problem is ill-posed. Small errors in the measured waveforms can give rise to large errors in the deconvolved input waveforms.

A different technique for evaluating the input waveform is presented in this paper. Rather than calculating the input from the measured output and step response waveforms directly, the approach uses a general functional form for the voltage waveform at the input of the measurement system and performs a convolution using the measured step response and the input waveform function. The parameters of the input waveform are adjusted so that the convolved input provides the best match to the measured output through a least-squares minimization procedure. The functional form of the input waveform is based upon a high-voltage circuit analysis. The sensitivity of the calculated input waveforms to noise is much less than the direct method because the numerical deconvolution calculation is unnecessary with the proposed model-based approach. This paper begins with a description of the proposed technique followed by a demonstration of its insensitivity to noise using computer-generated analytic waveforms with random noise added. The paper concludes with a discussion of the advantages and limitations of the proposed method.

Model-Based Deconvolution Technique

Let $x(t)$, $t \geq 0$, denote the (unknown) input to a divider system, $y(t)$ the system output, and $h(t)$ the system impulse response. The deconvolution problem is to recover $x(t)$ given the output $y(t)$ and the impulse response $h(t)$. In practice, $y(t)$ and the system step response $g(t)$ are observed over a time window $0 \leq l \leq T$ and consist of discrete, digitized measurements y_k and g_k, respectively, which contain digitization errors and system noise. Thus, y_k, g_k are each time series with the subscript k denoting the measured value at time t_k where the subscript $k = 1, ..., K$ and $t_K = T$.

The system step response $g(t)$ is usually measured rather than the impulse response $h(t)$ because it is easier to generate a signal that simulates an ideal voltage step than one that approximates an ideal delta function impulse. The time derivative of the step response can then be used to replace the impulse response in the convolution integral:

$$y(t) = \int_0^t \frac{d}{dt} (g(t-s))\ x(s)\ ds. \tag{2}$$

The standards specify how step responses are to be generated and place limits on the step response parameters, but do not allow for correction of measurement errors [1, 2]. A technique for estimating whether or not the errors introduced by the measurement system are significant is described in IEEE Standard 4 [1, 4]. It uses the measured step response with an input waveform having an assumed analytic functional form and applies the convolution integral directly. This method allows errors in the divider measurements to be evaluated at low voltage by comparing the input waveform with the calculated output.

In order to solve Eq. (1) numerically, it is replaced by the so-called convolution summation

$$y(k) = \sum_{i=1}^{k} h(k-i)\ x(i), \quad k = 1, \ldots, K, \tag{3}$$

where $x(k)$, $y(k)$, $h(k)$ denote true values at time t_k. Because of the presence of measurement system noise one has $y(k) = y_k + e_k$ where e_k is an unknown error vector. Thus, the deconvolution problem becomes that of estimating the true input $x(i)$ from estimates of the solution x_i to the linear system

$$y_k = \sum_{i=1}^{k} h_{k-i}\ x_i, \quad k = 1, \ldots, K, \tag{4}$$

where h_{k-i} is an approximate system impulse response vector calculated from the measured g_k. Eq. (4) may be written

$$\mathbf{y} = H\ \mathbf{x}, \tag{5}$$

where H is a $K \times K$ Toeplitz matrix with entries

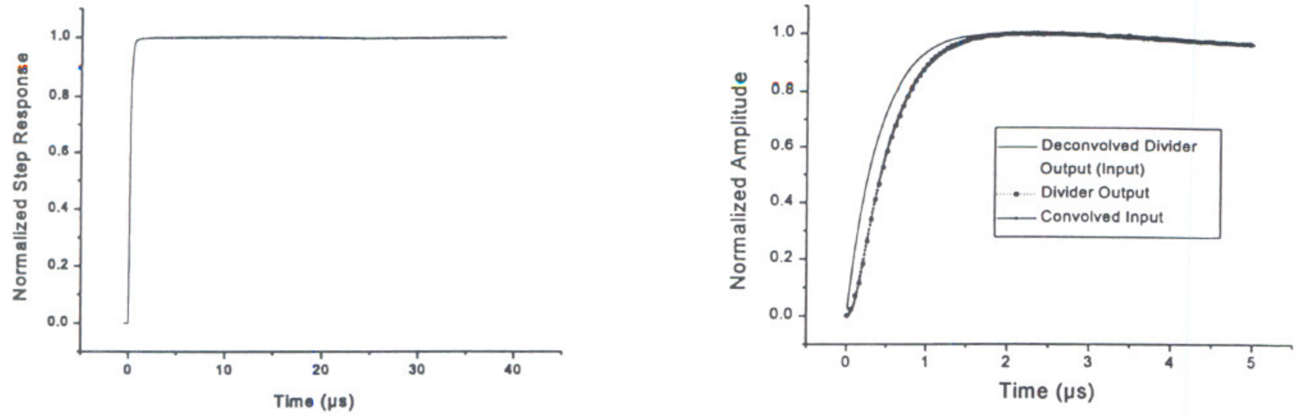
$$h_{ij} = \begin{cases} h_{i-j}, & 1 \leq j \leq i \leq K, \\ 0, & 1 \leq i \leq j \leq K. \end{cases} \tag{6}$$

It is well known that for the system given in Eq. (5) the matrix H can be singular or nearly so. Thus, small changes in the left side of Eq. (5) can lead to large changes in the solution x_i.

The deconvolution technique described in this paper for the reconstruction of full lightning impulses is insensitive to noise in the data proposed in this paper provides an accurate estimate of the unknown waveform. The algorithm is based on an assumed analytic form for the input voltage waveform, which is consistent with that predicted by a circuit analysis of the divider and generator system. The assumed structure of the unknown waveform is

$$x(t;\mathbf{p}) = A[e^{-\alpha t} - e^{-\beta t}(\cos(qt) + B\sin(qt))], \tag{7}$$

where $\mathbf{p}$ is the vector of parameters (A, B, q, α, β). Among all functions having the form of Eq. (7), the solution to



(a) normalized step response.

(b) noise-filled and deconvolved waveforms.

Figure 1. Deconvolution of Noise-Filled Waveforms

the deconvolution problem is defined to be the one which minimizes the quantity

$$Q(\mathbf{p}) = \sum_{k=1}^{K} \; y_k \; - [\; \sum_{i=1}^{k} \; h_{k-i} \; x(t_i;\mathbf{p})]^2, \qquad (8)$$

where the minimum is taken with respect to the vector p over a realistic range of parameter values defined by the restriction that all parameters are nonnegative.

A modification of the Levenberg-Marquardt algorithm is used to estimate the optimal values of the parameters. A feature of this algorithm, as implemented in the public-domain software package employed in our study [6], is the use of implicitly scaled variables in order to achieve scale invariance of the method and to limit the size of the correction in any direction where the objective function is changing rapidly. Under reasonable conditions on the objective function, this optimal choice of the correction enhances global convergence and results in a fast rate of convergence for problems with small residuals. However, even though global convergence is assured, the function $Q(\mathbf{p})$ defined by Eq. (8) may have multiple local minima. It is important that the starting value of **p** be sufficiently close to its optimal value in order that the algorithm converge to the optimal value of Q and not to some suboptimal local minimum.

Results and Discussion

To evaluate the sensitivity of the model-based deconvolution technique to random noise in the data, a test was carried out in which an analytic waveform was numerically convolved with the step response of the divider measurement system to calculate the output waveform. Different levels of computer-generated random noise were then added to the output waveforms which were then deconvolved using the proposed technique. A noise-filled waveform together with the deconvolved input waveform and the system step response is shown in Fig. 1. The resultant waveform parameters found in this manner are compared with the known input parameters in Table 1.

For this test, two waveform models were used. The first was a double-exponential type that arising from the simplified model for the Marx-type generator with a front capacitor and load resistor. The output voltage waveform is described by Eq. (7) with q set to zero. The deconvolved waveforms were evaluated for two time parameters, the front time T_f and time to crest T_c. The front time is defined according to IEEE Standard 4 and IEC Publication 60-1 [1, 2] as $1.67\text{x}T_{30\text{-}90}$. T_c is defined as the time from first deviation from zero voltage to the maximum voltage.

Only the time parameters were considered in this study. The maximum voltage and corresponding time are found from the waveform after smoothing is applied to the region of the voltage peak. Noise was added to the waveforms using the computer random number generator which produces a value between 0 and 1, and then scaling this number by 0.01, 0.03, or 0.05 of the normalized peak voltage. The deconvolved waveform parameters found from the noise-corrupted waveforms agree with those from the known input waveforms to

Table 1. Comparison of Time Parameters of Deconvolved Noise-Filled Waveforms

Waveform	*Noise Level*	$T_f(ns)$		$\delta_r(\%)$	$T_c(ns)$		$\delta_r(\%)$	*Period,* $q^{-1}(ns)$		$\delta_r(\%)$
		Input	**DW**		**Input**	**DW**		**Input**	**DW**	
DE	None	1202	1202	0.00	2335	2336	0.04	—	—	—
	1%	1202	1197	-0.42	2335	2325	-0.43	—	—	—
	5%	1202	1171	-2.58	2335	2290	-1.93	—	—	—
MDE	None	958	957	-0.10	1533	1528	-0.65	4198	4189	-0.21
	1%	958	958	0.00	1533	1534	-0.07	4198	4275	1.83
	5%	958	968	1.04	1533	1569	2.35	4198	5041	20.08

DE Double Exponential **MDE** Modified Double Exponential

within ±0.5% for added noise levels of 1%, which is well within the ±10% specified by the standards [1, 2].

The second waveform model used in the test was a sinusoidally-modified double exponential given in Eq. (2). It has an approximately double-exponential shape, but with an overshoot and oscillation at the voltage peak. The agreement between the parameters are better for this model: for 1% added noise, T_f and T_c agree within 0.1%. The frequency q is more sensitive than the front and crest times to the presence of noise, differing by 1.83% from that of the original input with a 1% noise level.It should be noted that the noise levels used in this study are at least twice as large as the intrinsic noise levels in the most commonly-used digitizers.

Conclusion

The model-based deconvolution method proposed here shows excellent insensitivity to noise with the analytic impulse waveforms used in this study. For the exaggerated noise levels of 1% and greater, the evaluation of the front time of the waveforms agreed with the known values to well within the tolerances prescribed by international high voltage test standards. Further evaluation of measured waveforms using different circuit models is planned.

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