Pulse parameter uncertainty analysis

N. G. Paulter and D. R. Larson

Abstract. A detailed uncertainty analysis is presented for the pulse parameter measurement service of the National Institute of Standards and Technology (NIST, USA). It relates to the new pulse parameter measurement and extraction processes. Uncertainties for pulse amplitude, transition duration, overshoot and undershoot (preshoot) are given. Effects of temperature variation, impulse response estimate, pulse parameter extraction algorithms, time-base distortion, calibration procedures and the waveform reconstruction process are included.

1. Introduction

The NIST supports a measurement service for high-speed (transition durations <20 ps) pulse generators that provides an estimate of the pulse parameters of amplitude and transition duration [1]. Overshoot and undershoot (preshoot) parameters were previously provided as well. However, support for these parameters was discontinued because of the lack of a viable uncertainty analysis [2], which is addressed by this paper.

The NIST is one of two national laboratories that provide a pulse parameter measurement service; the other being the National Physical Laboratory (NPL) in the United Kingdom. The NIST and NPL are performing a comparison of pulse parameter results, which includes measured data, corrected data (if applicable) and reconstructed data. The results to date indicate that both national laboratories are in close agreement.

The NIST measurement service currently uses commercial, high-bandwidth sampling oscilloscopes (3 dB attenuation, bandwidths of approximately 50 GHz) and pulse generators (3 dB attenuation, bandwidths of approximately 20 GHz) to measure the pulse parameters of short-transition-duration (high-speed) pulse generators and the step response of high-speed samplers. The purpose of this paper is to present our new uncertainty analysis for these parameters, which are pulse amplitude, transition duration, overshoot and undershoot.

For brevity, not all variables are described at the point of first use, but a list of variables and their description is provided in the glossary. Also, it should be pointed out that the step response of a device is equal to the convolution integral of the impulse response of that device with an ideal step. Both terms are used in this paper because transition durations and bandwidths of electronic systems are typically calculated from the step responses and, in waveform reconstruction, impulse responses are typically deconvolved.

N. G. Paulter and D. R. Larson: Electricity Division, National Institute of Standards and Technology (NIST), 100 Bureau Drive, Gaithersburg, MD 20899, USA.

2. Background

The pulse parameter measurement process that is used to acquire waveforms is briefly described in this section. The measurement process consists of a set of measurements of the customer's pulse generator or sampler (the device under test, or DUT) and a set of instrument calibration measurements. Some of the instrument calibration measurements are made during the DUT measurement sequence and some are not. The calibration measurements include time-base errors, sampler gain, jitter and sampler step response. Figure 1 is a diagram of the NIST pulse parameter measurement system.

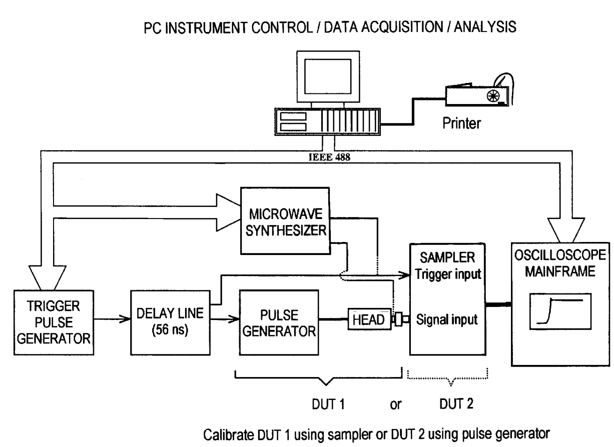

Figure 1. Diagram of NIST pulse measurement system. The dotted lines indicate insertion of instruments used in time-base calibration.

An estimate of the step response of the NIST 50 GHz sampler used to measure the DUT is obtained using the "nose-to-nose" method [3, 4], the results of which have been compared with results using swept frequency and optoelectronic methods [5]. We are currently examining the "nose-to-nose" method and its limitation in sampler calibration [6]. Measurements that are used to estimate the sampler step response, system jitter and dynamic gain of the sampler are taken routinely, but not necessarily as part of the DUT measurement sequence, and a control chart is maintained for the mean value and standard deviation of the transition durations of the sampler step response and the equivalent jitter step response and the sampler dynamic gain. The reason that the measurements of these parameters are not part of the DUT measurement sequence is that the sampler step response, the system jitter and the sampler dynamic gain are stable (small observable variation) and the variations in these parameters are represented in their associated control chart data. The DUT jitter is intentionally excluded from the system jitter because DUT jitter affects the DUT parameters of transition duration, overshoot and undershoot and will be exhibited in customer measurements.

Several sets of data are acquired for the customer's DUT. For brevity, the discussion that follows assumes that the DUT is a pulse generator. A set of data consists of M_1 sampler-acquired DUT waveforms and one measurement of the time-base errors. Measurements of the time-base errors [7-10] are a routine part of the DUT measurement procedure. The DUT measurement sequence is as follows:

1. measure time-base error: one independent measurement;
2. acquire waveforms: M_1 independent measurements of DUT output.

The DUT waveforms are subsequently corrected for gain and time-base errors only if these errors are large relative to the reported uncertainties. The corrected or uncorrected waveforms are then used in a reconstruction process to obtain a waveform that is an accurate estimate of the pulse measured by the sampler. The accuracy of this estimate (the reconstructed waveform) is dependent on the reconstruction process and the accuracy of the estimate of the sampler impulse response. The waveform reconstruction process uses an iterative deconvolution of the sampler impulse response from the measured data. From each reconstructed waveform, pulse parameter values are computed. The set of pulse parameter values thus computed is used to determine the mean value and standard deviation for the given parameter.

The pulse parameter computations are based on histogram methods [11]. The first step in the calculations is to compute the histogram of the waveform. Next the top-line, V_{S2}, and bottom-line, V_{S1}, values are obtained from the histogram. Then, using V_{S2} and V_{S1}, the pulse parameters are obtained for the waveform.

This uncertainty analysis, because it is applied to acquired waveforms, is applicable to both the measurement of the output of pulse generators and the step response of samplers with the appropriate change in reference measurements and waveforms. When measuring the output of a pulse generator it is assumed that the sampler step response is the reference, and when measuring the sampler step response it is assumed that the output of the pulse generator is the reference.

3. Uncertainty analysis

The reported pulse parameters are an average of the particular pulse parameters obtained from a set of M_1 pulse waveforms measured using the NIST pulse measurement systems. The average of a parameter, W for example, is given by

$$\overline{W} = \frac{1}{M_1}\sum_{i=1}^{M_1} W_i(\alpha_j), \tag{1}$$

where M_1 is the number of values for the parameter W, one value for each waveform, and W is dependent on a number of variables, α_j. The uncertainty for this average, $\overline{W}$ for example, is given by

$$u_{\overline{W}} = k_{\mathrm{eff}}\sqrt{\sum_{i=1}^{M_1}\left\{\left(\frac{\partial\overline{W}}{\partial W_i}\right)^2\left[\sum_j\left(\frac{\partial W_i(\alpha_j)}{\partial\alpha_j}\right)^2 u_j^2\right]\right\}}$$

$$= k_{\mathrm{eff}}\sqrt{\sum_{i=1}^{M_1}\left\{\frac{1}{M_1^2}\sum_j\left[\left(\frac{\partial W_i(\alpha_j)}{\partial\alpha_j}\right)^2 u_j^2\right]\right\}} \tag{2a}$$

$$= k_{\mathrm{eff}}\sqrt{\frac{1}{M_1}\sum_j\left(\frac{\partial W_i(\alpha_j)}{\partial\alpha_j}\right)^2 u_j^2}, \tag{2b}$$

where it is assumed in (2a) that the α_j values are uncorrelated, which is the reason why there are no cross terms in the partial derivatives with respect to the α_j . In (2b) it is further assumed that the u_j values are the same for every W_i; that is, the uncertainties in the variables for a given parameter are the same for every waveform. The term k_{eff} is the statistical weight [12] applied to the uncertainties of variables obtained from a limited number of trials. For a number of variables with different degrees of freedom, k_{eff} is found by first calculating the effective degrees of freedom using [12]

$$\nu_{\mathrm{eff}} = \frac{\left[\sum_{i=1}^{M}\left(\frac{\partial W(\alpha_1,\alpha_2,\ldots,\alpha_M)}{\partial\alpha_i}\right)^2 u_i^2\right]^2}{\sum_{i=1}^{M}\frac{c_i^4 u_i^4}{\nu_i}}, \tag{3}$$

Table 1a. Variables affecting pulse amplitude uncertainty.

Variable α_i	Uncertainty[1] u_i	Partial derivative $\left\|\frac{\partial W(\alpha_1,\alpha_2,\ldots,\alpha_{\mathrm{P}})}{\partial\alpha_i}\right\|$	Type	Degrees of freedom ν_i
$V_{\mathrm{S2,m}}$	$\sqrt{\sigma^2_{V_{\mathrm{S2,m}}}+\left(\frac{\partial V_{\mathrm{S2,m}}}{\partial H}u_H\right)^2}$	$\frac{1}{\sqrt{M_1}}\frac{1}{\overline{g}}$	A	M_1-1
$V_{\mathrm{S1,m}}$	$\sqrt{\sigma^2_{V_{\mathrm{S1,m}}}+\left(\frac{\partial V_{\mathrm{S1,m}}}{\partial H}u_H\right)^2}$	$\frac{1}{\sqrt{M_1}}\frac{1}{\overline{g}}$	A	M_1-1
$\overline{g}$	$\sqrt{\sigma^2_{V_{\mathrm{g}}}+\left(\frac{\partial \mathrm{g}}{\partial V,H}u_{V,H}\right)^2}$	$\frac{\|V_{\mathrm{A}}\|}{\overline{g}}$	A	$M_{10}-1$
$\overline{T}_{\mathrm{meas}}$	$\sigma_{T_{\mathrm{meas}}}$	$\frac{S_{\Delta V/\Delta T}}{\overline{g}}$	A	M_4-1
$\overline{T}_{\mathrm{ref}}$	$\sigma_{T_{\mathrm{ref}}}$	$\frac{S_{\Delta V/\Delta T}}{\overline{g}}$	A	M_5-1
$S_{\Delta V/\Delta T}$	$\sigma_{S_{\Delta V/\Delta T}}$	$\frac{\overline{T}_{\mathrm{meas}}-\overline{T}_{\mathrm{ref}}}{\overline{g}}$	A	M_6-2

1. The letter V is used to represent amplitude-related uncertainty contributions to $\overline{g}$, see (8) and Table 1c. The letter H is used to represent histogram-related uncertainty contributions, see (11) and Table 1b.

Table 1b. Variables affecting the uncertainty in V_{S2} obtained using a histogram method.

Variable α_i	Uncertainty u_i	Partial derivative $\left\|\frac{\partial V_{\mathrm{S2}}(\alpha_1,\alpha_2,\ldots,\alpha_{\mathrm{P}})}{\partial\alpha_i}\right\|$	Type	Degrees of freedom ν_i
$V_{\max}$	$\sigma_{V_{\max}}$ $(=0)$	$\frac{N_{\mathrm{S2}}}{N_{\mathrm{bins}}}$	A	∞
$V_{\min}$	$\sigma_{V_{\min}}$ $(=0)$	$\frac{N_{\mathrm{S2}}}{N_{\mathrm{bins}}}$	A	∞
N_{S2}	$u_{N_{\mathrm{S2}}}$ $(=0)$	$\frac{V_{\max}-V_{\min}}{N_{\mathrm{bins}}}$	B	∞
N_{bins}	$u_{N_{\mathrm{bins}}}$ $(\leq \pm 1/10 N_{\mathrm{bins}})$	$\frac{V_{\max}-V_{\min}}{N^2_{\mathrm{bins}}}$	B	∞

Table 1c. Variables affecting g.

Variable α_i	Uncertainty[1] u_i	Partial derivative $\left\|\frac{\partial W(\alpha_1,\alpha_2,\ldots,\alpha_{\mathrm{P}})}{\partial\alpha_i}\right\|$	Type	Degrees of freedom ν_i
$\overline{V}_{\mathrm{S2,m,r}}$	$\sqrt{\sigma^2_{V_{\mathrm{S2,m,r}}}+\left(\frac{\partial V_{\mathrm{S2,m,r}}}{\partial H}u_H\right)^2}$	$\frac{1}{\sqrt{M_{10}}}\left\|\frac{1}{\overline{V}_{\mathrm{S2,r}}-\overline{V}_{\mathrm{S1,r}}}\right\|$	A	M_2-1
$\overline{V}_{\mathrm{S1,m,r}}$	$\sqrt{\sigma^2_{V_{\mathrm{S1,m,r}}}+\left(\frac{\partial V_{\mathrm{S1,m,r}}}{\partial H}u_H\right)^2}$	$\frac{1}{\sqrt{M_{10}}}\left\|\frac{1}{\overline{V}_{\mathrm{S2,r}}-\overline{V}_{\mathrm{S1,r}}}\right\|$	A	M_2-1
$\overline{V}_{\mathrm{S2,r}}$	$\sqrt{\sigma^2_{V_{\mathrm{S2,r}}}+\left(\frac{\partial V_{\mathrm{S2,r}}}{\partial H}u_H\right)^2}$	$\frac{1}{\sqrt{M_{10}}}\frac{\|\overline{V}_{\mathrm{S2,m,r}}-\overline{V}_{\mathrm{S1,m,r}}\|}{\left(\overline{V}_{\mathrm{S2,r}}-\overline{V}_{\mathrm{S1,r}}\right)^2}$	A	M_3-1
$\overline{V}_{\mathrm{S1,r}}$	$\sqrt{\sigma^2_{V_{\mathrm{S1,r}}}+\left(\frac{\partial V_{\mathrm{S1,r}}}{\partial H}u_H\right)^2}$	$\frac{1}{\sqrt{M_{10}}}\frac{\|\overline{V}_{\mathrm{S2,m,r}}-\overline{V}_{\mathrm{S1,m,r}}\|}{\left(\overline{V}_{\mathrm{S2,r}}-\overline{V}_{\mathrm{S1,r}}\right)^2}$	A	M_3-1

1. The letter H is used to represent histogram-related uncertainty contributions see (11) and Table 1b.

Table 2a. Variables affecting pulse transition duration uncertainty.

Variable α_i	Uncertainty u_i	Partial derivative $\left\lvert\frac{\partial W(\alpha_1,\alpha_2,\ldots,\alpha_{\mathrm{P}})}{\partial\alpha_i}\right\rvert$	Type	Degrees of freedom ν_i
δt	$\sigma_{\delta t}$	$\frac{t_{\mathrm{d}}}{\delta t}$	A	$M_{13}-1$
X_{m}	$\sqrt{\sigma^2_{X_{\mathrm{m}}}+\left(\frac{\partial X_{\mathrm{m}}}{\partial H}u_H\right)^2+\left(\frac{\partial X_{\mathrm{m}}}{\partial T}u_T\right)^2}$ (see Tables 2b and 2d)	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\frac{1}{\delta t}\left\lvert\frac{\partial t_{\mathrm{d,R,i}}}{\partial t_{\mathrm{d,m,i}}}\right\rvert\right)^2}$	A	M_1-1
X_{r}	$\sqrt{\sigma^2_{X_{\mathrm{r}}}+\left(\frac{\partial X_{\mathrm{r}}}{\partial H}u_H\right)^2}$ (see Table 2b)	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\frac{1}{\delta t}\left\lvert\frac{\partial t_{\mathrm{d,R,i}}}{\partial t_{\mathrm{d,r,i}}}\right\rvert\right)^2}$	A	M_7-1
X_{j}	$\sqrt{\sigma^2_{X_{\mathrm{j}}}+\left(\frac{\partial X_{\mathrm{j}}}{\partial H}u_H\right)^2}$ (see Table 2b)	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\frac{1}{\delta t}\left\lvert\frac{\partial t_{\mathrm{d,R,i}}}{\partial t_{\mathrm{d,j}}}\right\rvert\right)^2}$	A	$M_{12}-1$

Table 2b. Variables affecting X value uncertainty.

Variable α_i	Uncertainty u_i	Partial derivative $\left\lvert\frac{\partial W(\alpha_1,\alpha_2,\ldots,\alpha_{\mathrm{P}})}{\partial\alpha_i}\right\rvert$	Type	Degrees of freedom ν_i
$m_{L_{1_-}}$	$\sigma_{\mathrm{m}_{L_{1_-}}}$ $(=0)$	$\frac{L_{1_+}-L_1}{L_{1_+}-L_{1_-}}$	A	∞
$m_{L_{1_+}}$	$\sigma_{\mathrm{m}_{L_{1_+}}}$ $(=0)$	$\frac{L_1-L_{1_-}}{L_{1_+}-L_{1_-}}$	A	∞
$m_{L_{2_-}}$	$\sigma_{\mathrm{m}_{L_{2_-}}}$ $(=0)$	$\frac{L_{2_+}-L_2}{L_{2_+}-L_{2_-}}$	A	∞
$m_{L_{2_+}}$	$\sigma_{\mathrm{m}_{L_{2_+}}}$ $(=0)$	$\frac{L_2-L_{2_-}}{L_{2_+}-L_{2_-}}$	A	∞
L_1	$\sqrt{\sigma^2_{L_1}+\left(\frac{\partial L_1}{\partial\alpha_i}u_{\alpha_i}\right)^2}$ (see Table 2c)	$(m_{L_{1_+}}-m_{L_{1_-}})\frac{1}{L_{1_+}-L_{1_-}}$	A	∞
L_{1_-}	$\sigma_{L_{1_-}}$ $(=0)$	$(m_{L_{1_+}}-m_{L_{1_-}})\frac{L_{1_+}-L_1}{(L_{1_+}-L_{1_-})^2}$	A	∞
L_{1_+}	$\sigma_{L_{1_+}}$ $(=0)$	$(m_{L_{1_+}}-m_{L_{1_-}})\frac{L_1-L_{1_-}}{(L_{1_+}-L_{1_-})^2}$	A	∞
L_2	$\sqrt{\sigma^2_{L_2}+\left(\frac{\partial L_2}{\partial\alpha_i}u_{\alpha_i}\right)^2}$ (see Table 2)	$(m_{L_{2_+}}-m_{L_{2_-}})\frac{1}{L_{2_+}-L_{2_-}}$	A	∞
L_{2_-}	$\sigma_{L_{2_-}}$ $(=0)$	$(m_{L_{2_+}}-m_{L_{2_-}})\frac{L_{2_+}-L_2}{(L_{2_+}-L_{2_-})^2}$	A	∞
L_{2_+}	$\sigma_{L_{2_+}}$ $(=0)$	$(m_{L_{2_+}}-m_{L_{2_-}})\frac{L_2-L_{2_-}}{(L_{2_+}-L_{2_-})^2}$	A	∞

where ν_i is the number of degrees of freedom for the parameters (shown in Tables 1 to 4) and c_i are the partial differential equations (shown in Tables 1 to 4). Weight k_{eff} is then found from ν_{eff} using the t-distribution [12].

The uncertainties in the variables, u_i (where the subscript refers to parameter i), are obtained from independent measurements that provide values for these particular variables. The tables list the source of u_i for the appropriate variables. To calculate the uncertainty of the different parameters, the partial derivatives of these parameters with respect to the independent variables must be calculated. These partial derivatives are also

Table 2c. Variables affecting uncertainty of L_1 and L_2.

Variable α_i	Uncertainty u_i	Partial derivative $\left\lvert\frac{\partial W(\alpha_1,\alpha_2,\ldots,\alpha_{\mathrm{P}})}{\partial\alpha_i}\right\rvert$	Type	Degrees of freedom ν_i
$V_{\mathrm{S1,m}}$	see Table 1a	$1-P\,\%$	A	∞
$V_{\mathrm{S2,m}}$	see Table 1a	$P\,\%$	A	∞

Table 2d. Variables affecting uncertainty of X_{m}.

Variable α_i	Uncertainty u_i	Partial derivative $\left\lvert\frac{\partial W(\alpha_1,\alpha_2,\ldots,\alpha_{\mathrm{P}})}{\partial\alpha_i}\right\rvert$	Type	Degrees of freedom ν_i
$S_{\Delta t/\Delta T}$	$\sigma_{S_{\Delta t/\Delta T}}$	$\overline{T}_{\mathrm{meas}}-\overline{T}_{\mathrm{ref}}$	A	M_8-2
$\overline{T}_{\mathrm{meas}}$	$\sigma_{T_{\mathrm{meas}}}$	$S_{\Delta t/\Delta T}$	A	M_4-1
$\overline{T}_{\mathrm{ref}}$	$\sigma_{T_{\mathrm{ref}}}$	$S_{\Delta t/\Delta T}$	A	M_5-1

Table 3. Variables affecting uncertainty in pulse overshoot.

Variable α_i	Uncertainty[1] u_i	Partial derivative $\left\lvert\frac{\partial W(\alpha_1,\alpha_2,\ldots,\alpha_{\mathrm{P}})}{\partial\alpha_i}\right\rvert$	Type	Degrees of freedom ν_i
$V_{\mathrm{max,m}}$	$\sigma_{V_{\mathrm{max,m}}}$ $(=0)$	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\beta_O\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\frac{1}{V_{\mathrm{A,R,i}}}\right)^2}$	A	∞
$V_{\mathrm{S2,m}}$	$\sigma'_{V_{\mathrm{S2,m}}}$	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\beta_O\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\frac{1}{V_{\mathrm{A,R,i}}}\right)^2}$	A	∞
$V_{\mathrm{S2,R}}$	$\sigma'_{V_{\mathrm{S2,R}}}$	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left[\left(\frac{1}{V^2_{\mathrm{A,R,i}}}\right)\left(\beta_O[V_{\mathrm{max,m,i}}-V_{\mathrm{S2,m,i}}]\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\right)\right]^2}$	A	∞
$V_{\mathrm{S1,R}}$	$\sigma'_{V_{\mathrm{S1,R}}}$	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left[\left(\frac{1}{V^2_{\mathrm{A,R,i}}}\right)\left(\beta_O[V_{\mathrm{max,m,i}}-V_{\mathrm{S2,m,i}}]\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\right)\right]^2}$	A	∞
X_{m}	$\sqrt{\sigma^2_{X_{\mathrm{m}}}+\left(\frac{\partial X_{\mathrm{m}}}{\partial H}u_H\right)^2}$ (see Tables 2b and 2c)	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\beta_O[V_{\mathrm{max,m,i}}-V_{\mathrm{S2,m,i}}]\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\frac{1}{V_{\mathrm{A,R,i}}}\right)^2}$	A	∞
X_{d}	$\sqrt{\sigma^2_{X_{\mathrm{R}}}+\left(\frac{\partial X_{\mathrm{R}}}{\partial H}u_H\right)^2}$ (see Tables 2b and 2c)	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\beta_O[V_{\mathrm{max,m,i}}-V_{\mathrm{S2,m,i}}]\frac{X_{\mathrm{m,i}}}{X^2_{\mathrm{R,i}}}\frac{1}{V_{\mathrm{A,R,i}}}\right)^2}$	A	∞
β_O	σ_{β_O}	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left([V_{\mathrm{max,m,i}}-V_{\mathrm{S2,m,i}}]\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\frac{1}{V_{\mathrm{A,R,i}}}\right)^2}$	A	M_9-2

1. The prime notation indicates that the uncertainty for these parameters must include histogram-dependent uncertainties, as for the example V_{S2} in Section 3.1. That is, the prime notation indicates

$$\sigma' = \sqrt{\sigma^2+\left(\frac{\partial V}{\partial H}u_H\right)^2}. \tag{33}$$

Table 4. Variables affecting uncertainty in pulse undershoot.

Variable α_i	Uncertainty[1] u_i	Partial derivative $\left\lvert\frac{\partial W(\alpha_1,\alpha_2,\ldots,\alpha_{\mathrm{P}})}{\partial\alpha_i}\right\rvert$	Type	Degrees of freedom ν_i
$V_{\mathrm{min,m}}$	$\sigma_{V_{\mathrm{min,m}}}$ $(=0)$	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\beta_U\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\frac{1}{V_{\mathrm{A,R,i}}}\right)^2}$	A	∞
$V_{\mathrm{S1,m}}$	$\sigma'_{V_{\mathrm{S1,m}}}$	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\beta_U\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\frac{1}{V_{\mathrm{A,R,i}}}\right)^2}$	A	∞
$V_{\mathrm{S2,R}}$	$\sigma'_{V_{\mathrm{S2,R}}}$	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left[\left(\frac{1}{V^2_{\mathrm{A,R,i}}}\right)\left(\beta_U[V_{\mathrm{min,m,i}}-V_{\mathrm{S1,m,i}}]\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\right)\right]^2}$	A	∞
$V_{\mathrm{S1,R}}$	$\sigma'_{V_{\mathrm{S1,r}}}$	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left[\left(\frac{1}{V^2_{\mathrm{A,R,i}}}\right)\left(\beta_U[V_{\mathrm{min,m,i}}-V_{\mathrm{S1,m,i}}]\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\right)\right]^2}$	A	∞
X_{m}	$\sqrt{\sigma^2_{X_{\mathrm{m}}}+\left(\frac{\partial X_{\mathrm{m}}}{\partial H}u_H\right)^2}$ (see Tables 2b and 2c)	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\beta_U[V_{\mathrm{min,m,i}}-V_{\mathrm{S1,m,i}}]\frac{1}{X_{\mathrm{R,i}}}\frac{1}{V_{\mathrm{A,R,i}}}\right)^2}$	A	∞
X_{d}	$\sqrt{\sigma^2_{X_{\mathrm{R}}}+\left(\frac{\partial X_{\mathrm{R}}}{\partial H}u_H\right)^2}$ (see Tables 2b and 2c)	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left(\beta_U[V_{\mathrm{min,m,i}}-V_{\mathrm{S1,m,i}}]\frac{X_{\mathrm{m,i}}}{X^2_{\mathrm{R,i}}}\frac{1}{V_{\mathrm{A,R,i}}}\right)^2}$	A	∞
β_U	σ_{β_U}	$\frac{1}{M_1}\sqrt{\sum_{i=1}^{M_1}\left([V_{\mathrm{min,m,i}}-V_{\mathrm{S1,m,i}}]\frac{X_{\mathrm{m,i}}}{X_{\mathrm{R,i}}}\frac{1}{V_{\mathrm{A,R,i}}}\right)^2}$	A	$M_{11}-2$

1. The prime notation indicates that the uncertainty for these parameters must include histogram-dependent uncertainties, as for the example V_{S2} in Section 3.1 (see also (33)).

shown in the tables. The tables for each variable include its type of uncertainty [12] and degrees of freedom, ν. For measured data, the number of degrees of freedom is given by $\nu = M_k - 1$, where M_k is the number of data elements used to compute the value of the k-th variable. For fits to data, ν is given by $\nu = M_k - m_k$, where m_k is the number of coefficients used to fit the data. The number of degrees of freedom for certain variables is equal to infinity ($\nu = \infty$) because the calculation of the value of these variables is based on a specific fixed waveform. Accordingly, every (an infinite set) computation of the value of that variable for that waveform yields the same result.

The variation in measurements, represented by the symbol σ in the tables and text, is unless otherwise indicated the standard deviation of a set of measurement values of a given parameter or the standard deviation of the residuals of a curve fitted to the data. For example, of the first case, $\sigma_{V_{\mathrm{S2,m}}}$ in Table 1a is the standard deviation of M_1 values, one $V_{\mathrm{S2,m}}$ value taken from each of the M_1 measured (acquired) waveforms. For example, of the second case, the $\sigma_{\mathrm{S}_{\Delta V/\Delta T}}$ in Table 1a is the standard deviation of the residuals to a fit to the amplitude versus temperature data.

3.1 Pulse amplitude

The pulse amplitude is obtained using a histogram-based algorithm (see Section 2). The pulse amplitude is the difference between V_{S2} and V_{S1}. Calculating the uncertainty in the pulse amplitude requires an equation that describes the reported pulse amplitude, V_{A}:

$$V_{\mathrm{A}} = \frac{\overline{V_{\mathrm{A,c/r}}}}{\overline{g}}, \tag{4}$$

where the horizontal bars indicate the arithmetic mean, $\overline{V}_{\mathrm{A,c/r}}$ is the mean of the set of M_1 pulse amplitudes corrected for sampler offset errors and waveform reconstruction errors, and $\overline{g}$ is the mean of the transient amplitude gain correction of the sampler. A common practice in oscilloscope calibrations is to use or include a static level gain-correction term in (4). However, since the signals being measured are transients (steps or impulses), as opposed to static levels, a transient gain term, the $\overline{g}$ in (4), should be used. The transient gain is affected by the impulse response of the sampler and the waveform epoch because of the settling response of the sampler. For example, if the sampler response has not settled by the end of the epoch, then g will be less than one for that epoch. Ideally, $\overline{g} = 1$ if the sampler exhibits no pulse gain or attenuation and the sampler has settled within the waveform epoch. The $\overline{V}_{\mathrm{A,c/r}}$ is given by

$$\begin{aligned}\overline{V}_{\mathrm{A,c/r}} &= \overline{V}_{\mathrm{A,c}} b_{\mathrm{r}} = (\overline{V}_{\mathrm{S2,c}} - \overline{V}_{\mathrm{S1,c}}) b_{\mathrm{r}} \\ &= [\overline{V}_{\mathrm{S2,m}} - \overline{V}_{S2\mathrm{,off,m}} - (\overline{V}_{\mathrm{S1,m}} - \overline{V}_{\mathrm{S1,off,m}}) + \\ &\quad \overline{V}_{\Delta T}] b_{\mathrm{r}},\end{aligned} \tag{5}$$

where $\overline{V}_{\mathrm{off}}$ is the voltage offset, which is a bias in the observed voltage, $\overline{V}_{\mathrm{A,c}}$ is the mean of the set of M_1 pulse amplitudes corrected for sampler offset errors, $\overline{V}_{\Delta T}$ is the mean of the amplitude corrections required for a change in measurement temperature, and β_{r} reflects the error in the amplitude of the reconstructed waveform caused by the reconstruction process. Ideally, β_{r} should be 1 because the sampler impulse response estimates integrate to 1. However, the reconstruction process introduces an error in the amplitude. This scaling error is exactly corrected by rescaling the pulse amplitude of the reconstructed waveform to equal $\overline{V}_{\mathrm{A,c}}$. The subscripts c and m refer to corrected and measured voltage values. We have observed that for the currently available high-bandwidth samplers, the voltage offset error is the same for both the top-line (S2) and bottom-line (S1) voltage levels; therefore, the offset voltage contribution can be ignored.

The temperature-correction term is obtained by measuring the change in the observed pulse amplitude with temperature [13]. The $\overline{V}_{\Delta T}$ term is therefore

$$\overline{V}_{\Delta T} = S_{\Delta V/\Delta T}(\overline{T}_{\mathrm{meas}} - \overline{T}_{\mathrm{ref}}), \tag{6}$$

where $\overline{T}_{\mathrm{meas}}$ is the average of M_4 sampler temperature values taken during the pulse measurement process, S is the slope of a straight-line fit through a set of previously acquired amplitude versus temperature data, and $\overline{T}_{\mathrm{ref}}$ is the average reference sampler temperature that is taken to be the mean of M_5 temperature values of the sampling head measured when the sampler impulse response was determined. The amplitude versus temperature data consist of a set of M_6 data pairs and are recorded over a temperature range between T_2 and T_1; the difference between these two temperatures is ΔT. Every sampler may exhibit a unique temperature-dependent response.

Amplitude V_{A} can now be rewritten using (5) and (6) in (4):

$$V_{\mathrm{A}} = \frac{(\overline{V}_{\mathrm{S2,m}} - \overline{V}_{\mathrm{S1,m}})}{\overline{g}} + \frac{S_{\Delta V/\Delta T}(\overline{T}_{\mathrm{meas}} - \overline{T}_{\mathrm{ref}})}{\overline{g}}. \tag{7}$$

The transient gain term, $\overline{g}$, is obtained by taking the ratio of the amplitude of the reference pulse as measured using the sampler and the amplitude of the reference pulse as measured using a reference instrument [14], which provides more accurate pulse amplitude measurements than high-speed samplers. As mentioned above, the gain term is obtained from a control chart and is given by

$$\begin{aligned}\overline{g} &= \frac{1}{M_{10}} \sum_{i=1}^{M_{10}} g_i \\ &= \frac{1}{M_{10}} \sum_{i=1}^{M_{10}} \frac{\overline{V}_{\mathrm{S2,m,r,i}} - \overline{V}_{\mathrm{S1,m,r,i}}}{\overline{V}_{\mathrm{S2,r,i}} - \overline{V}_{\mathrm{S1,r,i}}},\end{aligned} \tag{8}$$

where the subscript r denotes the reference pulse measurement and there are M_{10} independent gain terms. Using the ratio of average amplitudes in (8) is numerically more stable than an average of the amplitude ratios. Furthermore, the ratio of the average amplitudes relaxes the requirement that a reference measurement be made for each sampler-acquired measurement. A set of M_2 measurements of the reference pulse are used to obtain the reference-instrument-measured parameters and a set of M_3 measurements of the reference pulse are used to obtain the sampler-measured parameters. Temperature-dependent gain effects are included in $V_{\Delta T}$, which describes the change in pulse amplitude with change in temperature relative to T_{ref}. The g_i were computed from data taken at T_{ref}.

The variables shown in (8) contribute to the uncertainty in V_{A}. The uncertainty in V_{A}, $u_{\mathrm{V_A}}$, is dependent on the uncertainties of all the variables from which V_{A} is dependent, and these variables are listed in Tables 1a and 1b. The uncertainties, u_i, in the variables are obtained from independent measurements that provide values for these particular variables. Table 1a lists the source of u_i for the appropriate variables. The partial derivatives of V_{A} with respect to the independent variables are also shown in Table 1a, as are its associated degrees of freedom and uncertainty type [11]. $\overline{T}_{\mathrm{meas}}$, although obtained from M_1 averages of the average of M_4 temperature measurements taken during each waveform acquisition, has $\nu = M_1 - 1$. The uncertainties for $S_{\Delta V/\Delta T}$ require an empirical formula relating the pulse amplitude of the acquired waveform to temperature (see Figure 2) which is obtained by fitting a curve, typically a line, to the data.

In addition to measurement-related uncertainties, the reported amplitude values are also subject to uncertainties from the method used to calculate these values, in this case a histogram. The histogram-derived amplitude values, for example for V_{S2}, are given by

$$V_{\mathrm{S2}} = N_{\mathrm{S2}} \Delta V + V_{\mathrm{min}}, \tag{9}$$

where

$$\Delta V = \frac{V_{\mathrm{max}} - V_{\mathrm{min}}}{N_{\mathrm{bins}}}, \tag{10}$$

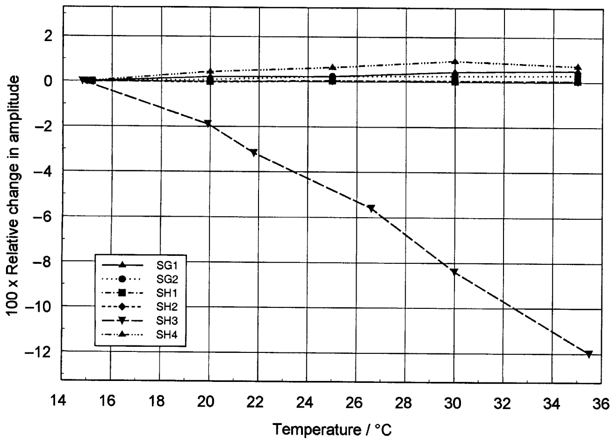

Figure 2. The percentage change in pulse amplitude with temperature relative to 15 °C. The designation SG refers to step generator and SH to sampling head. The number following the designation refers to models of devices made by different manufacturers.

and ΔV is the histogram bin size and N_{bins} is the number of histogram bins. The uncertainty contribution associated with V_{S2}, for example, is then

$$\frac{\partial V_{\mathrm{S2}}}{\partial H}u_H = \frac{1}{N_{\mathrm{bins}}} \times \sqrt{\begin{array}{l}(V_{\max} - V_{\min})^2 u_{N_{\mathrm{S2}}}^2 + N_{\mathrm{S2}}^2 \times \\ \left[\sigma_{V_{\max}}^2 + \sigma_{V_{\min}}^2 + \dfrac{1}{N_{\mathrm{bins}}^2}(V_{\max} - V_{\min})^2 u_{N_{\mathrm{bins}}}^2\right]\end{array}}. \quad (11)$$

The term H is used as a place holder to represent all the histogram-based dependences of a variable, such as V_{S2} in the example of (11). Table 1b, used as an example, lists variables affecting the V_{S2} amplitude variables. All amplitude values listed in the first column of Table 1a have an analogous list of variables. The degrees of freedom are infinite for N_{S2} and N_{bins} because they are extracted using a given pulse parameter algorithm and the output of this algorithm will not vary for a given waveform. The degrees of freedom are infinite for $V_{\max}$ and $V_{\min}$ because these values, for a given waveform, are fixed once a waveform has been acquired.

3.2 Transition duration

The transition duration is the difference between the occurrences of user-defined amplitude reference levels, for example, the 10 % and 90 % amplitude reference levels. The times at which the waveform crosses these reference levels are called reference-level instants. Accordingly, the 10 % to 90 % transition duration is the difference between the 90 % and 10 % reference level instants.

The reported waveform transition duration, t_{d}, is the average transition duration extracted from M_1 reconstructed pulse waveforms, $t_{\mathrm{d,R}}$. Duration t_{d} is related to the transition duration of the acquired waveform, $t_{\mathrm{d,m}}$, and the transition duration of the estimated sampler step response, $t_{\mathrm{d,r}}$:

$$t_{\mathrm{d}} = \frac{1}{M_1}\sum_{i=1}^{M_1} t_{\mathrm{d,R,i}} \times \overline{f_{\mathrm{dec}}(t_{\mathrm{d,m}} + \Delta t_{\mathrm{d},\Delta T} t_{\mathrm{d,r}})} + \Delta t_{\mathrm{d,rec}}, \quad (12)$$

where $\Delta t_{\mathrm{d},\Delta T}$ is the temperature-induced incremental change in transition duration [13] (described below), $\Delta t_{\mathrm{d,rec}}$ is the bias in the transition duration caused by the reconstruction process, and f_{dec} is used to indicate the deconvolution functional relationship between $t_{\mathrm{d,R}}$, $t_{\mathrm{d,m}}$ and $t_{\mathrm{d,r}}$. Duration $t_{\mathrm{d,R}}$ is found from a waveform that is obtained by deconvolving a waveform with transition duration $t_{\mathrm{d,r}}$ from a waveform with transition duration $t_{\mathrm{d,m}}$. The specific functional relationship, f_{dec}, between t_{d}, $t_{\mathrm{d,m}}$, and $t_{\mathrm{d,r}}$ is dependent on the type of waveform used and can only be derived for certain ideal waveforms. For example, for Gaussian waveforms, t_{d} is equal to the square root of the difference of the squares of transition durations of the measured and step response waveforms. For general waveforms, we can obtain an empirical relationship relating the three parameters. The first step in obtaining this empirical relationship is to vary either $t_{\mathrm{d,m}}$ or $t_{\mathrm{d,r}}$ in the waveform reconstruction process, keeping the other constant, and noting the variation in $t_{\mathrm{d,R}}$. This provides two sets of data, one relating $t_{\mathrm{d,R}}$ to $t_{\mathrm{d,m}}$ for a fixed $t_{\mathrm{d,r}}$ and another relating $t_{\mathrm{d,R}}$ to $t_{\mathrm{d,r}}$ for a fixed $t_{\mathrm{d,m}}$. We obtain our empirical relationship by fitting a curve (such as a polynomial) to these sets of data, $t_{\mathrm{d,m}}$ versus $t_{\mathrm{d,R}}$ and $t_{\mathrm{d,r}}$ versus $t_{\mathrm{d,R}}$. The reconstruction process we now use is described in [15]. However, we are also investigating various filtering methods, one of which is discussed in [16]. Using our present reconstruction algorithms on known waveforms, we have observed a bias, $\Delta t_{\mathrm{d,rec}}$, and a noise-dependent variation in $t_{\mathrm{d,R}}$. The noise-dependent variation is also contained in the empirical relationship, but the bias is not. Therefore, the uncertainty for the reconstruction process is computed by adding the absolute value of the bias to the computed uncertainty.

The parameters $t_{\mathrm{d,m}}$ and $t_{\mathrm{d,r}}$ can be expressed in terms of the sampling intervals:

$$\begin{aligned} t_{\mathrm{d,m}} &= X_{\mathrm{m}}\delta t, \\ t_{\mathrm{d,r}} &= X_{\mathrm{rj}}\delta t, \end{aligned} \quad (13)$$

where X_{m} and X_{rj} are the real-valued (non-integer) number of sampling intervals describing the transition duration for the measured sampler step response waveforms (including jitter) and δt is the duration of the equispaced sampling interval. X_{rj} is computed from waveforms that are the result of the convolution of the

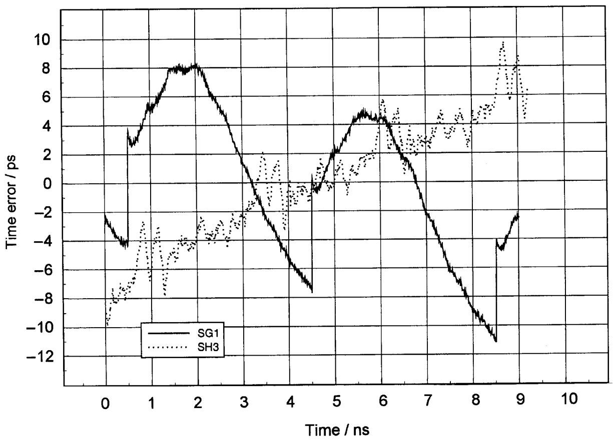

Figure 3. Time-base errors. The designation SG refers to step generator and SH to sampling head. The number following the designation refers to different models of devices made by different manufacturers.

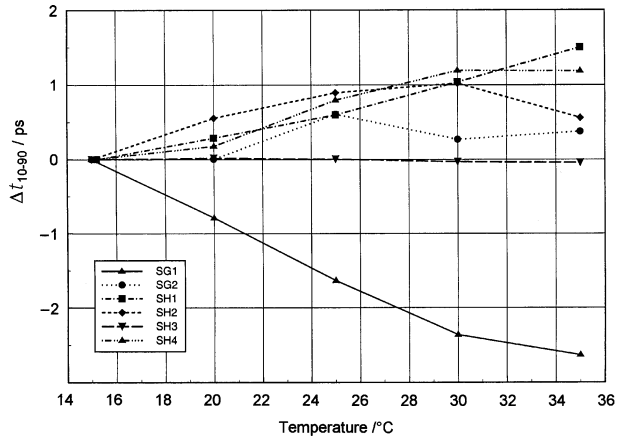

Figure 4. The change in transition duration, $\Delta t_{10\text{-}90}$, with temperature relative to 15 °C. The designation SG refers to step generator and SH to sampling head. The number following the designation refers to models of devices made by different manufacturers.

system jitter, represented by a Gaussian waveform, and the sampler step response, obtained from the sampler calibration method. The result of this convolution, estimated using the Gaussian approximation, is

$$X_{\mathrm{rj}} \approx \sqrt{X_{\mathrm{r}}^2 + X_{\mathrm{j}}^2}, \tag{14}$$

where X_{r} and X_{j} are the number of sampling intervals in the sampler step response and equivalent jitter step response transition durations. However, there are errors associated with this approximation [17]. For this uncertainty analysis the convolutions of the two waveforms are performed numerically and the resultant waveform used in subsequent processing. The measurement jitter typically has a normal distribution. We approximate X_{j} using a direct measurement taken by the sampler. We have observed that estimates of the jitter obtained employing a geometric method [18] are very sensitive to the transition region from which the estimate is made. Term X_{j} includes drift of the sampling aperture with respect to its trigger. Term δt is the average duration of the sampling intervals that span either the transition region of the waveform or the entire waveform [19] and is measured using sine-fit techniques [9] during the time-base calibration process. Figure 3 shows an example of time-base errors (vertical axis) versus measurement time. One time-base calibration is performed for each waveform; therefore the variation in δt is the variation of δt among the M_1 acquired waveforms. Similarly, the variation in X_{m} is dependent on the set of M_1 acquired waveforms. The variation in X_{r} is dependent on the set of M_7 acquired reference waveforms. The variation in X_{j} is dependent on the set of M_{12} jitter measurements. The temperature-dependent change in transition duration can be expanded:

$$\Delta t_{\mathrm{d},\Delta T} = S_{\Delta t/\Delta T}(\overline{T}_{\mathrm{meas}} - \overline{T}_{\mathrm{ref}}), \tag{15}$$

where $S_{\Delta t/\Delta T}$ is the slope of a straight-line fit to M_8 transition-duration-temperature data pairs (see Figure 4), measured independently of the M_1 acquired waveforms. Using (13) and (15) in (12) gives

$$t_{\mathrm{d}} = \overline{f_{\mathrm{dec}}[X_{\mathrm{m}}\delta t + S_{\Delta t/\Delta T}(\overline{T}_{\mathrm{meas}} - \overline{T}_{\mathrm{ref}})X_{\mathrm{rj}}\delta t]} + \Delta t_{\mathrm{d,rec}}. \tag{16}$$

For the samplers and pulse generators currently in use at the NIST, the value of $\Delta t_{\mathrm{d},\Delta T}$ is approximated as zero because it is much less than the reported uncertainties; however, it is retained here for completeness. As we did in the pulse amplitude uncertainty analysis, we generate a table for the variables contributing to the transition duration value (see Table 2a).

The values of X_{m}, X_{r} and X_{j} are determined by linearly interpolating to obtain the instant in time (the reference-level instant) corresponding to the given reference level. The value of X_{m} (and analogously for X_{r} and X_{j}), is

$$X_{\mathrm{m}} = \frac{t_{L_2} - t_{L_1}}{\delta t}, \tag{17}$$

where t_{L_1} and t_{L_2} are the time instances corresponding to the first (L_1) and second (L_2) reference levels of the transition duration. For example, in the 10 % to 90 % transition duration, L_1 is the 10 % reference-level instant and L_2 is the 90 % reference-level instant. Instant t_{L_1} is given by

$$t_{L_1} = t_{L_{1_-}} + \frac{t_{L_{1_+}} - t_{L_{1_-}}}{L_{1_+} - L_{1_-}}\left(L_1 - L_{1_-}\right), \tag{18}$$

where the subscripts + and – denote the actual sampling instances found on either side of the reference-level instant (either t_{L_1} or t_{L_2}) and the data values corresponding to the reference-level instants. Similarly, t_{L_2} is given by

$$t_{L_2} = t_{L_{2-}} + \frac{t_{L_{2+}} - t_{L_{2-}}}{L_{2+} - L_{2-}}(L_2 - L_{2-}). \tag{19}$$

Using (18) and (19) in (17) yields

$$\begin{aligned} X_\mathrm{m} &= \left[t_{L_{2-}} + \frac{t_{L_{2+}} - t_{L_{2-}}}{L_{2+} - L_{2-}}(L_2 - L_{2-}) - \right. \\ &\qquad \left. t_{L_{1-}} - \frac{t_{L_{1+}} - t_{L_{1-}}}{L_{1+} - L_{1-}}(L_1 - L_{1-}) \right] / \delta t \\ &= m_{L_{2-}} - m_{L_{1-}} + (m_{L_{2+}} - m_{L_{2-}}) \times \\ &\qquad \frac{L_2 - L_{2-}}{L_{2+} - L_{2-}} - (m_{L_{1+}} - m_{L_{1-}}) \frac{L_1 - L_{1-}}{L_{1+} - L_{1-}}, \end{aligned} \tag{20}$$

where the m_L terms are the time indices corresponding to the actual data found immediately above and below the reference levels. Using (20), we can obtain the uncertainty expansion for X_m (and, similarly, for X_r and X_j), which is shown in Table 2b. All the variables listed in Table 2b are extracted using our pulse parameter algorithm and, consequently, the degrees of freedom are infinite. The values of L_1 and L_2 can be expanded:

$$\begin{aligned} L_1 &= \overline{V}_{\mathrm{S1,m}} + P_1(V_{\mathrm{S2,m}} - V_{\mathrm{S1,m}}), \\ L_2 &= \overline{V}_{\mathrm{S1,m}} + P_2(V_{\mathrm{S2,m}} - V_{\mathrm{S1,m}}), \end{aligned} \tag{21}$$

where P_1 and P_2 are the percentage reference values, such as 10 % and 90 % or 20 % and 80 %. Table 2c provides an uncertainty assessment for L_1 and L_2.

3.3 Overshoot

Overshoot is, for waveforms with positive-going transitions, the maximum positive amplitude excursion relative to V_{S2} that the waveform makes near the transition region. On the other hand, for waveforms with negative-going transitions, overshoot is the maximum negative amplitude excursion relative to V_{S2} that the waveform makes near the transition region. For overshoot, we currently define "near the transition region" to be that period between the 50 % reference-level instant ($t_{50\%}$) and $t_{50\%} + 3t_\mathrm{d}$ (IEEE TC-10 Subcommittee on Pulse Techniques). Voltage offset errors are not considered here because they cancel, as they did for the uncertainty calculation of $V_{\mathrm{A,c}}$. The equation describing the calculation for the overshoot is

$$O = \overline{\left(\frac{V_{\mathrm{max,R}} - V_{\mathrm{S2,R}}}{V_{\mathrm{A,R}}} \right)} + \Delta O_{\mathrm{rec}}, \tag{22}$$

where ΔO_{rec} is the reconstruction-induced bias in the overshoot of the reconstructed waveform. $V_{\mathrm{max,R}}$ may be written

$$V_{\mathrm{max,R}} = V_{\mathrm{S2,R}} + V_{O,\mathrm{R}}, \tag{23}$$

where

$$V_{O,\mathrm{R}} = \frac{\beta_O V_{O,\mathrm{m}} t_{\mathrm{d,m}}}{t_{\mathrm{d,R}}}, \tag{24}$$

and β_O is a correction factor that is needed because of the reconstruction process and is found by fitting a curve to an M_9-element set of $t_{\mathrm{d,m}}V_{O,\mathrm{m}}$ versus $t_{\mathrm{d,R}}V_{O,\mathrm{R}}$ data. The $t_{\mathrm{d,m}}$ and $V_{O,\mathrm{m}}$ parameters are obtained from the acquired waveforms and $t_{\mathrm{d,R}}$ and $V_{O,\mathrm{R}}$ from the reconstructed waveforms. Equation (24) describes an empirical relationship between the overshoot and transition duration of the reconstructed (reported) waveform and those of the acquired waveform. As with transition duration (see Section 3.2), we have observed a bias and a noise-dependent variation in $V_{O,\mathrm{R}}$ caused by the waveform reconstruction: the noise-dependent variation is contained in the empirical relationship but the bias is not. Therefore, the uncertainty in overshoot is computed by adding the computed uncertainty to the absolute value of the bias. In (24), we assume that the product of the overshoot voltage and transition duration is not affected by an all-pass filter, which is how the sampler impulse response is expected to behave for an input signal that has a 3 dB attenuation bandwidth lower than that of the sampler. The uncertainties in $t_{\mathrm{d,r}}$ are included in the uncertainty estimate of $V_{O,\mathrm{R}}$ by propagation of uncertainties through $t_{\mathrm{d,R}}$. $V_{O,\mathrm{m}}$ can be expanded:

$$V_{O,\mathrm{m}} = V_{\mathrm{max,m}} - V_{\mathrm{S2,m}}. \tag{25}$$

Parameter $t_{\mathrm{d,R}}$ can be expanded in a similar way to $t_{\mathrm{d,m}}$ (13):

$$t_{\mathrm{d,R}} = X_\mathrm{R}\delta t, \tag{26}$$

where X_d is the non-integer number of sampling intervals describing the transition duration of the reconstructed waveform. Using (13), (23), (24), (25) and (26) in (22) yields for O:

$$O = \overline{\left(\frac{1}{V_{\mathrm{S2,R}} - V_{\mathrm{S1,R}}} \right) \left(\beta_O [V_{\mathrm{max,m}} - V_{\mathrm{S2,m}}] \frac{X_\mathrm{m}}{X_\mathrm{R}} \right)} + \Delta O_{\mathrm{rec}}. \tag{27}$$

The uncertainty in β_O is the standard deviation in the fitted curve relative to the set of corresponding $t_{\mathrm{d,m}}V_{O,\mathrm{m}}$ versus $t_{\mathrm{d,R}}V_{O,\mathrm{R}}$ data and the coverage factor is determined by the number of β_O values. Table 3 shows the uncertainty-related parameters for the variables affecting O.

3.4 Undershoot (preshoot)

Undershoot is, for waveforms with positive-going transitions, the maximum negative amplitude excursion relative to V_{S1} that the waveform makes near the transition region. On the other hand, for waveforms with negative-going transitions, undershoot is the maximum positive amplitude excursion relative to V_{S1} that the waveform makes near the transition region. For undershoot, we currently define "near the transition region" to be that period between the $t_{50\%} - 3t_d$ and $t_{50\%}$ (IEEE TC-10 Subcommittee on Pulse Techniques). Voltage offset errors are not considered here because they cancel, as they did for the uncertainty calculation of $V_{A,c}$. The equation describing the calculation for the undershoot is

$$U = \overline{\left(\frac{V_{\min,R} - V_{S1,R}}{V_{A,R}}\right)} + \Delta U_{\text{rec}}, \tag{28}$$

where ΔU_{rec} is the reconstruction-induced bias in the undershoot of the reconstructed waveform. $V_{\min,R}$ may be written

$$V_{\min,R} = V_{S1,R} + V_{U,R}, \tag{29}$$

where

$$V_{U,R} = \frac{\beta_U V_{U,m} t_{d,m}}{t_{d,R}}, \tag{30}$$

and β_U is a correction factor that is determined experimentally as for β_O. Equation (30) provides an empirical relationship between the undershoot and transition duration of the reconstructed (reported) waveform and that of the acquired waveform. As with overshoot (see Section 3.3), we have observed a bias and a noise-dependent variation in $V_{U,R}$ caused by the waveform reconstruction. The uncertainty in undershoot is computed by adding the computed uncertainty to the absolute value of the bias. We assume that the product of the undershoot voltage and transition duration is not affected by an all-pass filter, which is how the sampler impulse response is expected to behave for an input signal that has a 3 dB attenuation bandwidth lower than that of the sampler. $V_{U,m}$ can be expanded:

$$V_{U,m} = V_{\min,m} - V_{S1,m}. \tag{31}$$

Using (13), (26), (29), (30), and (31) in (28) yields for U:

$$U = \overline{\left(\frac{1}{V_{S2,R} - V_{S1,R}}\right)\left(\beta_U [V_{\min,m} - V_{S1,m}] \frac{X_m}{X_R}\right)} + \Delta U_{\text{rec}}. \tag{32}$$

The correction factor β_U is determined by fitting a curve to an M_9-element set of $t_{d,m} V_{U,m}$ versus $t_{d,R} V_{U,R}$ data. The uncertainty in β_U is the standard deviation in the fitted curve relative to the set of corresponding $t_{d,m} V_{U,m}$ versus $t_{d,R} V_{U,R}$ data and the coverage factor is determined by the number of β_U values. Table 4 shows the uncertainty-related parameters for the variables affecting U.

4. Summary

A detailed uncertainty analysis of the parameters transition duration, overshoot, undershoot and amplitude of step-like waveforms was performed. This analysis included a consideration of effects that can affect the value of the reported parameters, such as temperature, computation algorithms, history of instrument performance, equipment limitations and estimates of the response characteristics of the instrument. Our present published uncertainties, which are the result of the new measurement process and associated uncertainty analysis, for high-speed (7 ps < t_d < 350 ps) pulse generators and samplers are ±2 mV for pulse amplitude and ±1.5 ps for transition duration. Our measured uncertainties, however, are smaller.

Acknowledgements. We would like to thank W. F. Guthrie and G. N. Stenbakken of the NIST, Gaithersburg, Md., and D. C. DeGroot of the NIST, Boulder, Colo., for technical comments and B. A. Bell of the NIST, Gaithersburg, for administrative support.

Note: Electricity Division, Electronics and Electrical Engineering Laboratory, Technology Administration, Department of Commerce. Official contribution of the National Institute of Standards and Technology, not subject to copyright in the USA.

References

1. *NIST Calibration Services Users Guide,* NIST Spec. Publ. SP250, Washington, D.C., U.S. Dept. of Commerce, 1998, 189-193.
2. Paulter N. G., Larson D. R., *Improving the uncertainty analysis of NIST's pulse parameter measurement service*, Automatic Radio Frequency Techniques Group, 56th Conference, Boulder, Colo., 30 November to 1 December 2000.
3. Rush K., Draving S., Kerley J., *IEEE Spectrum,* 1990, 38-39.
4. Verspecht J., *IEEE Trans. Instrum. Meas.*, 1995, **44**, 991-997.
5. Henderson D., Roddie A. G., Smith A. J. A., *IEE Proceedings-A*, 1992, **139**, 254-260.
6. Larson D. R., Paulter N. G., *Proc. 17th IEEE Instrumentation and Measurement Technology Conference, IMTC 2000,* Baltimore, USA, 1-4 May 2000, 1425-1428.
7. Verspecht J., *IEEE Trans. Instrum. Meas.,* 1994, **43**, 210-215.
8. Pintelon R., Schoukens J., *IEEE Trans. Instrum. Meas.,* 1996, **45**, 588-593.

9. Stenbakken G. N., Deyst J. P., *IEEE Trans. Instrum. Meas.,* 1998, **47**, 1056-1061.
10. Wang C. M., Hale P. D., Coakley K. J., *IEEE Trans. Instrum. Meas.*, 1999, **48**, 1324-1332.
11. IEC 469-2: *International Electrotechnical Commission, Pulse Techniques and Apparatus, Part 2: Pulse Measurement and Analysis, General Considerations*, 1987.
12. Taylor B. N., Kuyatt C. E., *Guidelines for evaluating and expressing the uncertainty of NIST measurement results*, NIST Technical Note 1297, Washington, D.C., U.S. Dept. of Commerce, 1994.
13. Larson D. R., Paulter N. G., *Temperature effects on the high-speed response of digitizing sampling oscilloscopes*, NCSL International, 2000 Workshop and Symposium, Toronto, Canada, 16-20 July 2000.
14. Souders T. M., Waltrip B. C., Laug O. B., Deyst J. P., *IEEE Trans. Instrum. Meas.*, 1997, **46**, 947-953.
15. Dabózi T., Kollár I., *IEEE Trans. Instrum. Meas.,* 1996, **45**, 417-421.
16. Roy S., Souders T. M., *Proc. 17th IEEE Instrumentation and Measurement Technology Conference, IMTC 2000*, Baltimore, USA, 1-4 May 2000, 1429-1434.
17. Gans W. L., *IEEE Trans. Instrum. Meas.*, 1983, **IM-32**, 126-133.
18. Mittermayer C., Steininger A., *IEEE Trans. Instrum. Meas.*, 1999, **48**, 1103-1107.
19. Paulter N. G., Larson D. R., *Time-base setting dependence of pulse parameters using high-bandwidth digital sampling oscilloscopes*, NCSL International, 2000 Workshop and Symposium, Toronto, Canada, 16-20 July 2000.

Received on 27 September 2001 and in revised form on 3 January 2002.

Glossary of terms

Synonymous meanings: measured, acquired, or sampled waveform

Not-reported variables

c_i	partial differential equation
g	transient amplitude gain-correction term
L_1	low reference level (for example, the 10 % reference level)
$L_{1\_}$	data value found at $t_{L_{1\_}}$
L_{1_+}	data value found at $t_{L_{1_+}}$
L_2	high reference level (for example, the 90 % reference level)
$L_{2\_}$	data value found at $t_{L_{2\_}}$
L_{2_+}	data value found at $t_{L_{2_+}}$
M_1	number of waveforms in DUT measurement set
M_2	number of reference pulse waveforms measured with sampler and used in sampler gain calibration
M_3	number of reference pulse waveforms measured with reference instrument and used in sampler gain calibration
M_4	number of temperature measurements taken during DUT measurement process
M_5	number of temperature measurements performed during sampler impulse response characterization
M_6	number of amplitude-temperature data pairs used to determine temperature effects on sampler's amplitude response or pulse generator's output amplitude
M_7	number of independent waveforms used to estimate transition duration of sampler step response, from control chart
M_8	number of transition duration-temperature data pairs used to determine temperature effects on sampler's transition duration response or pulse generator's output transition duration
M_9	number of $t_{\mathrm{d,m}}V_{O,\mathrm{m}}$ versus $t_{\mathrm{d,R}}V_{O,\mathrm{R}}$ data pairs used to calculate β_O curve
M_{10}	number of gain terms, from control chart
M_{11}	number of $t_{\mathrm{d,m}}V_{U,\mathrm{m}}$ versus $t_{\mathrm{d,R}}V_{U,\mathrm{R}}$ data pairs used to calculate β_U curve
M_{12}	number of jitter measurements, from control chart
M_{13}	number of amplitude versus time data pairs used to calculate duration of sampling interval
$m_{L_{1\_}}$	time index for data having value closest to but $\leq L_1$
$m_{L_{1_+}}$	time index for data having value closest to but $\geq L_1$
$m_{L_{2\_}}$	time index for data having value closest to but $\leq L_2$
$m_{L_{2_+}}$	time index for data having value closest to but $\geq L_2$
N_{bins}	number of bins in histogram
N_{S1}	bin number in histogram corresponding to lower state level mode bin
N_{S2}	bin number in histogram corresponding to upper state level mode bin
P_1	percentage of pulse amplitude for reference level L_1
P_2	percentage of pulse amplitude for reference level L_2
$S_{\delta t/\Delta T}$	temperature-dependent change in pulse transition duration
$S_{\Delta V/\Delta T}$	temperature-dependent change in pulse amplitude
T	temperature
T_{meas}	temperature at which a particular waveform was recorded
T_{ref}	average temperature over which a set of waveforms was recorded
$t_{\mathrm{d,m}}$	transition duration, acquired waveform
$t_{\mathrm{d,r}}$	transition duration, sampler step response estimate convolved with jitter
$t_{\mathrm{d,R}}$	transition duration, reconstructed waveform
t_{L_1}	L_1 reference-level instant
$t_{L_{1\_}}$	data instant preceding t_{L_1}
$t_{L_{1_+}}$	data instant following t_{L_1}
t_{L_2}	L_2 reference-level instant

$t_{L_{2-}}$	data instant preceding t_{L_2}
$t_{L_{2+}}$	data instant following t_{L_2}
$V_{\mathrm{A,c}}$	pulse amplitude, corrected waveform
$V_{\mathrm{A,m}}$	pulse amplitude, acquired waveform
$V_{\mathrm{A,m,r}}$	pulse amplitude, acquired waveform, reference pulse
$V_{\mathrm{A,R}}$	pulse amplitude, reconstructed waveform
$V_{\mathrm{A,r}}$	pulse amplitude, reference-instrument acquired waveform, reference pulse
$V_{\mathrm{S1,c}}$	lower state level, corrected waveform
$V_{\mathrm{S1,m}}$	lower state level, acquired waveform
$V_{\mathrm{S1,m,r}}$	lower state level, acquired waveform, using reference pulse and sampler
$V_{\mathrm{S1,R}}$	lower state level, reconstructed waveform
$V_{\mathrm{S1,r,r}}$	lower state level, acquired waveform, using reference pulse and reference measurement instrument
$V_{\mathrm{max,m}}$	maximum value, acquired waveform
$V_{\mathrm{max,R}}$	maximum value, reconstructed waveform
$V_{\mathrm{min,m}}$	minimum value, acquired waveform
$V_{\mathrm{min,R}}$	minimum value, reconstructed waveform
$V_{\mathrm{S2,c}}$	upper state level, corrected waveform
$V_{\mathrm{S2,m}}$	upper state level, acquired waveform
$V_{\mathrm{S2,m,r}}$	upper state level, acquired waveform, using reference pulse and sampler
$V_{\mathrm{S2,R}}$	upper state level, reconstructed waveform
$V_{\mathrm{S2,r,r}}$	upper state level, acquired waveform, using reference pulse and reference measurement instrument
$V_{O,\mathrm{m}}$	overshoot value, acquired waveform
$V_{O,\mathrm{R}}$	overshoot value, reconstructed waveform
$V_{U,\mathrm{m}}$	undershoot value, acquired waveform
$V_{U,\mathrm{R}}$	undershoot value, reconstructed waveform
$V_{\Delta T}$	temperature-induced incremental change in pulse amplitude
X_{d}	non-integer number of sampling intervals in transition duration of reconstructed waveform
X_{j}	non-integer number of sampling intervals in transition duration of effective jitter step response
X_{m}	non-integer number of sampling intervals in transition duration of acquired waveform
X_{r}	non-integer number of sampling intervals in transition duration of sampler step response waveform
β_O	overshoot correction factor relating transition duration of measured and reconstructed waveforms
β_U	undershoot correction factor relating transition duration of measured and reconstructed waveforms
$\Delta t_{\mathrm{d},\Delta T}$	temperature-induced incremental change in transition duration
ΔV	histogram bin size
δt	sampling interval
ν_{eff}	effective degrees of freedom
ν_i	degrees of freedom

Reported variables

O	overshoot value
U	undershoot (preshoot) value
t_{d}	transition duration
V_{A}	pulse amplitude, reported
u_i	uncertainty of the i-th variable