Pulse parameter uncertainty analysis

N. G. Paulter and D. R. Larson

Abstract. A detailed uncertainty analysis is presented for the pulse parameter measurement service of the National Institute of Standards and Technology (NIST, USA). It relates to the new pulse parameter measurement and extraction processes. Uncertainties for pulse amplitude, transition duration, overshoot and undershoot (preshoot) are given. Effects of temperature variation, impulse response estimate, pulse parameter extraction algorithms, time-base distortion, calibration procedures and the waveform reconstruction process are included.

1. Introduction

The NIST supports a measurement service for high-speed (transition durations <20 ps) pulse generators that provides an estimate of the pulse parameters of amplitude and transition duration [1]. Overshoot and undershoot (preshoot) parameters were previously provided as well. However, support for these parameters was discontinued because of the lack of a viable uncertainty analysis [2], which is addressed by this paper.

The NIST is one of two national laboratories that provide a pulse parameter measurement service; the other being the National Physical Laboratory (NPL) in the United Kingdom. The NIST and NPL are performing a comparison of pulse parameter results, which includes measured data, corrected data (if applicable) and reconstructed data. The results to date indicate that both national laboratories are in close agreement.

The NIST measurement service currently uses commercial, high-bandwidth sampling oscilloscopes (3 dB attenuation, bandwidths of approximately 50 GHz) and pulse generators (3 dB attenuation, bandwidths of approximately 20 GHz) to measure the pulse parameters of short-transition-duration (high-speed) pulse generators and the step response of high-speed samplers. The purpose of this paper is to present our new uncertainty analysis for these parameters, which are pulse amplitude, transition duration, overshoot and undershoot.

For brevity, not all variables are described at the point of first use, but a list of variables and their description is provided in the glossary. Also, it should be pointed out that the step response of a device is equal to the convolution integral of the impulse response of that device with an ideal step. Both terms are used in this paper because transition durations and bandwidths of electronic systems are typically calculated from the step responses and, in waveform reconstruction, impulse responses are typically deconvolved.

2. Background

The pulse parameter measurement process that is used to acquire waveforms is briefly described in this section. The measurement process consists of a set of measurements of the customer’s pulse generator or sampler (the device under test, or DUT) and a set of instrument calibration measurements. Some of the instrument calibration measurements are made during the DUT measurement sequence and some are not. The calibration measurements include time-base errors, sampler gain, jitter and sampler step response. Figure 1 is a diagram of the NIST pulse parameter measurement system.

![Figure 1. Diagram of NIST pulse measurement system. The dotted lines indicate insertion of instruments used in time-base calibration.](image-url)
An estimate of the step response of the NIST 50 GHz sampler used to measure the DUT is obtained using the “nose-to-nose” method [3, 4], the results of which have been compared with results using swept frequency and optoelectronic methods [5]. We are currently examining the “nose-to-nose” method and its limitation in sampler calibration [6]. Measurements that are used to estimate the sampler step response, system jitter and dynamic gain of the sampler are taken routinely, but not necessarily as part of the DUT measurement sequence, and a control chart is maintained for the mean value and standard deviation of the transition durations of the sampler step response and the equivalent jitter step response and the sampler dynamic gain. The reason that the measurements of these parameters are not part of the DUT measurement sequence is that the step response, the system jitter and the sampler dynamic gain are stable (small observable variation) and the variations in these parameters are represented in their associated control chart data. The DUT jitter is intentionally excluded from the system jitter because DUT jitter affects the DUT parameters of transition duration, overshoot and undershoot and will be exhibited in customer measurements.

Several sets of data are acquired for the customer’s DUT. For brevity, the discussion that follows assumes that the DUT is a pulse generator. A set of data consists of $M_1$ sampler-acquired DUT waveforms and one measurement of the time-base errors. Measurements of the time-base errors [7-10] are a routine part of the DUT measurement procedure. The DUT measurement sequence is as follows:

1. measure time-base error: one independent measurement;
2. acquire waveforms: $M_1$ independent measurements of DUT output.

The DUT waveforms are subsequently corrected for gain and time-base errors only if these errors are large relative to the reported uncertainties. The corrected or uncorrected waveforms are then used in a reconstruction process to obtain a waveform that is an accurate estimate of the pulse measured by the sampler. The accuracy of this estimate (the reconstructed waveform) is dependent on the reconstruction process and the accuracy of the estimate of the sampler impulse response. The waveform reconstruction process uses an iterative deconvolution of the sampler impulse response from the measured data. From each reconstructed waveform, pulse parameter values are computed. The set of pulse parameter values thus computed is used to determine the mean value and standard deviation for the given parameter.

The pulse parameter computations are based on histogram methods [11]. The first step in the calculations is to compute the histogram of the waveform. Next the top-line, $V_{52}$, and bottom-line, $V_{51}$, values are obtained from the histogram. Then, using $V_{52}$ and $V_{51}$, the pulse parameters are obtained for the waveform.

This uncertainty analysis, because it is applied to acquired waveforms, is applicable to both the measurement of the output of pulse generators and the step response of samplers with the appropriate change in reference measurements and waveforms. When measuring the output of a pulse generator it is assumed that the sampler step response is the reference, and when measuring the sampler step response it is assumed that the output of the pulse generator is the reference.

3. Uncertainty analysis

The reported pulse parameters are an average of the particular pulse parameters obtained from a set of $M_1$ pulse waveforms measured using the NIST pulse measurement systems. The average of a parameter, $W$ for example, is given by

$$W = \frac{1}{M_1} \sum_{i=1}^{M_1} W_i(\alpha_j), \quad (1)$$

where $M_1$ is the number of values for the parameter $W$, one value for each waveform, and $W$ is dependent on a number of variables, $\alpha_j$. The uncertainty for this average, $\overline{u_W}$ for example, is given by

$$\overline{u_W} = k_{eff} \sqrt{\sum_{i=1}^{M_1} \left\{ \left( \frac{\partial W}{\partial W_i} \right)^2 \sum_j \left( \frac{\partial W_i(\alpha_j)}{\partial \alpha_j} \right)^2 u_j^2 \right\}}$$

$$= k_{eff} \sqrt{\frac{1}{M_1} \sum_{i=1}^{M_1} \left[ \left( \frac{\partial W_i(\alpha_j)}{\partial \alpha_j} \right)^2 u_j^2 \right]} \quad (2a)$$

$$= k_{eff} \sqrt{\frac{1}{M_1} \sum_j \left( \frac{\partial W_i(\alpha_j)}{\partial \alpha_j} \right)^2 u_j^2} \quad (2b)$$

where it is assumed in (2a) that the $\alpha_j$ values are uncorrelated, which is the reason why there are no cross terms in the partial derivatives with respect to the $\alpha_j$. In (2b) it is further assumed that the $u_j$ values are the same for every $W_i$; that is, the uncertainties in the variables for a given parameter are the same for every waveform. The term $k_{eff}$ is the statistical weight [12] applied to the uncertainties of variables obtained from a limited number of trials. For a number of variables with different degrees of freedom, $k_{eff}$ is found by first calculating the effective degrees of freedom using [12]

$$\nu_{eff} = \frac{M \sum_{i=1}^{M} \left( \frac{\partial W_i(\alpha_1,\alpha_2,...,\alpha_M)}{\partial \alpha_1} \right)^2 u_i^2}{\sum_{i=1}^{M} \frac{c_i u_i^4}{\nu_i}}, \quad (3)$$
### Table 1a. Variables affecting pulse amplitude uncertainty.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
<th>Partial derivative</th>
<th>Type</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_i )</td>
<td>( u_i )</td>
<td>( \frac{\partial W(0_1, \ldots, 0_p)}{\partial \alpha_i} )</td>
<td>A</td>
<td>( M_i - 1 )</td>
</tr>
</tbody>
</table>

\[
V_{S2,m} \quad \sqrt{\sigma_{S2,m}^2 + \left( \frac{\partial k_{S2,m}}{\partial H} \right)^2 u_H^2} \quad \frac{1}{\sqrt{M_i}} \quad \frac{1}{\tilde{g}} \quad A \quad M_i - 1
\]

\[
V_{S1,m} \quad \sqrt{\sigma_{S1,m}^2 + \left( \frac{\partial k_{S1,m}}{\partial H} \right)^2 u_H^2} \quad \frac{1}{\sqrt{M_i}} \quad \frac{1}{\tilde{g}} \quad A \quad M_i - 1
\]

\[
\tilde{g} \quad \sqrt{\sigma_{g}^2 + \left( \frac{\partial k_{g}}{\partial V, H} \right)^2 u_{H,V}^2} \quad \frac{V_{S1}}{\tilde{g}} \quad A \quad M_{i0} - 1
\]

\[
\mathcal{T}_{\text{means}} \quad \sigma_{\mathcal{T}_{\text{means}}} \quad S_{\Delta V/\Delta T} \quad A \quad M_{i0} - 1
\]

\[
\mathcal{T}_{\text{ref}} \quad \sigma_{\mathcal{T}_{\text{ref}}} \quad S_{\Delta V/\Delta T} \quad A \quad M_{i} - 1
\]

\[
S_{\Delta V/\Delta T} \quad \sigma_{S_{\Delta V/\Delta T}} \quad \frac{\tilde{g}}{\mathcal{T}_{\text{means}} - \mathcal{T}_{\text{ref}}} \quad A \quad M_{i0} - 2
\]

1. The letter \( \tilde{g} \) is used to represent amplitude-related uncertainty contributions to \( \tilde{g} \), see (8) and Table 1c. The letter \( H \) is used to represent histogram-related uncertainty contributions, see (11) and Table 1b.

### Table 1b. Variables affecting the uncertainty in \( V_{S2} \) obtained using a histogram method.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
<th>Partial derivative</th>
<th>Type</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_i )</td>
<td>( u_i )</td>
<td>( \frac{\partial W(0_1, \ldots, 0_p)}{\partial \alpha_i} )</td>
<td>A</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

\[
V_{\text{max}} \quad \sigma_{V_{\text{max}}} \quad (= 0) \quad \frac{N_{S2}}{N_{\text{hist}}} \quad A \quad \infty
\]

\[
V_{\text{min}} \quad \sigma_{V_{\text{min}}} \quad (= 0) \quad \frac{N_{S2}}{N_{\text{hist}}} \quad A \quad \infty
\]

\[
N_{S2} \quad u_{N_{S2}} \quad (= 0) \quad \frac{V_{\text{max}} - V_{\text{min}}}{N_{\text{hist}}} \quad B \quad \infty
\]

\[
N_{\text{hist}} \quad u_{N_{\text{hist}}} \quad (\leq \pm 1/10 N_{\text{hist}}) \quad \frac{V_{\text{max}} - V_{\text{min}}}{N_{\text{hist}}} \quad B \quad \infty
\]

### Table 1c. Variables affecting \( g \).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
<th>Partial derivative</th>
<th>Type</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_i )</td>
<td>( u_i )</td>
<td>( \frac{\partial W(0_1, \ldots, 0_p)}{\partial \alpha_i} )</td>
<td>A</td>
<td>( M_i - 1 )</td>
</tr>
</tbody>
</table>

\[
V_{S2,m,r} \quad \sqrt{\sigma_{S2,m,r}^2 + \left( \frac{\partial k_{S2,m,r}}{\partial H} \right)^2 u_H^2} \quad \frac{1}{\sqrt{M_i}} \quad \frac{1}{V_{S2,m,r} - V_{S1,m,r}} \quad A \quad M_i - 1
\]

\[
V_{S1,m,r} \quad \sqrt{\sigma_{S1,m,r}^2 + \left( \frac{\partial k_{S1,m,r}}{\partial H} \right)^2 u_H^2} \quad \frac{1}{\sqrt{M_i}} \quad \frac{1}{V_{S2,m,r} - V_{S1,m,r}} \quad A \quad M_i - 1
\]

\[
V_{S2} \quad \sqrt{\sigma_{S2}^2 + \left( \frac{\partial k_{S2}}{\partial H} \right)^2 u_H^2} \quad \frac{1}{\sqrt{M_i}} \quad \frac{V_{S2,m,r} - V_{S1,m,r}}{V_{S2,m,r} - V_{S1,m,r}} \quad A \quad M_i - 1
\]

\[
V_{S1} \quad \sqrt{\sigma_{S1}^2 + \left( \frac{\partial k_{S1}}{\partial H} \right)^2 u_H^2} \quad \frac{1}{\sqrt{M_i}} \quad \frac{V_{S2,m,r} - V_{S1,m,r}}{V_{S2,m,r} - V_{S1,m,r}} \quad A \quad M_i - 1
\]

1. The letter \( H \) is used to represent histogram-related uncertainty contributions see (11) and Table 1b.
Table 2a. Variables affecting pulse transition duration uncertainty.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty $\sigma_u$</th>
<th>Partial derivative $\frac{\partial V}{\partial \alpha}$</th>
<th>Type $\nu_i$</th>
<th>Degrees of freedom $\nu_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta t$</td>
<td>$\sigma_{\delta t}$</td>
<td>$\frac{\partial \delta t}{\partial \alpha}$</td>
<td>A</td>
<td>$M_{13} - 1$</td>
</tr>
<tr>
<td>$X_m$</td>
<td>$\sqrt{\sigma^2_{X_m} + \left( \frac{\partial X_m}{\partial \alpha} \right)^2 + \left( \frac{\partial X_m}{\partial \alpha} \right)^2}$</td>
<td>$\frac{1}{M_i} \sum_{j=1}^{M_i} \left( \frac{1}{M_i} \frac{\partial \delta t_{i,j}}{\partial \delta t_{i,j}} \right)^2$</td>
<td>A</td>
<td>$M_i - 1$</td>
</tr>
<tr>
<td>$X_i$</td>
<td>$\sqrt{\sigma^2_{X_i} + \left( \frac{\partial X_i}{\partial \alpha} \right)^2}$</td>
<td>$\frac{1}{M_i} \sum_{j=1}^{M_i} \left( \frac{1}{M_i} \frac{\partial \delta t_{i,j}}{\partial \delta t_{i,j}} \right)^2$</td>
<td>A</td>
<td>$M_{12} - 1$</td>
</tr>
</tbody>
</table>

(see Tables 2b and 2d)

Table 2b. Variables affecting $X$ value uncertainty.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty $\sigma_u$</th>
<th>Partial derivative $\frac{\partial V}{\partial \alpha}$</th>
<th>Type $\nu_i$</th>
<th>Degrees of freedom $\nu_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{1-}$</td>
<td>$\sigma_{\alpha_{1-}}$</td>
<td>$\left( \alpha_{1-} - \alpha_{1-} \right)$</td>
<td>A</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\alpha_{1+}$</td>
<td>$\sigma_{\alpha_{1+}}$</td>
<td>$\left( \alpha_{1+} - \alpha_{1+} \right)$</td>
<td>A</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\alpha_{2-}$</td>
<td>$\sigma_{\alpha_{2-}}$</td>
<td>$\left( \alpha_{2-} - \alpha_{2-} \right)$</td>
<td>A</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\alpha_{2+}$</td>
<td>$\sigma_{\alpha_{2+}}$</td>
<td>$\left( \alpha_{2+} - \alpha_{2+} \right)$</td>
<td>A</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$L_1$</td>
<td>$\sqrt{\sigma^2_{L_1} + \left( \frac{\partial L_1}{\partial \alpha} \right)^2}$</td>
<td>$\left( \alpha_{1-} - \alpha_{1-} \right) \frac{1}{L_1 - L_1}$</td>
<td>A</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$L_2$</td>
<td>$\sqrt{\sigma^2_{L_2} + \left( \frac{\partial L_2}{\partial \alpha} \right)^2}$</td>
<td>$\left( \alpha_{1+} - \alpha_{1+} \right) \frac{1}{L_1 - L_1}$</td>
<td>A</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

(see Table 2c)

| $L_3$ | $\sigma_{L_3}$ | $\left( \alpha_{2-} - \alpha_{2-} \right)$ | A | $\infty$ |
| $L_4$ | $\sigma_{L_4}$ | $\left( \alpha_{2+} - \alpha_{2+} \right)$ | A | $\infty$ |

where $\nu_i$ is the number of degrees of freedom for the parameters (shown in Tables 1 to 4) and $\sigma_u$ are the partial differential equations (shown in Tables 1 to 4). Weighted $k_{eff}$ is then found from $k_{eff}$ using the $f$-distribution [12].

The uncertainties in the variables, $u_i$ (where the subscript refers to parameter $\gamma$), are obtained from independent measurements that provide values for these particular variables. The tables list the source of $u_i$ for the appropriate variables. To calculate the uncertainty of the different parameters, the partial derivatives of these parameters with respect to the independent variables must be calculated. These partial derivatives are also
Table 2c. Variables affecting uncertainty of $L_1$ and $L_2$.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
<th>Partial derivative</th>
<th>Type</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{S1,m}$</td>
<td>see Table 1a</td>
<td>$1 - P%$</td>
<td>A</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$V_{S2,m}$</td>
<td>see Table 1a</td>
<td>$P%$</td>
<td>A</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Table 2d. Variables affecting uncertainty of $X_m$.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
<th>Partial derivative</th>
<th>Type</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\Delta t/\Delta T}$</td>
<td>$\sigma_{S_{\Delta t/\Delta T}}$</td>
<td>$\frac{\partial W}{\partial \sigma_{S_{\Delta t/\Delta T}}}$</td>
<td>A</td>
<td>$M_6 - 2$</td>
</tr>
<tr>
<td>$T_{\text{meas}}$</td>
<td>$\sigma_{T_{\text{meas}}}$</td>
<td>$\frac{\partial W}{\partial T_{\text{meas}}}$</td>
<td>A</td>
<td>$M_4 - 1$</td>
</tr>
<tr>
<td>$T_{\text{ref}}$</td>
<td>$\sigma_{T_{\text{ref}}}$</td>
<td>$\frac{\partial W}{\partial T_{\text{ref}}}$</td>
<td>A</td>
<td>$M_5 - 1$</td>
</tr>
</tbody>
</table>

Table 3. Variables affecting uncertainty in pulse overshoot.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
<th>Partial derivative</th>
<th>Type</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{max,s},m}$</td>
<td>$\sigma_{V_{\text{max},m}}$</td>
<td>$\frac{1}{M} \left[ \sum_{i=1}^{M} \beta_0 \frac{X_{m,i}}{X_{R,i}} \frac{1}{V_{A,R,i}} \right]^2$</td>
<td>A</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$V_{S2,m}$</td>
<td>$\sigma_{V_{S2,m}}$</td>
<td>$\frac{1}{M} \left[ \sum_{i=1}^{M} \beta_0 \frac{X_{m,i}}{X_{R,i}} \frac{1}{V_{A,R,i}} \right]^2$</td>
<td>A</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$V_{S2,R}$</td>
<td>$\sigma_{V_{S2,R}}$</td>
<td>$\frac{1}{M} \left[ \sum_{i=1}^{M} \left( \frac{1}{V_{A,R,i}} \frac{1}{V_{R,i}} \frac{1}{V_{A,R,i}} \frac{1}{V_{R,i}} \frac{1}{V_{A,R,i}} \frac{1}{V_{R,i}} \frac{1}{V_{A,R,i}} \frac{1}{V_{R,i}} \right)^2 \beta_0</td>
<td>V_{\text{max},m,i} - V_{S2,m,i}</td>
<td>\frac{X_{m,i}}{X_{R,i}}</td>
</tr>
<tr>
<td>$V_{S1,R}$</td>
<td>$\sigma_{V_{S1,R}}$</td>
<td>$\frac{1}{M} \left[ \sum_{i=1}^{M} \left( \frac{1}{V_{A,R,i}} \frac{1}{V_{R,i}} \frac{1}{V_{A,R,i}} \frac{1}{V_{R,i}} \frac{1}{V_{A,R,i}} \frac{1}{V_{R,i}} \frac{1}{V_{A,R,i}} \frac{1}{V_{R,i}} \right)^2 \beta_0</td>
<td>V_{\text{max},m,i} - V_{S2,m,i}</td>
<td>\frac{X_{m,i}}{X_{R,i}}</td>
</tr>
</tbody>
</table>

| $X_m$ | $\sqrt{\sigma_{X_m}^2 + \left( \frac{\partial X_m}{\partial \theta} \right)^2}$ | $\frac{1}{M} \left[ \sum_{i=1}^{M} \beta_0 | V_{\text{max},m,i} - V_{S2,m,i} | \frac{X_{m,i}}{X_{R,i}} | \frac{1}{V_{A,R,i}} \right]^2$ | A | $\infty$ |
| $X_d$ | $\sqrt{\sigma_{X_d}^2 + \left( \frac{\partial X_d}{\partial \theta} \right)^2}$ | $\frac{1}{M} \left[ \sum_{i=1}^{M} \beta_0 | V_{\text{max},m,i} - V_{S2,m,i} | \frac{X_{m,i}}{X_{R,i}} | \frac{1}{V_{A,R,i}} \right]^2$ | A | $\infty$ |

1. The prime notation indicates that the uncertainty for these parameters must include histogram-dependent uncertainties, as for the example $V_{S2}$ in Section 3.1. That is, the prime notation indicates

$$\sigma' = \sqrt{\sigma^2 + \left( \frac{\partial V}{\partial \theta} \right)^2},$$

(33)

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shown in the tables. The tables for each variable include its type of uncertainty \([12]\) and degrees of freedom, \(\nu\). For measured data, the number of degrees of freedom is given by \(\nu = M_k - 1\), where \(M_k\) is the number of data elements used to compute the value of the \(k\)-th variable. For fits to data, \(\nu\) is given by \(\nu = M_k - m_k\), where \(m_k\) is the number of coefficients used to fit the data. The number of degrees of freedom for certain variables is equal to infinity \((\nu = \infty)\) because the calculation of the value of these variables is based on a specific fixed waveform. Accordingly, every (an infinite set) computation of the value of that variable for that waveform yields the same result.

The variation in measurements, represented by the symbol \(\sigma\) in the tables and text, is unless otherwise indicated the standard deviation of a set of measurement values of a given parameter or the standard deviation of the residuals of a curve fitted to the data. For example, of the first case, \(\sigma_{V_{S2,m}}\) in Table 1a is the standard deviation of \(M_1\) values, one \(V_{S2,m}\) value taken from each of the \(M_1\) measured (acquired) waveforms. For example, of the second case, the \(\sigma_{S_{AV/\Delta T}}\) in Table 1a is the standard deviation of the residuals to a fit to the amplitude versus temperature data.

### 3.1 Pulse amplitude

The pulse amplitude is obtained using a histogram-based algorithm (see Section 2). The pulse amplitude is the difference between \(V_{S2}\) and \(V_{S1}\). Calculating the uncertainty in the pulse amplitude requires an equation that describes the reported pulse amplitude, \(V_{\bar{A}}\):

\[
V_{\bar{A}} = \frac{V_{Ac}/\bar{f}}{\bar{g}},
\]  

(4)

where the horizontal bars indicate the arithmetic mean, \(V_{Ac}/\bar{f}\) is the mean of the set of \(M_1\) pulse amplitudes corrected for sampler offset errors and waveform reconstruction errors, and \(\bar{g}\) is the mean of the transient amplitude gain correction of the sampler. A common practice in oscilloscope calibrations is to use or include a static level gain-correction term in (4). However, since the signals being measured are transients (steps or pulses), as opposed to static levels, a transient gain term, the \(\bar{g}\) in (4), should be used. The transient gain is affected by the impulse response of the sampler and the waveform epoch because of the settling response of the sampler. For example, if the sampler response...
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has not settled by the end of the epoch, then \( g \) will be less than one for that epoch. Ideally, \( \bar{g} = 1 \) if the sampler exhibits no pulse gain or attenuation and the sampler has settled within the waveform epoch. The \( \bar{V}_{\Delta T} \) is given by

\[
\bar{V}_{\Delta T} = \frac{1}{M_{\Delta T}} \sum_{i=1}^{M_{\Delta T}} V_{\Delta T,i},
\]

(5)

where \( V_{\Delta T} \) is the voltage offset, which is a bias in the observed voltage, \( V_{\Delta T} \) is the mean of the set of \( M_{\Delta T} \) pulse amplitudes corrected for sampler offset errors, \( V_{\Delta T} \) is the mean of the amplitude corrections required for a change in measurement temperature, and \( \beta_k \) reflects the error in the amplitude of the reconstructed waveform caused by the reconstruction process. Ideally, \( \beta_k \) should be 1 because the sampler impulse response estimates integrate to 1. However, the reconstruction process introduces an error in the amplitude. This scaling error is exactly corrected by rescaling the pulse amplitude of the reconstructed waveform to equal \( V_{\Delta T} \).

The subscripts c and m refer to corrected and measured variables. We have observed that for the currently available high-bandwidth samplers, the voltage offset error is the same for both the top-line (S2) and bottom-line (S1) voltage levels; therefore, the offset voltage contribution can be ignored.

The temperature-correction term is obtained by measuring the change in the observed pulse amplitude with temperature [13]. The \( S_{\Delta V/\Delta T} \) term is therefore

\[
V_{\Delta T} = S_{\Delta V/\Delta T}(T_{\text{meas}} - T_{\text{ref}}),
\]

(6)

where \( T_{\text{meas}} \) is the average of \( M_{\Delta T} \) sampler temperature values taken during the pulse measurement process, \( S \) is the slope of a straight-line fit through a set of previously acquired amplitude versus temperature data, and \( T_{\text{ref}} \) is the average reference sampler temperature that is taken to be the mean of \( M_{\Delta T} \) temperature values of the sampling head measured when the sampler impulse response was determined. The amplitude versus temperature data consist of a set of \( M_{\Delta T} \) data pairs and are recorded over a temperature range between \( T_2 \) and \( T_1 \); the difference between these two temperatures is \( \Delta T \). Every sampler may exhibit a unique temperature-dependent response.

Amplitude \( V_{\Delta T} \) can now be rewritten using (5) and (6) in (4):

\[
V_{\Delta T} = \frac{(V_{\text{S2,m,min}} - V_{\text{S1,m,min}})}{\bar{g}} + S_{\Delta V/\Delta T}(T_{\text{meas}} - T_{\text{ref}}),
\]

(7)

where the subscript \( r \) denotes the reference pulse measurement and there are \( M_{\Delta T} \) independent gain terms. Using the ratio of average amplitudes in (8) is numerically more stable than an average of the amplitude ratios. Furthermore, the ratio of the average amplitudes relaxes the requirement that a reference measurement be made for each sampler-acquired measurement. A set of \( M_{\Delta T} \) measurements of the reference pulse are used to obtain the reference-instrument-measured parameters and a set of \( M_{\Delta T} \) measurements of the reference pulse are used to obtain the sampler-measured parameters. Temperature-dependent gain effects are included in \( V_{\Delta T} \), which describes the change in pulse amplitude with change in temperature relative to \( T_{\text{ref}} \). The \( g_i \) were computed from data taken at \( T_{\text{ref}} \).

The variables shown in (8) contribute to the uncertainty in \( V_{\Delta T} \). The uncertainty in \( V_{\Delta T} \), \( u_{V_{\Delta T}} \), is dependent on the uncertainties of all the variables from which \( V_{\Delta T} \) is dependent, and these variables are listed in Tables 1a and 1b. The uncertainties, \( u_i \), in the variables are obtained from independent measurements that provide values for these particular variables. Table 1a lists the source of \( u_i \) for the appropriate variables. The partial derivatives of \( V_{\Delta T} \) with respect to the independent variables are also shown in Table 1a, as are its associated degrees of freedom and uncertainty type [11]. \( V_{\text{meas}} \), although obtained from \( M_{\Delta T} \) averages of the average of \( M_{\Delta T} \) temperature measurements taken during each waveform acquisition, has \( \nu = M_{\Delta T} - 1 \). The uncertainties for \( S_{\Delta V/\Delta T} \) require an empirical formula relating the pulse amplitude of the acquired waveform to temperature (see Figure 2) which is obtained by fitting a curve, typically a line, to the data.

In addition to measurement-related uncertainties, the reported amplitude values are also subject to uncertainties from the method used to calculate these values, in this case a histogram. The histogram-derived amplitude values, for example for \( V_{\text{S2}} \), are given by

\[
V_{\text{S2}} = N_{\text{bins}}(V_{\text{max}} - V_{\text{min}}),
\]

(9)

where

\[
\Delta V = \frac{V_{\text{max}} - V_{\text{min}}}{N_{\text{bins}}},
\]

(10)

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and $\Delta V$ is the histogram bin size and $N_{\text{bins}}$ is the
number of histogram bins. The uncertainty contribution
associated with $V_{\text{S2}}$, for example, is then

$$
\frac{\partial V_{\text{S2}}}{\partial H} u_H = \frac{1}{N_{\text{bins}}} \times
\left[ \frac{(V_{\text{max}} - V_{\text{min}})^2 u_{V_{\text{max}}}^2 + N_{\text{bins}}^2}{\sigma_{V_{\text{max}}}^2 + \sigma_{V_{\text{min}}}^2 + \frac{1}{N_{\text{bins}}^2} (V_{\text{max}} - V_{\text{min}})^2 u_{N_{\text{bins}}}^2} \right].
$$

The term $H$ is used as a place holder to represent all the
histogram-based dependences of a variable, such as $V_{\text{S2}}$
in the example of (11). Table 1b, used as an example,
lists variables affecting the $V_{\text{S2}}$ amplitude variables. All
amplitude values listed in the first column of Table 1a
have an analogous list of variables. The degrees
of freedom are infinite for $N_{\text{S2}}$ and $N_{\text{bins}}$ because they are
extracted using a given pulse parameter algorithm and
the output of this algorithm will not vary for a given
waveform. The degrees of freedom are infinite for $V_{\text{max}}$
and $V_{\text{min}}$ because these values, for a given waveform,
are fixed once a waveform has been acquired.

### 3.2 Transition duration

The transition duration is the difference between the
occurrences of user-defined amplitude reference levels,
for example, the 10 % and 90 % amplitude reference
levels. The times at which the waveform crosses
these reference levels are called reference-level instants.
Accordingly, the 10 % to 90 % transition duration is
the difference between the 90 % and 10 % reference
level instants.

The reported waveform transition duration, $t_d$, is the average transition duration extracted from $M_1$
reconstructed pulse waveforms, $t_{\text{dLR}}$. Duration $t_d$ is related to the transition duration of the acquired
waveform, $t_{\text{dLM}}$, and the transition duration of the
estimated sampler step response, $t_{\text{dJR}}$:

$$
t_d = \frac{1}{M_1} \sum_{i=1}^{M_1} t_{\text{dLR},i} \times
\frac{f_{\text{dec}}(t_{\text{dLM}} + \Delta t_{\text{d},\Delta T} t_{\text{dLR}})}{\Delta t_{\text{d},\text{dec}}},
$$

where $\Delta t_{\text{d},\Delta T}$ is the temperature-induced incremental
change in transition duration [13] (described below),
$\Delta t_{\text{d},\text{dec}}$ is the bias in the transition duration caused by
the reconstruction process, and $f_{\text{dec}}$ is used to indicate
the deconvolution functional relationship between $t_{\text{dLR}},$
$t_{\text{dLM}}$ and $t_{\text{dJR}}$. Duration $t_{\text{dLR}}$ is found from a waveform
that is obtained by deconvolving a waveform with
transition duration $t_{\text{dJR}}$ from a waveform with transition
duration $t_{\text{dLM}}$. The specific functional relationship, $f_{\text{dec}}$,
between $t_d$, $t_{\text{dLM}}$, and $t_{\text{dJR}}$ is dependent on the type of
waveform used and can only be derived for certain ideal
waveforms. For example, for Gaussian waveforms, $t_d$
is equal to the square root of the difference of the
squares of transition durations of the measured and
step response waveforms. For general waveforms, we
can obtain an empirical relationship relating the three
parameters. The first step in obtaining this empirical
relationship is to vary either $t_{\text{dLM}}$ or $t_{\text{dJR}}$ in the waveform
reconstruction process, keeping the other constant, and
noting the variation in $t_{\text{dLR}}$. This provides two sets
of data, one relating $t_{\text{dLR}}$ to $t_{\text{dLM}}$ for a fixed $t_{\text{dJR}}$
and another relating $t_{\text{dLR}}$ to $t_{\text{dJR}}$ for a fixed $t_{\text{dLM}}$. We obtain
our empirical relationship by fitting a curve (such as
a polynomial) to these sets of data, $t_{\text{dLM}}$ versus $t_{\text{dJR}}$
and $t_{\text{dJR}}$ versus $t_{\text{dLM}}$. The reconstruction process we
now use is described in [15]. However, we are also
investigating various filtering methods, one of which
is discussed in [16]. Using our present reconstruction
algorithms on known waveforms, we have observed a
bias, $\Delta t_{\text{d},\text{rec}}$, and a noise-dependent variation in $t_{\text{dJR}}$.
The noise-dependent variation is also contained in the
empirical relationship, but the bias is not. Therefore, the
uncertainty for the reconstruction process is computed
by adding the absolute value of the bias to the computed
uncertainty.

The parameters $t_{\text{dLM}}$ and $t_{\text{dJR}}$ can be expressed in
terms of the sampling intervals:

$$
t_{\text{dLM}} = X_{\text{m}} \delta t,
$$

$$
t_{\text{dJR}} = X_{\text{JR}} \delta t,
$$

where $X_{\text{m}}$ and $X_{\text{JR}}$ are the real-valued (non-integer)
number of sampling intervals describing the transition
duration for the measured sampler step response
waveforms (including jitter) and $\delta t$ is the duration of
the equispaced sampling interval. $X_{\text{JR}}$ is computed from
waveforms that are the result of the convolution of the
system jitter, represented by a Gaussian waveform, and the sampler step response, obtained from the sampler calibration method. The result of this convolution, estimated using the Gaussian approximation, is

\[ X_{ij} \approx \sqrt{X_i^2 + X_j^2}, \]  

where \( X_i \) and \( X_j \) are the number of sampling intervals in the sampler step response and equivalent jitter step response transition durations. However, there are errors associated with this approximation [17]. For this uncertainty analysis the convolutions of the two waveforms are performed numerically and the resultant waveform used in subsequent processing. The measurement jitter typically has a normal distribution. We approximate \( X_j \) using a direct measurement taken by the sampler. We have observed that estimates of the jitter obtained employing a geometric method [18] are very sensitive to the transition region from which the estimate is made. Term \( X_j \) includes drift of the sampling aperture with respect to its trigger. Term \( \delta t \) is the average duration of the sampling intervals that span either the transition region of the waveform or the entire waveform [19] and is measured using sine-fit techniques [9] during the time-base calibration process. Figure 3 shows an example of time-base errors (vertical axis) versus measurement time. One time-base calibration is performed for each waveform; therefore the variation in \( \delta t \) is the variation of \( \delta t \) among the \( M_j \) acquired waveforms. Similarly, the variation in \( X_m \) is dependent on the set of \( M_j \) acquired waveforms. The variation in \( X_m \) is dependent on the set of \( M_j \) acquired reference waveforms. The variation in \( X_j \) is dependent on the set of \( M_j \) acquired reference waveforms. The temperature-dependent change in transition duration can be expanded:

\[ \Delta t_{i,j} = S_{\Delta t/\Delta T}(T_{\text{meas}} - T_{\text{ref}}), \]  

where \( S_{\Delta t/\Delta T} \) is the slope of a straight-line fit to \( M_j \) transition-duration-temperature data pairs (see Figure 4), measured independently of the \( M_j \) acquired waveforms. Using (13) and (15) in (12) gives

\[ t_d = \frac{f_{\text{dec}} \cdot X_m \delta t + S_{\Delta t/\Delta T}(T_{\text{meas}} - T_{\text{ref}})X_j \delta t}{S_{\Delta t/\Delta T}(T_{\text{meas}} - T_{\text{ref}})X_j \delta t} + \Delta t_{\text{rec}}. \]  

For the samplers and pulse generators currently in use at the NIST, the value of \( \Delta t_{\text{rec}} \) is approximated as zero because it is much less than the reported uncertainties; however, it is retained here for completeness. As we did in the pulse amplitude uncertainty analysis, we generate a table for the variables contributing to the transition duration value (see Table 2a).

The values of \( X_m \), \( X_j \), and \( X_j \) are determined by linearly interpolating to obtain the instant in time (the reference-level instant) corresponding to the given reference level. The value of \( X_m \) (and analogously for \( X_i \) and \( X_j \)) is

\[ X_m = \frac{t_{L_2} - t_{L_1}}{\delta t}, \]  

where \( t_{L_1} \) and \( t_{L_2} \) are the time instances corresponding to the first \( (L_1) \) and second \( (L_2) \) reference levels of the transition duration. For example, in the 10 % to 90 % transition duration, \( L_1 \) is the 10 % reference-level instant and \( L_2 \) is the 90 % reference-level instant. Instant \( t_{L_1} \) is given by

\[ t_{L_1} = t_{L_1} + \frac{t_{L_1} - t_{L_1}^-}{t_{L_1}^- - t_{L_2}^+}(L_1 - L_1^-), \]  

where the subscripts + and − denote the actual sampling instances found on either side of the reference-level instant (either \( t_{L_1} \) or \( t_{L_2} \)) and the data values
corresponding to the reference-level instants. Similarly, $t_{L_2}$ is given by

$$t_{L_2} = t_{L_2-} + \frac{t_{L_2+} - t_{L_2-}}{L_{2+} - L_{2-}} (L_2 - L_2-) \tag{19}.$$ 

Using (18) and (19) in (17) yields

$$X_m = [t_{L_4+} + \frac{t_{L_4+} - t_{L_4-}}{L_{4+} - L_{4-}} (L_4 - L_4-) - t_{L_4+} - \frac{t_{L_4+} - t_{L_4-}}{L_{4+} - L_{4-}} (L_4 - L_4-) / \delta t]$$

$$= m_{L_2+} - m_{L_2-} + (m_{L_2+} - m_{L_2-}) \times$$

$$\frac{L_2 - L_2-}{L_{2+} - L_{2-}} - \frac{(m_{L_1+} - m_{L_1-}) (L_1 - L_1-)}{L_{1+} - L_{1-}} \tag{20}.$$ 

where the $m_L$ terms are the time indices corresponding to the actual data found immediately above and below the reference levels. Using (20), we can obtain the uncertainty expansion for $X_m$ (and, similarly, for $X_f$ and $X_d$), which is shown in Table 2b. All the variables listed in Table 2b are extracted using our pulse parameter algorithm and, consequently, the degrees of freedom are infinite. The values of $L_1$ and $L_2$ can be expanded:

$$L_1 = V_{S1,m} + P_1 (V_{S2,m} - V_{S1,m}) \tag{21};$$

$$L_2 = V_{S1,m} + P_2 (V_{S2,m} - V_{S1,m}) \tag{21};$$

where $P_1$ and $P_2$ are the percentage reference values, such as 10 % and 90 % or 20 % and 80 %. Table 2c provides an uncertainty assessment for $L_1$ and $L_2$.

### 3.3 Overshoot

Overshoot is, for waveforms with positive-going transitions, the maximum positive amplitude excursion relative to $V_{S2}$ that the waveform makes near the transition region. On the other hand, for waveforms with negative-going transitions, overshoot is the maximum negative amplitude excursion relative to $V_{S2}$ that the waveform makes near the transition region. For overshoot, we currently define “near the transition region” to be that period between the 50 % reference-level instant ($t_{50\%}$) and $t_{50\%} + 3t_d$ (IEEE TC-10 Subcommittee on Pulse Techniques). Voltage offset errors are not considered here because they cancel, as they did for the uncertainty calculation of $V_{A,c}$. The equation describing the calculation for the overshoot is

$$O = \left( \frac{V_{\text{max},R} - V_{S2,R}}{V_{A,R}} \right) + \Delta O_{\text{rec},} \tag{22}.$$ 

where $\Delta O_{\text{rec}}$ is the reconstruction-induced bias in the overshoot of the reconstructed waveform. $V_{\text{max},R}$ may be written

$$V_{\text{max},R} = V_{S2,R} + V_{O,R}, \tag{23}$$

where

$$V_{O,R} = \frac{\beta_O V_{O,m} t_{d,m}}{t_{d,R}}, \tag{24}$$

and $\beta_O$ is a correction factor that is needed because of the reconstruction process and is found by fitting a curve to an $M_5$-element set of $t_{d,m} V_{O,m}$ versus $t_{d,R} V_{O,R}$ data. The $t_{d,m}$ and $V_{O,m}$ parameters are obtained from the acquired waveforms and $t_{d,R}$ and $V_{O,R}$ from the reconstructed waveforms. Equation (24) describes an empirical relationship between the overshoot and transition duration of the reconstructed (reported) waveform and those of the acquired waveform. As with transition duration (see Section 3.2), we have observed a bias and a noise-dependent variation in $V_{O,R}$ caused by the waveform reconstruction: the noise-dependent variation is contained in the empirical relationship but the bias is not. Therefore, the uncertainty in overshoot is computed by adding the computed uncertainty to the absolute value of the bias. In (24), we assume that the product of the overshoot voltage and transition duration is not affected by an all-pass filter, which is how the sampler impulse response is expected to behave for an input signal that has a 3 dB attenuation bandwidth lower than that of the sampler. The uncertainties in $t_{d,R}$ are included in the uncertainty estimate of $V_{O,R}$ by propagation of uncertainties through $t_{d,m}$. $V_{O,m}$ can be expanded:

$$V_{O,m} = V_{\text{max},m} - V_{S2,m}, \tag{25}$$

Parameter $t_{d,R}$ can be expanded in a similar way to $t_{d,m}$ (13):

$$t_{d,R} = X_R \delta t, \tag{26}$$

where $X_d$ is the non-integer number of sampling intervals describing the transition duration of the reconstructed waveform. Using (13), (23), (24), (25) and (26) in (22) yields for $O$:

$$O = \left( \frac{1}{V_{S2,R} - V_{S1,R}} \right) \left( V_{\text{max},R} - V_{S2,m} \right) \frac{X_m}{X_R} + \Delta O_{\text{rec},} \tag{27}.$$ 

The uncertainty in $\beta_O$ is the standard deviation in the fitted curve relative to the set of corresponding $t_{d,m} V_{O,m}$ versus $t_{d,R} V_{O,R}$ data and the coverage factor is determined by the number of $\beta_O$ values. Table 3 shows the uncertainty-related parameters for the variables affecting $O$. 

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3.4 Undershoot (preshoot)

Undershoot is, for waveforms with positive-going transitions, the maximum negative amplitude excursion relative to $V_{SI}$ that the waveform makes near the transition region. On the other hand, for waveforms with negative-going transitions, undershoot is the maximum positive amplitude excursion relative to $V_{SI}$ that the waveform makes near the transition region. For undershoot, we currently define “near the transition region” to be that period between the $t_{3\Delta R} - 3t_d$ and $t_{3\Delta R}$ (IEEE TC-10 Subcommittee on Pulse Techniques). Voltage offset errors are not considered here because they cancel, as they did for the uncertainty calculation of $V_{Ae}$. The equation describing the calculation for the undershoot is

$$U = \left( \frac{V_{\text{min},R} - V_{SI,R}}{V_{A,R}} \right) + \Delta U_{\text{reconst}}, \quad (28)$$

where $\Delta U_{\text{reconst}}$ is the reconstruction-induced bias in the undershoot of the reconstructed waveform. $V_{\text{min},R}$ may be written

$$V_{\text{min},R} = V_{SI,R} + V_{UR}, \quad (29)$$

where

$$V_{UR} = \frac{\beta_U V_{\text{U,sm}} t_{d,sm}}{t_{d,R}}, \quad (30)$$

and $\beta_U$ is a correction factor that is determined experimentally as for $\beta_O$. Equation (30) provides an empirical relationship between the undershoot and transition duration of the reconstructed (reported) waveform and that of the acquired waveform. As with overshoot (see Section 3.3), we have observed a bias and a noise-dependent variation in $V_{UR}$ caused by the waveform reconstruction. The uncertainty in undershoot is computed by adding the computed uncertainty to the absolute value of the bias. We assume that the product of the undershoot voltage and transition duration is not affected by an all-pass filter, which is how the sampler impulse response is expected to behave for an input signal that has a 3 dB attenuation bandwidth lower than that of the sampler. $V_{U,sm}$ can be expanded:

$$V_{U,sm} = V_{\text{min},m} - V_{SI,sm}. \quad (31)$$

Using (13), (26), (29), (30), and (31) in (28) yields for $U$:

$$U = \left( \frac{1}{V_{SI,sm} - V_{SI,R}} \right) \left( \frac{\beta_U [V_{\text{min},m} - V_{SI,sm}]}{V_{A,R}} \right) + \Delta U_{\text{reconst}}. \quad (32)$$

The correction factor $\beta_U$ is determined by fitting a curve to an $M_0$-element set of $t_{d,sm} V_{U,sm}$ versus $t_{d,R} V_{UR}$ data. The uncertainty in $\beta_U$ is the standard deviation in the fitted curve relative to the set of corresponding $t_{d,sm} V_{U,sm}$ versus $t_{d,R} V_{UR}$ data and the coverage factor is determined by the number of $\beta_U$ values. Table 4 shows the uncertainty-related parameters for the variables affecting $U$.

4. Summary

A detailed uncertainty analysis of the parameters transition duration, overshoot, undershoot and amplitude of step-like waveforms was performed. This analysis included a consideration of effects that can affect the value of the reported parameters, such as temperature, computation algorithms, history of instrument performance, equipment limitations and estimates of the response characteristics of the instrument. Our present published uncertainties, which are the result of the new measurement process and associated uncertainty analysis, for high-speed (7 ps < $t_d$ < 350 ps) pulse generators and samplers are ±2 mV for pulse amplitude and ±1.5 ps for transition duration. Our measured uncertainties, however, are smaller.

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References


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Glossary of terms

Synonymous meanings: measured, acquired, or sampled waveform

Not-reported variables

- $c_t$: partial differential equation
- $q$: transient amplitude gain-correction term
- $L_1$, $L_2$, $L_3$, $L_4$, $L_5$: low reference level (for example, the 10% reference level)
- $L_{1,2}$: data value found at $t_{1,2}$
- $L_{2,2}$: data value found at $t_{2,2}$
- $L_{2,3}$: high reference level (for example, the 90% reference level)
- $L_{3,3}$: data value found at $t_{3,3}$
- $M_i$: number of waveforms in DUT measurement set
- $M_k$: number of reference pulse waveforms measured with DUT and used in sampler gain calibration
- $M_1$: number of reference pulse waveforms measured with reference instrument and used in sampler gain calibration
- $M_2$: number of temperature measurements taken during DUT measurement process
- $M_3$: number of temperature measurements performed during sampler impulse response characterization
- $M_4$: number of amplitude-temperature data pairs used to determine temperature effects on sampler’s amplitude response or generator’s output amplitude
- $M_5$: number of independent waveforms used to estimate transition duration of sampler step response, from control chart
- $M_6$: number of transition duration-temperature data pairs used to determine temperature effects on sampler’s transition duration response or pulse generator’s output transition duration
- $M_{1,0}$: number of $t_{1,0}V_{0,R}$ versus $t_{1,0}V_{0,R}$ data pairs used to calculate $\beta_0$ curve
- $M_{1,1}$: number of $t_{1,1}V_{1,R}$ versus $t_{1,1}V_{1,R}$ data pairs used to calculate $\beta_1$ curve
- $M_{1,2}$: number of jitter measurements, from control chart
- $M_{1,3}$: number of amplitude versus time data pairs used to calculate duration of sampling interval
- $m_{1,1}$, $m_{1,2}$, $m_{1,3}$: time index for data having value closest to but $\leq L_1$
- $m_{2,1}$, $m_{2,2}$, $m_{2,3}$: time index for data having value closest to but $\geq L_2$
- $N_{0,m}$: number of bins in histogram
- $N_{0,2}$: bin number in histogram corresponding to lower state level mode bin
- $N_{0,2}$: bin number in histogram corresponding to upper state level mode bin
- $P_i$: percentage of pulse amplitude for reference level $L_i$
- $S_{\delta_0/\Delta T}$: temperature-dependent change in pulse transition duration
- $S_{\Delta V/\Delta T}$: temperature-dependent change in pulse amplitude
- $T$: temperature
- $T_{\text{mean}}$: temperature at which a particular waveform was recorded
- $T_{\text{ave}}$: average temperature over which a set of waveforms was recorded
- $t_{d,0}$: transition duration, acquired waveform
- $t_{d,0}$: transition duration, sampler step response estimate convolved with jitter
- $t_{d,1}$: transition duration, reconstructed waveform
- $t_{d,1}$: $L_1$ reference-level instant
- $t_{d,1}$: data instant preceding $t_{d,1}$
- $t_{d,1}$: data instant following $t_{d,1}$
- $t_{d,2}$: $L_2$ reference-level instant

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\[ t_{c,2} \] data instant preceding \( t_{c,2} \)
\[ t_{c,2^+} \] data instant following \( t_{c,2} \)
\[ V_{A,c} \] pulse amplitude, corrected waveform
\[ V_{A,m} \] pulse amplitude, acquired waveform
\[ V_{A,m,r} \] pulse amplitude, acquired waveform, reference pulse
\[ V_{A,R} \] pulse amplitude, reconstructed waveform
\[ V_{A,r} \] pulse amplitude, reference-instrument acquired waveform, reference pulse
\[ V_{S1,c} \] lower state level, corrected waveform
\[ V_{S1,m} \] lower state level, acquired waveform
\[ V_{S1,m,r} \] lower state level, acquired waveform, using reference pulse and sampler
\[ V_{S1,R} \] lower state level, reconstructed waveform
\[ V_{S1,r,r} \] lower state level, acquired waveform, using reference pulse and reference measurement instrument
\[ V_{\text{max},m} \] maximum value, acquired waveform
\[ V_{\text{max},R} \] maximum value, reconstructed waveform
\[ V_{\text{min},m} \] minimum value, acquired waveform
\[ V_{\text{min},R} \] minimum value, reconstructed waveform
\[ V_{S2,c} \] upper state level, corrected waveform
\[ V_{S2,m} \] upper state level, acquired waveform
\[ V_{S2,m,r} \] upper state level, acquired waveform, using reference pulse and sampler
\[ V_{S2,R} \] upper state level, reconstructed waveform
\[ V_{S2,r,r} \] upper state level, acquired waveform, using reference pulse and reference measurement instrument
\[ V_{O,m} \] overshoot value, acquired waveform
\[ V_{O,R} \] overshoot value, reconstructed waveform
\[ V_{U,m} \] undershoot value, acquired waveform
\[ V_{U,R} \] undershoot value, reconstructed waveform
\[ V_{\Delta T} \] temperature-induced incremental change in pulse amplitude
\[ \Delta T \] non-integer number of sampling intervals in transition duration of reconstructed waveform
\[ \Delta T_{s} \] non-integer number of sampling intervals in transition duration of effective jitter step response
\[ \Delta T_{a} \] non-integer number of sampling intervals in transition duration of acquired waveform
\[ \beta_{O} \] overshoot correction factor relating transition duration of measured and reconstructed waveforms
\[ \beta_{U} \] undershoot correction factor relating transition duration of measured and reconstructed waveforms
\[ \Delta \delta \] temperature-induced incremental change in transition duration
\[ \delta \] histogram bin size
\[ \nu \] effective degrees of freedom
\[ \nu \] degrees of freedom

**Reported variables**

- **\( O \)** overshoot value
- **\( U \)** undershoot (preshoot) value
- **\( \delta \)** transition duration
- **\( V_{A} \)** pulse amplitude, reported
- **\( u \)** uncertainty of the \( i \)-th variable