

# Electronic limitations in phase meters for heterodyne interferometry

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*Limitations imposed by the phase meters used in heterodyne interferometers are evaluated. These instruments measure the phase relationship between electrical signals generated by the heterodyning process, allowing the interferometers to resolve fractions of an optical fringe. Measurements indicate that the phase meters used in currently available heterodyne interferometers probably limit achievable accuracy to a greater extent than barriers imposed by the optics. We show that a new class of time interval counters offers a means of greatly improving accuracy in these instruments.*

**Keywords:** Phase meters; heterodyne interferometry

## Introduction

Commercial heterodyne interferometers claim displacement measurement accuracies of approximately 10 nm, and because the interferometers are based on well-known optical wavelengths, such accuracies have been accepted with little questioning. However, more demanding requirements, which push displacement measurement uncertainties down to 1 nm and below, have led to the investigation of remaining sources of error. Reasonable attention has been given to the fidelity of the heterodyning process, which converts the optical path difference between beams that have traversed the test and reference legs to a phase difference between electrical signals from the test and reference photodetectors.<sup>1-9</sup> This article deals with the errors introduced in the next step: the measurement of the electrical phase angle.

## Heterodyne interferometers

A heterodyne interferometer, shown schematically in *Figure 1*, uses a dual-frequency laser to produce two electrical signals: a reference signal,  $V_R$ , which is at the laser difference frequency ( $\delta f = f_1 - f_2$ ), and a measurement signal,  $V_M$ , which is at the difference frequency but Doppler-shifted by the motion of the test reflector ( $\delta f \pm \Delta f$ ). This Doppler shift

can equivalently be thought of as a time-varying phase shift,  $\phi(t) = 2\pi \int \Delta f(t) dt$ . If  $v(t)$  is the test mirror or reflector speed, then

$$\Delta f(t) = \frac{2nv(t)}{\lambda}$$

where  $n$  is the index of refraction of the medium along the light path and  $\lambda$  is the wavelength of the light.

For commercial heterodyne interferometers, the difference frequency is typically between 250 kHz and 20 MHz. At rest, the reference and measurement signals have the same frequency and their phase relationship is proportional to the position of the test reflector. The ability to measure this phase angle determines how accurately one knows the change in position of the test reflector. In a simple Michelson interferometer, the phase is related to displacement by

$$\phi(t) = 4\pi nL(t)/\lambda,$$

where  $L(t)$  is the displacement of the test reflector relative to the reference. From the above relationship,  $1^\circ$  of phase represents a test reflector displacement of  $\lambda/720$ , or about 0.9 nm for He-Ne laser wavelengths.

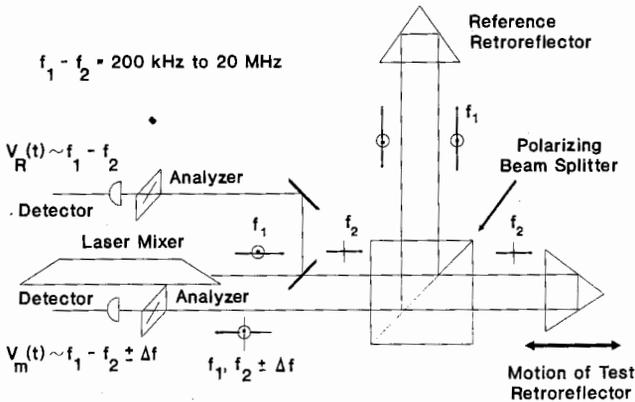
## Angular resolution

Most phase meters measure phase angle by measuring time ratios. In interferometry, absolute accuracy is not necessary, but stability, linearity, and resolution are. The angular resolution obtainable depends on the signal frequency and the timing resolution:

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Typical Heterodyne Interferometer



**Figure 1** Block diagram of a typical heterodyne interferometer system.  $f_1$  and  $f_2$  are the frequencies of the two-frequency laser source;  $V_R(t)$  is the reference signal;  $V_M(t)$  is the measurement signal. Polarization states perpendicular to the plane of the page,  $\odot$ , and parallel to the plane of the page,  $\otimes$ , are indicated

$$\Delta\alpha = 360^\circ f \Delta\tau$$

where  $\Delta\alpha$  is the angular resolution in degrees,  $f$  is the signal frequency in hertz, and  $\Delta\tau$  is the timing resolution in seconds.

This relationship is displayed graphically in *Figure 2*, where it can be seen that to resolve  $0.1^\circ$  at 200 kHz requires a time resolution of about 1 nsec. At 2 MHz the same  $0.1^\circ$  resolution requires a time resolution of 100 psec. For a 20 MHz signal, 10 psec resolution is required. In the application that led to this study,<sup>10</sup> the objective was to resolve 0.1 nm using a laser with a difference frequency of about 500 kHz and using an optical configuration that gives a fringe spacing of  $\lambda/8$ . This translates to a phase measurement resolution of  $0.4^\circ$  or a time resolution of 2 nsec. The chosen heterodyne frequency limits the update rate to  $2 \mu\text{sec}$ , which limits the motional slow rate to about 10 mm/s. (The slow rate is calculated by assuming a conservative maximum of one fourth of a fringe per measurement update cycle.) The phase meter must meet or exceed this update rate to maintain the slow rate.

Before this investigation, standards for phase angle (the solid curve crossing *Figure 2*) were only available up to 50 kHz,<sup>11</sup> although the measurement uncertainties were  $\pm 0.01^\circ$  at that frequency. Extending this capability out to 1 MHz at  $0.1^\circ$  required an improvement in timing resolution by a factor of two. More challenging was the need to make new phase measurements every microsecond (existing standards required seconds of averaging). Fortunately, at the time we were beginning this study, several manufacturers had recently introduced a new class of counter/timer known generically as time interval analyzers (TIAs). These instruments

can be configured as fast, high-frequency phase meters.

Phase meters

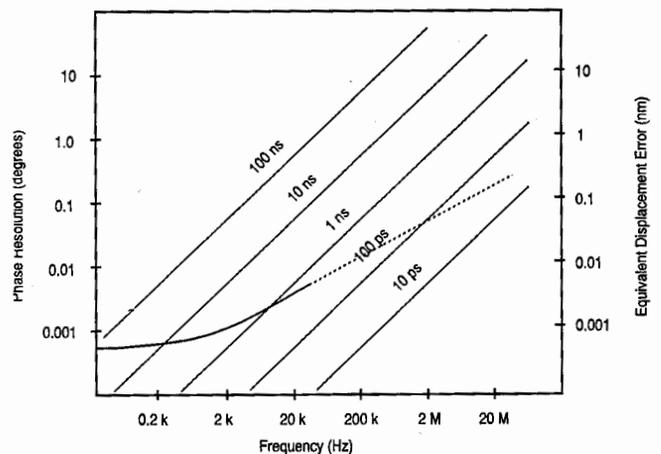
Time interval analyzers measure phase by measuring the ratio of the time between zero crossings of the two signals to the period of one of the signals. As shown in *Figure 3*, the phase relationship between  $V_R$  and  $V_M$  (both the same frequency), is given by

$$\Phi = (T_+/T_R)360^\circ$$

where  $T_+$  is the time interval between the positive going zero-crossings of  $V_R$  and  $V_M$ , and  $T_R$  is the period of  $V_R$ .

Unlike conventional digital timers, TIAs can resolve times in the picosecond range by counting periods of a reference clock and then interpolating on the ends. A common time interpolation scheme uses an analog ramp generator to charge a circuit at a known rate,  $I$ , during the interpolation period. The height of the ramp,  $V$ , at the end of the period is proportional to the interpolation interval,  $T_i \propto V/I$ . Several methods are then used to determine the ramp height,  $V$ . One way is to measure the time it takes to discharge the circuit back to its starting point using a lower discharge rate,  $I/k$ . This technique converts a short time interval to a longer one that can be measured using the digital timer. However, to enhance time interval resolution by a factor

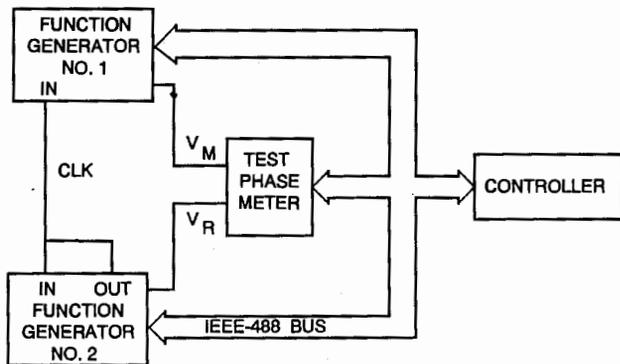
ANGULAR RESOLUTION AS A FUNCTION OF TIME RESOLUTION AND SIGNAL FREQUENCY



**Figure 2** Angular phase resolution as a function of time resolution and signal frequency. Equivalent reflector-displacement error is also plotted, assuming a simple Michelson interferometer and a 633-nm wavelength. The solid curve represents the NIST phase-angle calibration standards capability before this study. The dotted extension of this curve shows the extended capability developed as a result of this study



DUAL CHANNEL SOURCE USED TO TEST PHASE METERS



**Figure 4** Block diagram of the dual channel source used to test phase meters. Sinusoidal signals are synthesized using two phase-adjustable function generators connected to a common frequency reference. Phase resolution is  $\pm 0.1^\circ$ , whereas uncertainties as low as  $\pm 0.03^\circ$  may be achieved by characterizing the source

the more typical case of phase measurements made "on the fly" while the test reflector is moving. In this mode, new phase measurements should ideally be made at every zero crossing of the measurement signal. This phase standard and the techniques for characterizing it are described elsewhere in greater detail.<sup>12,13</sup>

**Static results**

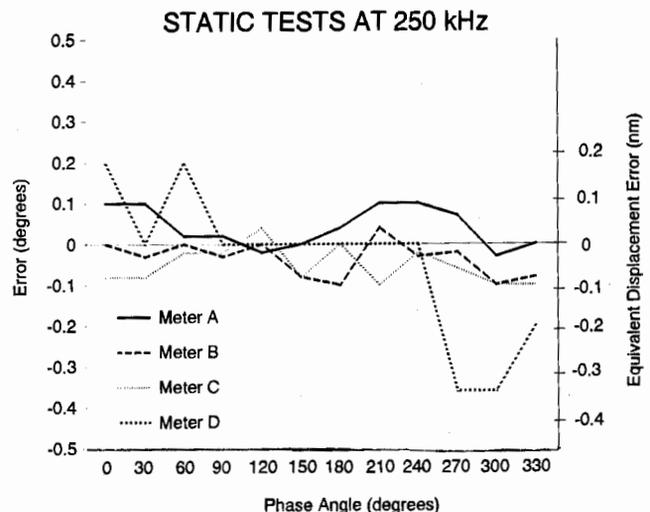
Results of measurements performed at 250 kHz under static conditions for four different meters are plotted in *Figure 5*. Each point on each of the plots represents an average of 1,000 readings; however, the amplitude of the errors was about the same even for a single reading, indicating that the errors are not primarily due to random noise. Results shown in this plot are from a digital phase meter (B), two TIAs (A and C), and a phase meter used with a commercial interferometer (D). Phase meter D has a resolution of a part in 1,000 and is linear to  $\pm 1$  count ( $\pm 0.4^\circ$ ). The other three meters have higher resolution and show nonlinearities on the order of  $\pm 0.1^\circ$ . The scale on the right shows the equivalent reflector displacement error (in nm) for a simple Michelson interferometer. The results indicate that, ignoring optical sources of error, the interferometer's phase meter will limit its resolution and linearity to about  $\pm 0.4$  nm. However, the electronic limitation could be improved by a factor of 4, to 0.1 nm, by using one of the other meters.

At 2 MHz, static measurements were performed on two TIAs (A and C) and a phase meter from a different commercial interferometer (E). The results of these tests are plotted in *Figure 6A*. Here the

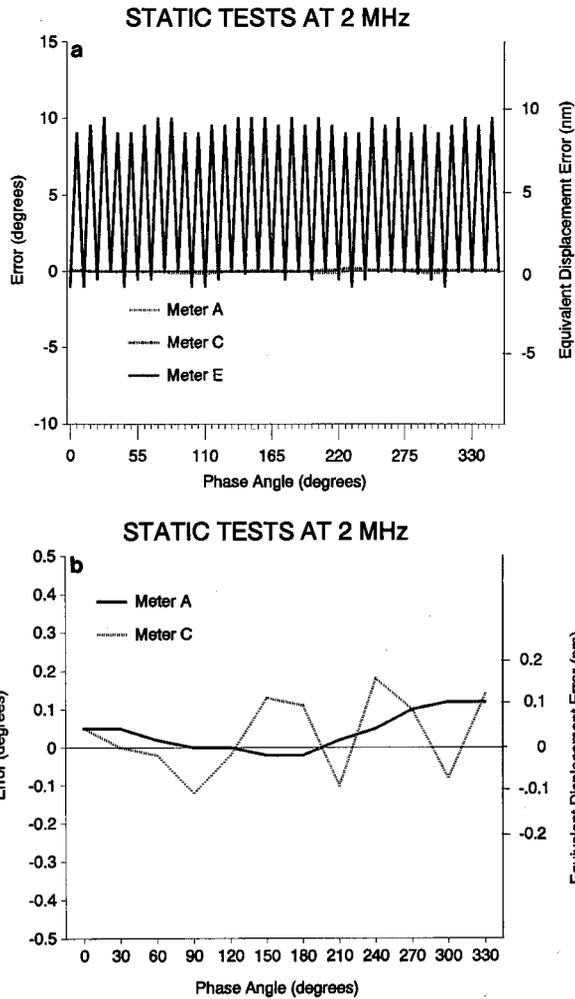
interferometer phase meter (E) has an angular resolution of only about  $11^\circ$  but is still linear to  $\pm 1$  count (about 10 nm), which is its specification. Note that the phase error is well behaved in that it builds up linearly between digitization steps. The TIA errors are so small that they cannot be resolved on this plot. Expanding the scale in *Figure 6B* shows that the TIA nonlinearities are  $\pm 0.2^\circ$  (which corresponds to about 0.2 nm, 50 times better).

At 20 MHz, two TIAs (A and C) and the phase meter from a third commercial interferometer (F) were tested. The results are plotted in *Figure 7*. Here the interferometer phase meter (F) has a resolution of  $2.8^\circ$ ; however, a peak nonlinearity in excess of  $10^\circ$  was observed. (There were no convenient input connectors on this instrument, so signals were applied to test points on the circuit board. It is not clear what influence this may have had on the measurements.) Once again, the TIAs had better resolution and were more linear ( $\pm 1^\circ$ ), indicating that it should be possible to improve the accuracy of this system by as much as a factor of eight by substituting a TIA for the built-in phase meter.

The tests described up to this point were performed using relatively pure synthesized sine waves simulating the signals that would be produced in an interferometer. To test the performance with optically generated signals, the setup illustrated in *Figure 8* was used. Here a  $360^\circ$  phase angle change between the reference and test signals was generated by moving the retroreflector along a track

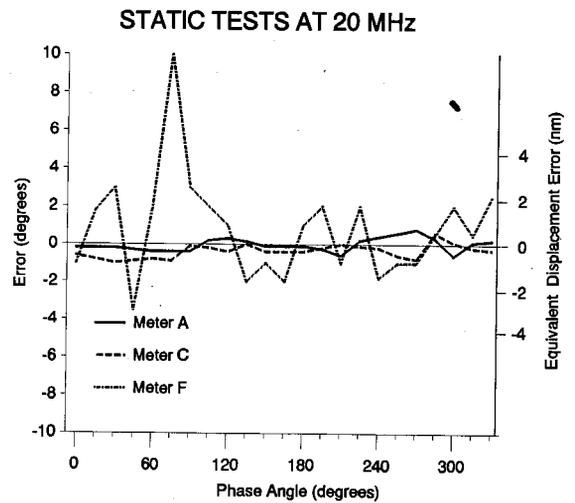


**Figure 5** A comparison of the errors in phase measurement of several phase meters at 250 kHz over a full period. These tests were conducted with a fixed (static) phase difference, and each point represents an average of 1,000 measurements. The scale on the right axis represents the equivalent reflector displacement error for  $\lambda = 633$  nm and a fringe spacing of  $\lambda/2$ . Meter D is the interferometer phase meter

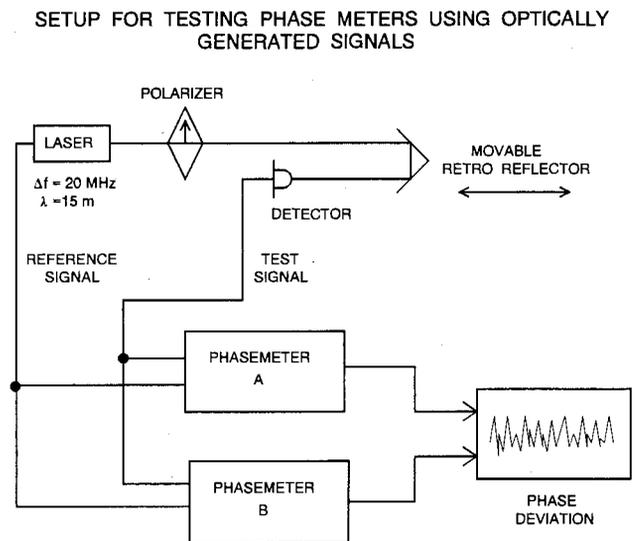


**Figure 6** (A) A comparison of the errors in phase measurement of several phase meters at 2 MHz using a static phase difference as in *Figure 5*. Meter E is the interferometer phase meter. (B) An expanded y-axis plot of (A) showing the much smaller nonlinearities in meters A and C

a distance equal to half the wavelength of the heterodyne frequency. Thus, for a 20 MHz difference frequency, moving the retroreflector 7.5 m produced a change in path length of 15 m and a 360° phase shift. Highly resolved phase changes are easily produced, because a 20 mm motion results in a phase shift of only 1°. The phase differences were measured simultaneously using an interferometer phase meter (F) and a TIA (A). The results of these tests are shown in *Figure 9*. The upper plot is the difference between the mechanically measured phase change and the change measured by the interferometer phase meter (F). The middle plot shows the same difference for the TIA (phase meter A). As with electrically synthesized signals, the linearity of the TIA is about four times better for these measurements. The plot at the bottom of the graph shows the error signature obtained by testing the



**Figure 7** A comparison of the errors in phase measurements of several phase meters at 20 MHz using a static phase difference. Meter F is the interferometer phase meter



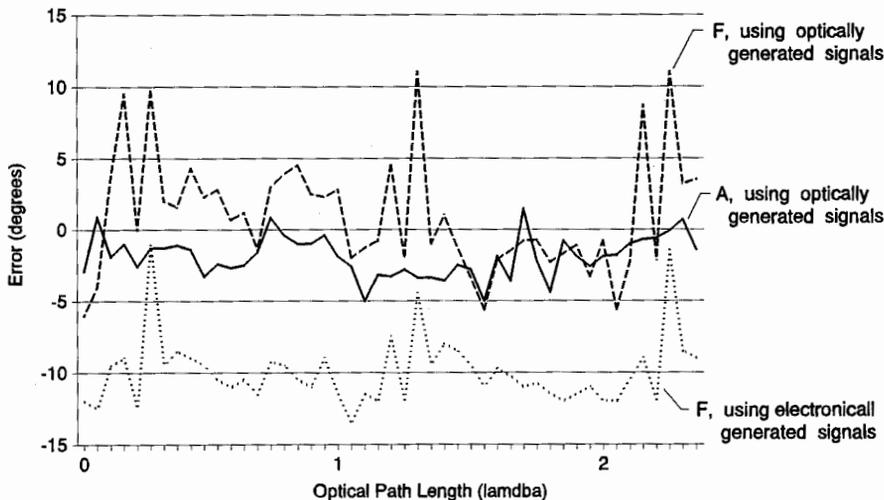
**Figure 8** A block diagram of the setup used to optically generate precisely controllable phase differences. As the retroreflector is moved through half a wavelength (7.5 m for a 20 MHz heterodyne frequency), the phase of the test signal shifts 360° relative to the reference signal

interferometer phase meter (F) with synthesized signals, as in *Figure 7*. These error patterns are obviously correlated and give confidence that the measured error originates in the phase meter in both setups.

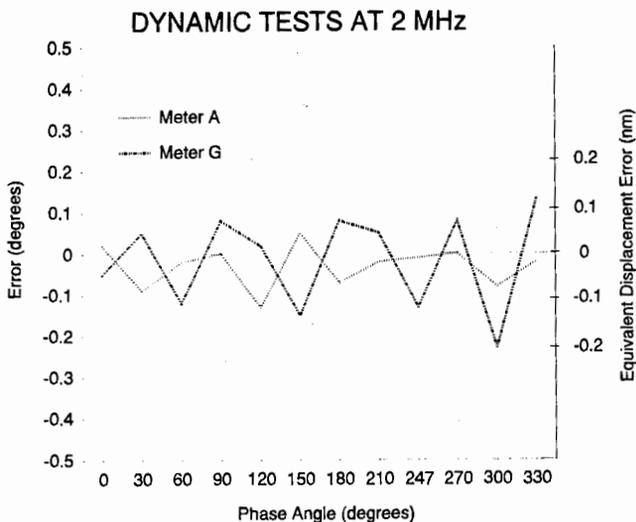
**Dynamic results**

The interferometer electronics are not only able to make static measurements, but also dynamic

STATIC TESTS AT 20 MHz



**Figure 9** Comparison of phase measurement errors of a time interval analyzer (A) and a commercial interferometer phase meter (F). The phase shift in the abscissa is the optical phase shift generated as shown in *Figure 8*. Also shown for comparison are the errors measured for meter F relative to the electronically generated phase standard (offset by  $-10^\circ$ )



**Figure 10** Comparison of the errors in dynamic phase measurement of two high-speed phase meters. The reference signal was 2 MHz and the test signal was 2.2 MHz so that the relative phase shifted through  $360^\circ$  in  $5 \mu\text{sec}$ . No averaging was used on these data

measurements, made on the fly while the test reflector is moving. In fact, this is the mode in which they are normally operated. On two of the interferometers, these high-speed data are available on another output port. Due to interfacing complications, the dynamic performance of the interferometer phase meters was not tested. However, it is unlikely that it would be any better than the static performance. Shown in *Figure 10* are results of dynamic

tests on two TIAs (meters A and G) that can operate at megahertz update rates. For this test, the frequency of the reference signal was 2 MHz and that of the test signal was 2.2 MHz. This simulates the test reflector moving at about 60 mm/s. No averaging was used for the data in this plot. Nonlinearities were  $\pm 0.3^\circ$ , indicating that even in this mode, the TIA-imposed limit to dynamic resolution would only be about 0.3 nm.

These data indicate that substituting a TIA for the interferometer phase meters may improve dimensional resolution. However, in the dynamic mode there are some problems that complicate the direct replacement of a built-in phase meter with a TIA.

The most critical problem is gaining access to high-speed timing and phase measurements. Only one of the TIAs that were tested (meter A) has a data bus that can be used in a real-time servo loop. Interfacing with this data bus is itself a nontrivial task. The others store individual readings in memory that can only be read off-line.

The other caveat is that some care must be taken in the definition of phase angle where the two signals are at different frequencies. In at least one of the tested TIAs (A), the internal phase calculation was sometimes flawed in circumstances where two zero-crossings of one signal fell within one period of the other. Clearly, the larger the Doppler shift, the more serious the error. This subject will be treated in more detail in a future paper.

As an alternative to using a phase standard, intercomparisons between phase meters could give some indication of errors. For example, two receivers and phase meters could view the same interferometer beam, and the resulting phase

measurements could be compared. By then rotating one of the detector polarizers by  $90^\circ$ , or by introducing a delay line, the relative phase offset could be changed by  $180^\circ$  or by any arbitrary amount, respectively, and the differences in phase measurements could again be compared.

### Conclusions

One of the major sources of uncertainty in commercial heterodyne interferometers appears to be the phase measurement. At higher heterodyne frequencies it may be the primary source of uncertainty. Data from these instruments are quantized between  $0.4^\circ$  (in the low frequency instruments) to as much as  $11^\circ$ .

On the other hand, limitations imposed by the new time interval analyzers (under ideal signal conditions) are on the order of  $\pm 0.1^\circ$  below 2 MHz. That corresponds to  $\lambda/7,000$  or about 0.1 nm for a simple Michelson interferometer.

Based on these measurements, the new time interval analyzers should be capable of enhancing the phase-measuring performance of these interferometers. Improvement by a factor of four to 50 may be possible if optical limitations do not begin to dominate the improved resolution. The major obstacle to taking advantage of this increased performance (aside from the cost of the TIA) is the difficult, but not insurmountable, problem of accessing the TIA data in a real-time mode.

### Acknowledgment

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