SPHERICAL NEAR-FIELD SCANNING: DETERMINING THE INCIDENT FIELD NEAR A ROTATABLE PROBE

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Introduction. Many RCS, EMI/EMC, and antenna measurements require a known incident field within a test volume. To evaluate systems designed to produce a specific incident field (compact ranges, for example), we must measure the actual illumination for comparison with design specifications. Beyond its diagnostic value, this incident field data can also be used for error estimation and for calculating first order corrections.

In this paper, we develop a spherical near-field scanning algorithm for determining incident fields inside a probe's "minimum sphere." This differs from the well-known spherical near-field scanning formulation which determines fields outside the source's minimum sphere [1]. The scanner size depends on the extent of the region of interest and not on the extent of the (possibly much larger) source. The data may be collected using a standard roll-over-azimuth positioner.

The Incident Field can be expanded in multipoles near the origin:

$$\mathbf{E}_i(\mathbf{r}) = \sum_{\nu=1}^{N} \sum_{\mu=-\nu}^{\nu} [a^H_{\nu\mu}\, \mathbf{m}_{\nu\mu}(\mathbf{r}) + a^E_{\nu\mu}\, \mathbf{n}_{\nu\mu}(\mathbf{r})] \tag{1}$$

$$\mathbf{m}_{nm}(\mathbf{r}) \equiv i^n\, j_n(kr)\, \mathbf{X}_{nm}(\hat{\mathbf{r}}) \; , \qquad \mathbf{n}_{nm}(\mathbf{r}) \equiv \frac{1}{k}\, \nabla \times \mathbf{m}_{nm}(\mathbf{r}) \; . \tag{2}$$

$\mathbf{X}_{nm}$ is a spherical harmonic and j_n is a spherical Bessel function [2, chapter 16]. Equation (1) describes $\mathbf{E}_i(\mathbf{r})$ for $r < a \sim N/k$.

The Receiving Pattern is defined so that an incident plane wave

$$\frac{1}{2\pi}\, \mathbf{E}_0 \exp(i\mathbf{k} \cdot \mathbf{r})$$

produces the probe response

$$\mathbf{r}(\hat{\mathbf{k}}) \cdot \mathbf{E}_0 \exp(i\mathbf{k} \cdot \mathbf{r}) \; .$$

$\mathbf{r}(\hat{\mathbf{k}})$ has the spherical harmonic expansion:

$$\mathbf{r}(\hat{\mathbf{k}}) = \sum_{\nu=1}^{N} \sum_{\mu=-\nu}^{\nu} [r^H_{\nu\mu}\, \mathbf{X}_{\nu\mu}(\hat{\mathbf{k}}) + r^E_{\nu\mu}\, i\hat{\mathbf{k}} \times \mathbf{X}_{\nu\mu}(\hat{\mathbf{k}})] \; . \tag{3}$$

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For practical purposes we may take $N \sim ka$, where a is the radius of the smallest (minimum) sphere that is centered on the origin and encloses the probe. The incident field cannot be reliably calculated outside the probe's minimum sphere because it is not experimentally feasible to determine a^H_{nm} or a^E_{nm} when the probe is insensitive to the nth multipole ($r^H_{n\mu} \approx 0$ or $r^E_{n\mu} \approx 0$).

The rotated probe is described by the receiving function

$$\mathbf{r}'(\hat{\mathbf{k}}) = \sum_{\nu\mu\mu'} D^{\nu}_{\mu'\mu}(\phi,\theta,\chi)\ [r^H_{\nu\mu}\ \mathbf{X}_{\nu\mu'}(\hat{\mathbf{k}}) + r^E_{\nu\mu}\ i\hat{\mathbf{k}} \times \mathbf{X}_{\nu\mu'}(\hat{\mathbf{k}})]\ . \quad (4)$$

The D's are rotation functions [3]. Orientation is specified by Euler angles defined as follows: Starting with the probe in its reference orientation 1) rotate the probe by an angle χ about the z axis; 2) then rotate by θ about the y axis; 3) finally, rotate by ϕ about the z axis.

<u>The Transmission Formula</u> gives the response of the rotated probe (located at the origin) to the incident field:

$$W(\phi,\theta,\chi) = \frac{1}{2} \sum_{\nu\mu\mu'} (-)^{\mu+1}\ D^{\nu}_{\mu\mu'}(\phi,\theta,\chi)\ [r^H_{\nu\mu'}\ a^H_{\nu,-\mu} - r^E_{\nu\mu'}\ a^E_{\nu,-\mu}]\ . \quad (5)$$

This follows from plane-wave representations for $\mathbf{m}_{nm}$ and $\mathbf{n}_{nm}$ [4].

<u>The "Symmetric" Probe.</u> In general, rotation function orthogonalities can be used to solve the transmission equation for the unknown coefficients; however, application of (5) is simplified by the use of a symmetric ($\mu = \pm 1$) probe [5] for which $r^H_{n\mu} = r^E_{n\mu} = 0$ if $\mu \neq \pm 1$. Such probes can be constructed by incorporating a section of open-ended circular waveguide designed so that only the TE_{11} modes propagate (see Figure 1).

With a symmetric probe (5) can be written

$$\mathbf{W}(\hat{\mathbf{r}}) = \sum_{\nu\mu} [A^H_{\nu\mu}\ \mathbf{X}_{\nu\mu}(\hat{\mathbf{r}}) + A^E_{\nu\mu}\ i\hat{\mathbf{r}} \times \mathbf{X}_{\nu\mu}(\hat{\mathbf{r}})] \quad (6)$$

$$\mathbf{W}(\hat{\mathbf{r}}) \equiv W(\phi,\theta,\chi{=}-\pi/2)\ \hat{\boldsymbol{\theta}} + W(\phi,\theta,\chi{=}0)\ \hat{\boldsymbol{\phi}}$$

$$A^H_{nm} \equiv i \sqrt{\frac{\pi}{2n+1}} \left[(r^H_{n1} - r^H_{n,-1})\ a^H_{nm} - (r^E_{n1} - r^E_{n,-1})\ a^E_{nm}\right] \quad (7a)$$

$$A^E_{nm} \equiv -i \sqrt{\frac{\pi}{2n+1}} \left[(r^H_{n1} + r^H_{n,-1})\ a^H_{nm} - (r^E_{n1} + r^E_{n,-1})\ a^E_{nm}\right] \quad (7b)$$

Measurements are needed at two "spin" orientations ($\chi = -\pi/2,\ 0$) for each pointing direction (θ, ϕ).

Equation (6) can be inverted to give

$$A^H_{nm} = \int \mathbf{W}(\hat{\mathbf{r}}) \cdot \mathbf{X}^*_{nm}(\hat{\mathbf{r}})\, d\hat{\mathbf{r}} \tag{8a}$$

$$A^E_{nm} = \int \mathbf{W}(\hat{\mathbf{r}}) \cdot [i\hat{\mathbf{r}} \times \mathbf{X}_{nm}(\hat{\mathbf{r}})]^*\, d\hat{\mathbf{r}} \ . \tag{8b}$$

The modal coefficients a^H_{nm} and a^E_{nm} can then be found by solving the simultaneous equations (7). (When (7a) and (7b) are not linearly independent, as for a circularly polarized probe, an additional probe must be used.)

<u>Practical Implementation.</u> The incident field can be determined over a region roughly corresponding to the volume of the probe's minimum sphere. If the area of interest is larger, then it is necessary to make measurements with the probe in different locations, or to use a bigger probe. Increasing the probe size is not always practical; for example, multiple interactions with the source can become a problem. Figure 2 shows a larger probe with a reasonably small cross section. The probe consists of a small transducer on a rotating arm in an arrangement resembling the usual spherical scanning geometry turned inside out.

While it can be measured directly, it is often easier to obtain the receiving pattern for the probe of Figure 2 from the receiving pattern $\mathbf{r}_0(\hat{\mathbf{k}})$ for the transducer alone:

$$\mathbf{r}(\hat{\mathbf{k}}) = \mathbf{r}_0(\hat{\mathbf{k}}) \exp(i\mathbf{k} \cdot \rho\hat{\mathbf{z}}) \ , \tag{9}$$

where ρ is the length of the arm. We have

$$r^H_{nm} = \int \mathbf{r}_0(\hat{\mathbf{k}}) \cdot \mathbf{X}^*_{nm}(\hat{\mathbf{k}}) \exp(i\mathbf{k} \cdot \rho\hat{\mathbf{z}})\, d\hat{\mathbf{k}} \tag{10a}$$

$$r^E_{nm} = \int \mathbf{r}_0(\hat{\mathbf{k}}) \cdot [i\hat{\mathbf{k}} \times \mathbf{X}_{nm}(\hat{\mathbf{k}})]^* \exp(i\mathbf{k} \cdot \rho\hat{\mathbf{z}})\, d\hat{\mathbf{k}} \ , \tag{10b}$$

r^H_{nm} and r^E_{nm} may be calculated using numerical routines developed to evaluate (8), or they may be calculated from $r^H_{0\nu m}$ and $r^E_{0\nu m}$ [4].

<u>References</u>

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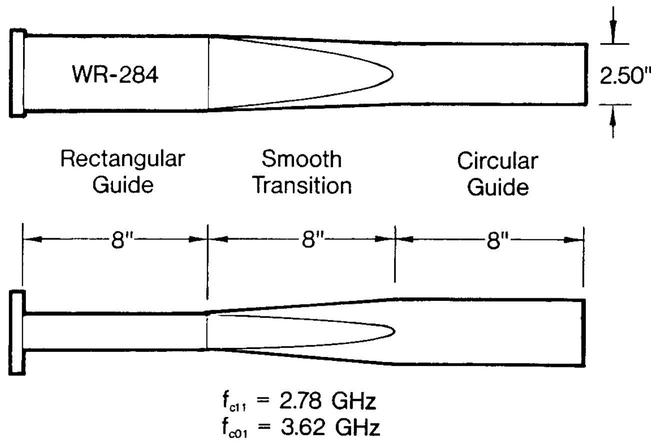

Figure 1: A $\mu = \pm 1$ probe for use at frequencies near 3 GHz.

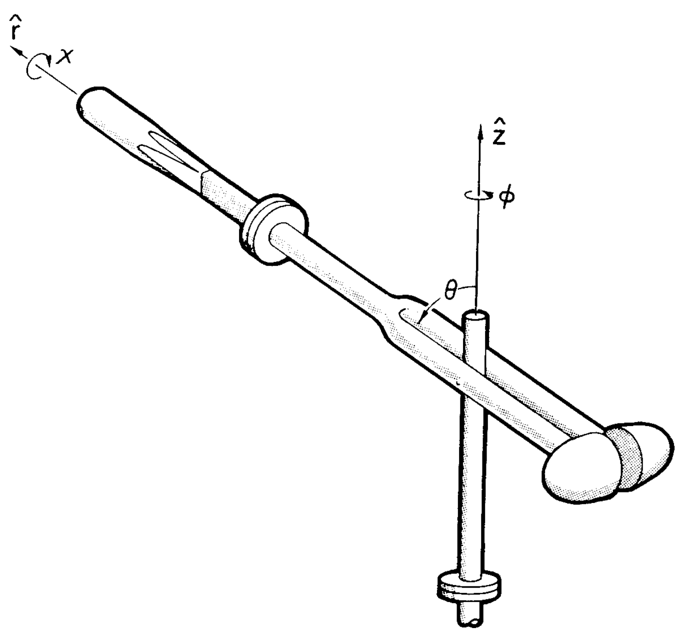

Figure 2: A scanner for probing incident fields.