# Re-examination of Quantum Hall Plateaus

C. T. Van Degrift, Kazuo Yoshihiro, E. C. Palm, J. Wakabayashi, and S. Kawaji

Abstract—Even though the unit of electrical resistance was based on the quantum Hall effect starting January 1, 1990, our understanding of the fundamental physics of current flow, contacting, and impurity effects in quantum Hall systems remains incomplete. This paper examines some recently discovered effects which may affect quantum Hall resistance determinations. We also describe improvements to the NIST potentiometric measurement system and present new data comparing the i =4 plateaus of a Si-MOSFET and a GaAs heterostructure with a room temperature reference resistance.

#### I. BACKGROUND

#### A. Introduction

DURING the few years immediately following the discovery of the quantum Hall effect, there was much activity centered on testing the universality of the effect in different samples, for different filling factors, and for different semiconductor systems. The result was that the quantum Hall effect was established to be universal to within a few tenths of a part per million for "well-behaved" samples measured under restricted conditions. In contrast to the Josephson voltage steps which have no observed material-dependent corrections, the quantum Hall plateaus are not only temperature and current dependent [1], but also can have irregularities even when the longitudinal resistance is negligibly small [2]–[3]. Furthermore, imperfections in the contacts can lead to contactdependent corrections to the Hall resistance [4].

Recently, Hartland *et al.* [5] made a comparison of the quantized resistances of Si-MOSFET and GaAs heterostructure devices and found no discrepancies at the 0.0003 ppm (parts per million) level for a particular set of samples. Delahaye and Dominguez [6] carried out an extensive intercomparison of "high-quality" heterostructure devices, finding that the plateaus were device-independent at the 0.004 ppm level. Nevertheless, anomalies seen even in apparently "imperfect" devices, including Si-MOSFET's, must be understood since these effects may be present in "high-quality" devices at a level more difficult to discern. It is important to continue to pursue investigations of physical phenomena which might lead to deviations from the quantized values. As the attainable precision and accuracy of quantum Hall resistance mea-

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surements improves beyond the 0.01 ppm level, it is necessary to carefully reexamine, both experimentally and theoretically, questions related to the material and parameter independence of the effect. No conclusions tested at the 0.01 ppm level can be relied upon at the 0.001 ppm level.

#### **B.** Plateau Flatness

We recently observed [3] irregularities in the i = 4 plateau of a Si-MOSFET (grown by Sony Corp. and configured at Gakushuin University) which otherwise appeared to be an excellent candidate for use as a resistance standard. It had an exceedingly small  $\rho_{xx}$  (less than 0.002 ppm of  $R_H$ ) for nearly 3% of its gate voltage at the i = 4 plateau. Yet, its plateau undulated (with changing gate voltage) about the quantized value by up to  $\pm 0.04$  ppm. Measurements on a similar device at the Electrotechnical Laboratory also revealed a comparable lack of flatness. Heinonen and Johnson [7] have offered a possible theoretical explanation for irregularities in the plateaus of Si-MOSFET's. They show how scattering between discrete degenerate states at the sample edges can lead to Hall resistance anomalies without introducing dissipation.

## C. Offset Plateaus an advisor addeniate test site as the

In 1988, Kawaji et al. [2] reported measurements of flat plateaus in Si-MOSFET's that were offset from the GaAs heterostructure value by 0.16 ppm. These samples satisfied all known criteria for standards quality devices including having  $\rho_{xx}$  values smaller than 0.015 ppm of the Hall resistance, a value not expected to shift the plateau by more than 0.003 ppm. On the other hand, Delahaye and Bournaud [8] reported no anomalies in their examination of a similar Sony sample. They found that their Sony Si-MOSFET sample was in agreement with their GaAs heterostructure samples to within 0.010 ppm. Measurements at NIST on our Sony sample and at ETL on two other Sony samples all revealed similar offsets [3] to those seen by Kawaji. At NIST, however, the offset plateaus were less stable, being induced by thunderstorm activity and cleared by warming to room temperature.

### D. Contacts and Edges

In 1988, Büttiker [9] published a remarkable paper which emphasized the importance of edge currents and presented a model for their interaction with device contacts. Although his model actually applies only to currents less than those normally used for precision resistance measurements, it points to effects which can cause significant corrections to plateau resistance values. Komiyama and Hirai [4] have used the Büttiker model to calculate corrections to measured plateau values which are dependent upon the resistance and arrangement of the sample contacts. Their corrections assume that all current flows in edge states, and consequently will overestimate contact effects present at the higher currents used for high precision measurements. During the last few years, there has been a flood of experimental and theoretical work focused on clarifying the exact role of edge currents and contacts in quantum Hall systems.

#### E. Attainable Precision

Intercomparisons between quantum Hall resistances or between a quantum Hall resistance and a room temperature reference resistor will ultimately be limited by a combination of the Johnson noise of the room temperature resistances, the intrinsic noise of the quantum Hall device(s), and the noise figure of the null detector. Modern instrumentation is adequate to allow sufficient control of ac common-mode interference, ground-loop currents, vibration-induced voltages, and thermal voltages, so that experiments need be limited only by fundamental noise sources and switching transient decay times.

Consider a single comparison between the *i*<sup>th</sup> plateau of a quantum Hall sample and a reference resistance *R* held at temperature *T*. If the measurement requires  $\tau$  seconds, the effective noise bandwidth is  $(2\tau)^{-1}$ . Assuming further that the dominant noise source is the Johnson noise of the reference resistance, and that the measurement current is *I*, then the best attainable precision is

$$\frac{\delta R_H}{R_H} = \frac{\sqrt{4kTR/(2\tau)}}{Ih/(ie^2)} \tag{1}$$

where k is Boltzman's constant, h is Planck's constant, and e is the electronic charge. Using T = 295 K,  $R = 6453.2 \Omega$ ,  $I = 25 \mu$ A, i = 4, and  $\tau = 250$  s, we find that the relative standard deviation of each such single measurement can be no less than 0.0028 ppm.

In actuality, the avoidance of thermal voltage and other system drifts, and of "1/f" noise in the null detector will require frequent current reversal and, in our case, periodic replacement of the Hall sample by a transportable reference resistance  $R_R$ . Furthermore, when  $R_R$  is in place, it also contributes to the effective value of the noise resistance R (in our case doubling it). Thus to attain the same precision as in the  $R_H$  measurement, the measurement of  $R_R$  actually requires a longer measurement time. Additional time will be required to accomplish the switching and for the decay of switching transients. The switching frequency depends on the system drift-rates and on the "1/f" knee frequency of the null detector noise characteristic. For our system, an individual measurement comparing a quantum Hall resistance and a room-temperature resistance takes 650 seconds and has an experimental standard deviation of 0.008 ppm. Two days of averaging

are required to reduce the standard deviation of the mean to 0.0005 ppm. We find that we can quantitatively analyze our system noise and use (1) to obtain Boltzman's constant to within a few percent. At that level, vibrationinduced noise becomes important.

#### II. RECENT IMPROVEMENTS

#### A. Measurement System Changes

The measurement system that has been used to transfer the quantum Hall resistance to transportable reference resistors for subsequent step-down to the NIST 1  $\Omega$  resistance standard is that described by Marullo-Reedtz and Cage [10]. Our earlier 8.9  $\mu$ A measurements [3] on the Sony Si-MOSFET sample using that system required a full day of averaging to reduce the uncertainty arising from random effects to 0.004 ppm. Further investigation required more rapid and more precise measurements. Consequently, in 1991, the L&N 9829 null detector [11] originally used in that system was replaced by an EM Electronics N11 [11] which had a significantly lower noise figure.

As a continuation of our earlier investigations of Si-MOSFET plateaus [3], we examined a "sister" Si-MOS-FET sample to that used by Hartland *et al.* [5]. Our initial examination at 25  $\mu$ A revealed that that Southampton/ Nottingham sample had a plateau that was flat and seemed in agreement with our GaAs sample to within our system uncertainty of about 0.01 ppm. Measurements at 8.9  $\mu$ A, however, turned out to be complicated by an "interchange" error [12] nearly ten times larger than we had previously seen. Although the mean value of the "interchanged" and "normal" measurements also seemed to be in agreement with the GaAs value, we considered such a large "interchange error" to be unacceptable. Two changes reduced that interchange error to a level even less than observed at 25  $\mu$ A.

First, we improved the symmetry of the measurement system as illustrated in Fig. 1. When switching between the sample  $R_H$  and reference resistance  $R_R$ , the system had an asymmetry which caused residual ac currents to be connected differently to the null detector depending on whether  $R_H$  or  $R_R$  was being balanced. This effect was exaggerated for low-current measurements when a 390 k $\Omega$ additional resistance (not shown) was placed in series with the 6 k $\Omega$  trim resistor. In contrast to the L&N 9829, the new null detector was quite sensitive to these ac currents. In a system that is perfectly symmetric to ac as well as dc currents, there can be no "interchange error." Thus moving the current reversal switching closer to the null detector as shown in the lower diagram, greatly reduced the problem.

We then replaced the N11 with a Keithley 182 nanovoltmeter (K182) [11]. Whereas it was necessary to use the internal batteries of the L&N 9829 and EM N11 for optimal noise performance, the K182 has sufficient power line isolation to render battery operation unnecessary. Furthermore, its input circuit is designed to minimize



Fig. 1. Schematic diagrams of measurement system before and after improvements were made to reduce the "interchange" error which is evident when  $R_H$  and  $R_R$  are interchanged.

"pump-out noise" (sometimes called "dragon's breath"), noise emitted from the input terminals of a digital meter which can cause havoc when measuring sensitive samples such as Josephson and quantum Hall devices.

To correct for drifts, the K182 automatically performs a zero and gain check after each measurement. Consequently, it is only sampling the experiment for about 1/3of its measurement cycle, and the experiment will require three times longer to attain the same precision as would be the case if the averaging were continuous. We solved this problem by using sequential external triggering to interlace three Keithley 182's. We could not observe any deleterious side-effects from this arrangement, and since the effective duty cycle was about 95%, our measurement time was reduced by nearly a factor of 3 [13]. We found that the automatic zero and gain calibration of these meters prevented any noticeable "1/f" noise in their readings. Thus, we could average large numbers of readings for each null determination.

Fig. 2 shows the scatter of 131 complete intercomparisons between a GaAs heterostructure sample and a transportable 6453.2  $\Omega$  reference resistor using this arrangement. The Johnson noise of the room temperature resistors ( $R_p$  and  $R_R$  of Fig. 1) is the primary source of this system noise.

#### B. Barometric Pressure Effect

Soon after we started using a single Keithley 182 and were freed for the first time from the daily eight-hour dead time which had been required for charging the batteries of the null detector, we noticed an apparent dependence upon barometric pressure. Our measurements comparing



Fig. 2. System noise for 11-minute measurements taken over a 24-hour period.



Fig. 3. Data taken over a 10-day period (points and left-hand scale) showing the correlation with atmospheric pressure variations (upper curve and right-hand scale). Smoothed running means of the data are shown with (dark curve) and without (light curve) compensation for the pressure variations.

the Si-MOSFET with a transportable reference resistance (identified as E1) had irregularities that were correlated with changes in barometric pressure. This is shown in Fig. 3 where 11 days of measurements are displayed. The scattered points are values of the difference between the Si-MOSFET and E1 (left scale). The barometric pressure (right scale) is the curve in the upper portion of the figure. The quantum Hall data were found to be correlated with pressure with a coefficient of  $(\pm 0.0045 \pm 0.0006)$ ppm/kPa. This is presumably the pressure coefficient of E1. Measurements against a different reference resistor, E4, turned out to have a similar pressure coefficient,  $(\pm 0.0050 \pm 0.0002)$  ppm/kPa.

#### III. RESULTS

#### A. GaAs-Si-MOSFET Intercomparison

After most of the improvements had been made to the measurement system, a failure of the E1 transportable resistor's air bath controller forced us to switch to another

transportable resistor, E4. We then compared E4 against our usual GaAs heterostructure device, GaAs(7) at 6 T for 25 days, against the Nottingham Si-MOSFET at 14 T for 45 days, and finally again against GaAs(7) for 22 days. There was sufficient variation in barometric pressure  $P_{\rm baro}$ during that time to allow us to extract a reliable barometric pressure coefficient for E4. Also, variations in wiring connections and magnetic field direction were made to test for systematic errors. The measurements were made with sample temperatures below 500 mK (usually below 350 mK) and were least-squares fitted to the following sixparameter equation:

$$\{R_{H} - R_{E4}\}_{\text{fit}}/R_{H} = a_{1} + a_{2}(t - 145 \text{ day}) + a_{3}(P_{\text{baro}} - 100 \text{ kPa}) + a_{4}X + a_{5}S_{\text{center}} + a_{6}S_{\text{end}}.$$
(2)

Here X is +1 for "normal" configuration of the measurement system and -1 for the "interchanged" configuration.  $S_{center}$  is 1 for points measured against the Si-MOSFET sample using its central pair of contacts and zero otherwise;  $S_{end}$  is 1 for points measured using an outer set of voltage contacts of the Si-MOSFET sample and zero otherwise. Thus for the GaAs sample,  $S_{center} = S_{end} = 0$ ; for the Si-MOSFET sample using its central set of voltage contacts,  $S_{center} = 1$  and  $S_{end} = 0$ ; and for the Si-MOSFET sample using its end set of voltage contacts,  $S_{center} = 0$ and  $S_{end} = 1$ . Only one set of voltage contacts on GaAs(7), an end set, was usable, and a voltage contact at the drain end of the Si-MOSFET was open and could not be used.

The second term on the right-hand side of (2) allows for the gradual drift of the value of E4 vs time t, measured in days. By fitting to this single equation, all data irrespective of sample and measurement configuration contribute to determining the value  $a_1$  of E4, its drift rate  $a_2$ , the pressure coefficient  $a_3$ , and the interchange error  $a_4$ . These parameters fully characterize our measurement system and reference resistor E4, but are not of general interest. The parameters  $a_5$  and  $a_6$ , however, quantify the difference between the plateau values of GaAs(7) and this Southampton/Nottingham Si-MOSFET device. The results were independent of field direction and consequently no field-direction parameter appears in (2).

The 7500 data points, the fit parameters, and the resulting curve given by (2) are shown in Fig. 4. Here and elsewhere in this paper, the quoted uncertainties are estimates of the standard uncertainties arising from random effects. The slight drift of E4, its variability with atmospheric pressure, and the interchange differences, make it difficult to distinguish clearly if the quantum Hall resistances of the Si-MOSFET and GaAs samples are in agreement or not. Some sets of points, shown in boxes, were excluded from the fit because they were taken at temperatures above 1 K or because the system accidentally had multiple grounds. The residual errors between our data and the fit to (2) are plotted in Fig. 5. The residuals appear





Fig. 4. The full 100 days of raw data comparing the i = 4 plateaus of GaAs(7) (days 117–145 and days 194–216) and the Si-MOSFET (days 146–191) against room-temperature reference resistor E4. Except as indicated, the sample temperature was below 500 mK (usually below 350 mK). Experimentation with the measurement system timing caused the variations in system noise prior to day 137. The broad, solid line through the data was made by plotting the values at each measurement that are calculated from the fitted equation. Its irregularities reflect barometric pressure changes and changes between "normal" and "interchanged" measurement configurations. The boxed data was excluded from the fit for the reasons shown.



Fig. 5. Residuals between the fitted and measured values for the data of Fig. 4. The random scatter of these residuals shows that (2) gives an excellent representation of the behavior of the data for a variety of sample, measurement circuit, magnetic field, and gate voltage variations.

random, suggesting that laboratory humidity and temperature variations, reference resistor temperature controller errors, liquid helium bath level changes, etc., have no substantial effect on our measurements.

From the fitted value of  $a_5$ , the Si-MOSFET plateau resistance as measured at its central voltage contacts is seen to be in agreement with the GaAs value to within 0.0004 ppm [ $a_5 = (0.0000 \pm 0.0004)$  ppm]. On the other hand, the Si-MOSFET plateau resistance as measured by the voltage contact pair next to the source is seen to be 0.004 ppm above the GaAs value [ $a_6 = (0.0040 \pm 0.0003)$  ppm]. Under these same conditions, we also checked  $\rho_{xx}$  and found it to be less than 0.002 ppm of  $R_H$ when measured between an end voltage contact near the source and one near the drain. Since breakdown at the source and drain tends to extend further into Si-MOSFET devices than into GaAs heterostructures and since contact errors [4] are much greater for contacts adjacent to the source or drain, it is not too surprising that the end voltage contact pair might yield anomalous values. This is particularly true for this Si-MOSFET sample which has its outer voltage contact pairs quite close to the source and drain.

The primary purpose of this paper, however, is not to present this comparison, but rather to highlight some pitfalls encountered in pushing quantum Hall measurements to the 0.001 ppm level and below. The three groups of data in Figs. 4 and 5 which are boxed and identified as being taken with accidental multiple grounds illustrate one such problem. The first small group at day 141 is clearly below the nominal value. In this case, a multimeter used to check the thermometer monitoring E4 was accidentally left connected during the measurement. The middle boxed group of data were taken before it was discovered that the sample header had partially come out of its socket causing one sample lead to touch a wall. Even though this lead was nominally at the same potential as the surrounding metal, it appears that this second ground caused disruptive ac currents and led to the offset shown. The sample was then warmed and repositioned only to have a different lead touch (group of data at day 162). In this case, no offset can be seen. Finally, the sample was warmed again, the problem was permanently resolved, and the remaining data taken.

Extra sets of measurements were performed with the sample temperature between 1100 and 1400 mK. These are the boxed data at days 161–162 and 188–189. Any difference between these plateau values and those taken at temperatures below 500 mK is less than 0.002 ppm. In contrast to our earlier study of a Sony Si-MOSFET [3], this Southampton/Nottingham Si-MOSFET was insensitive to electrical disturbances, and we were able to continue making measurements even during violent thunderstorms.

We plan further measurements on this Si-MOSFET at both higher and lower currents, and on a variety of other Si-MOSFET and heterostructure samples.

#### **B.** Plateau Flatness

Although not apparent in Fig. 5, the gate voltage of the Si-MOSFET was set to four different values during days 147–151 to investigate the flatness of its plateau. Using the residuals of these data from our fit to (2), we determined separately the means of each of these data sets. The result is plotted in Fig. 6, where the ordinate values are referenced to the overall mean of these values. The error bars for each point, which are typically  $\pm 0.0008$  ppm, represent the statistical error inferred from  $\approx 120$ , 11-minute measurements at each gate voltage. The Si-MOS-FET plateau is clearly very flat.

A similar analysis was performed to examine the flatness of the GaAs heterostructure sample. Data taken at different field settings during days 193–216 are shown in Fig. 7. Here, the error bars, which are typically  $\pm 0.0006$ 



Fig. 6. Relative deviations of Si-MOSFET values vs gate voltage. These measurements, taken during days 148–153, show a flat plateau over a 2.2% change in gate voltage. The error bars represent the statistical error of each point as inferred from the scatter of  $\approx$  120, 11-minute measurements. The numbers attached to each point indicate the time sequencing of the measurements.



Fig. 7. The plateau of GaAs(7) vs magnetic field. The error bars represent the statistical error of each point as inferred from the scatter of  $\approx 240$ , 11-minute measurements. The numbers attached to each point indicate the time sequencing of the measurements. It appears that a shift may have occurred between points 4 and 5.

ppm, represent the statistical uncertainty of the mean inferred from  $\approx 240$ , 11-minute measurements comprising each plotted point. The time sequence of these measurements is given by the numbers attached to each point. If the plateau is assumed to be perfectly flat, we can treat these 2-day means as 10 independent determinations and use their actual scatter to obtain a different measure of their statistical uncertainty. This latter method gives 0.0015 ppm, more than twice that calculated from the fluctuations of the 11-minute measurements. The exact cause of this is unknown, but it appears that a shift occurred between points 4 and 5 when we replaced the batteries in the unit controlling the triggering of the K182 meters.

#### **IV.** CONCLUSIONS

We have re-examined our quantum Hall measurement system using both our usual GaAs sample and a Si-MOS-FET sample similar to that studied by Hartland. A variety of difficulties were uncovered which affect measurements at the parts in  $10^9$  level, but none could explain the larger anomalies we observed in our earlier studies of a Sony Si-MOSFET.

The quantum Hall effect in existing 2-D systems is a very complex phenomenon that is only partially understood. Not only are the irregularities in the Sony Si-MOS-FET samples not understood, but other basic questions regarding the current distribution, exact effect of impurities, the effect of edge-currents at medium current levels, and the breakdown of quantization at the highest currents all remain unresolved. These questions need to be fully understood in order to confidently rely on the quantum Hall effect at the 0.001 ppm level. Furthermore, metrologists must take care to retest all sources of possible measurement error. Every minute aspect of the measurement system must be *quantitatively* understood.

Establishing guidelines that can guarantee accuracy of a quantum Hall resistance standard is very difficult. Extremely subtle measurement system and sample errors exist which can require months of testing to characterize and/ or eliminate. Guidelines that seem to work at the 0.01 ppm level must be exhaustively retested at the 0.001 ppm level. This symbiotic relationship between probing new physical effects and advancing the technology of precision measurement continues to yield new physics and valuable metrology.

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- [11] Identification by manufacture and model number is for completeness in scientific description and does not constitute an endorsement by NIST.
- [12] The "interchange error" is the difference observed when the sample and reference resistors are interchanged. As long as leakage resistances are greater than  $10^{13} \Omega$  and the quantum Hall device responds linearly to the applied current, a dc analysis of the system dictates that there can be no interchange error. In fact, however, a difference of typically 0.006 ppm is observed. The final value for the comparison has been taken as the mean of the "interchanged" and "normal" values. See [10] for additional details. This interchange error appears to be caused by the response of the null detector to stray ac signals.
- [13] This interlacing avoided simultaneous sampling by more than one K182 at a time. In principle, if the "pump-out noise" and injection currents are sufficiently low, simultaneous measurements may be practical. Then, if the input noise of the different units remained uncorrelated, averaging of their readings would reduce the effect of that instrumentation noise. The effective input impedance would, of course, be reduced in accordance with the number of simultaneously connected input circuits.

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