Pulse-Driven Josephson Digital/Analog Converter

Samuel P. Benz, Clark A. Hamilton, Charles J. Burroughs
National Institute of Standards and Technology, Boulder, CO 80303 USA

Lawrence A. Christian
New Zealand Institute for Industrial Research and Development, Lower Hutt, New Zealand

John X. Przybysz
Northrop Grumman STC, Pittsburgh, PA 15235 USA

Abstract–We have designed and demonstrated a pulse-driven Josephson digital/analog converter. When used as a programmable voltage standard, this device can synthesize metrologically accurate ac waveforms as well as stable dc voltages. We show through simulations that Josephson quantization produces a nearly ideal quantization noise spectrum when a junction is driven with a typical waveform produced by a digital code generator. This technique has been demonstrated in preliminary experiments with arrays of 1000 junctions clocked at frequencies up to 12 Gbit/s where sine waves of a few millivolts in amplitude were synthesized at frequencies up to 1 MHz.

I. INTRODUCTION

In this paper we describe recent progress in Josephson D/A converters for ac wave form synthesis. NIST is developing Josephson D/A converters as fast programmable voltage standards for the following applications: (1) calibration of dc reference standards and digital voltmeters, (2) characterization of commercial D/A and A/D converters, and (3) generation of digitally synthesized ac waveforms with calculable rms voltages. In 1995 Hamilton et al. proposed a Josephson D/A converter based on a binary sequence of series arrays of resistively shunted tunnel junctions [1]. When biased with a microwave frequency f, each junction exhibits constant voltage steps at $V = nf/K_J$, where $K_J = 483\ 597.9$ GHz/V. The quantum step number $n = -1$, 0, or +1 corresponds to the number of junction pulses per period $1/f$, and is determined by the array bias. The average dc voltage of each array of N junctions is

$$V = nNf/K_J. \tag{1}$$

To improve the performance of these arrays, NIST has developed a superconductor-normal-super-conductor (SNS) trilayer junction technology [2], [3]. These SNS junctions have large critical currents, $I_c > 1$ mA, that provide better stability against noise and higher output current. Their internal resistance results in a nonhysteretic current-voltage curve that is inherently stable without the use of external shunt resistors. This junction technology and circuit design has advanced to the level where a binary sequence circuit with 32 768 SNS junctions in 9 independently selectable arrays on a single chip has demonstrated stable accurate voltages over the range from −1.2 V to +1.2 V [2]-[6].

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Although this binary-sequence design is adequate for programming *fast dc* voltages, the first two applications listed above, it has not proved practical for generating *fast ac* waveforms. Switching the bias of selected arrays produces switching transients that result in a substantial uncertainty to the generated waveform [7]. However, instead of changing the quantum number n or number of junctions N, the output voltage can also be controlled by changing the excitation frequency f. Recently we have shown that if a pulse excitation is used instead of a sine wave, the step amplitude is independent of the pulse repetition frequency for all frequencies below a characteristic frequency $f_c = I_cRK_J$, where R is the junction's normal state resistance [8], [9].

II. VOLTAGE PULSE QUANTIZATION

As shown in Fig. 1, a pulse-driven Josephson D/A converter consists of a single large array of N junctions distributed along a wide bandwidth transmission line. When a pulse propagates down the line, it induces a quantized voltage pulse with a time-integrated area n/K_J across each junction it passes. As in (1), n is an integer that corresponds to the number of junction pulses for each input pulse. Thus a pulse train of frequency f propagating down the line generates an average voltage nNf/K_J across the array. A complex output waveform can be generated by gating the input pulse train with a long digital word generator. A knowledge of the digital code, the clock frequency, and the number of junctions in the array is sufficient to precisely calculate the output waveform. Since there is only one array, the uncertainty associated with switching between arrays is eliminated and the number of Josephson pulses occurring in any time increment is calculable.

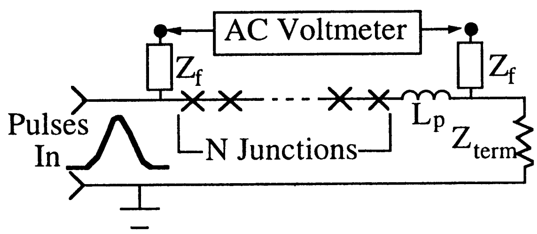

Fig. 1: Josephson array pulse quantizer for a D/A converter.

Figure 2 is a block diagram of the process that is used to synthesize a sine wave of frequency f or any other periodic waveform from quantized Josephson pulses. The modulator algorithm block is a computer program that digitizes m periods of an input sine wave $S(t)$ at a sampling frequency f_S. There are mN_s two state (−1 or +1) digital samples, where $N_s = f_s/f$ is the number of samples per period. The modulator output $S(i)$ is therefore a long digital sequence where the density and sign of the 1's are proportional to the input (see the qualitative bit pattern inset in Fig. 2). For a repetitive waveform, this code is calculated just once and stored in a circulating memory. The digital code generator re-creates the waveform as an output voltage in real time $S_D(t)$ by clocking the memory at the sampling frequency. Each output digit generates either a negative, or a positive current pulse that is launched into the Josephson array stripline.

The Josephson array quantizes the time integral of the pulses producing a time-dependent output voltage $S_J(t)$. The Fourier spectrum of this quantized waveform consists of the desired waveform plus additional harmonics of f (and f/m for >1 period) that are known as quantization noise. We chose a special type of algorithm called a delta-sigma modulator to minimize the quantization noise spectrum near f thus giving a very high signal-to-noise ratio in the filter pass band [10]. Systems like that shown in Fig. 2 (without the Josephson pulse quantizer) that generate analog signals by summing many pulses are called delta-sigma D/A converters. Since most of the quantization noise occurs at frequencies far above f, a low pass filter can eliminate it, leaving only the desired waveform $S'(t) \approx S(t)$.

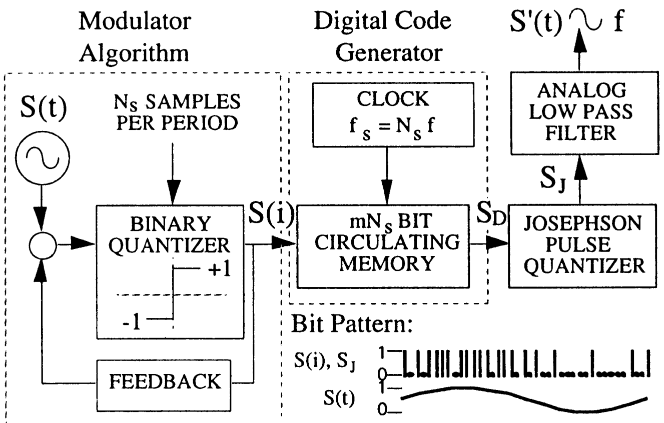

Fig. 2 A block diagram of a delta-sigma digital-to-analog converter based on pulsed Josephson junctions.

We will now demonstrate why the Josephson quantizer is an essential component of an accurate D/A converter by considering the Fourier spectra of the intermediate waveforms in Fig. 3. Consider a 1 MHz offset sine wave having 4 periods and sampled at 2.56 GHz. Since the sine wave is generated by repeating the digital sequence indefinitely, the Fourier transform of the digital code $S(i)$ is a line spectrum with a dominant line at f = 1 MHz. The line spectrum for this "ideal digital code" is shown in Fig. 3(a), where for clarity we have connected the points representing the power in each harmonic. The peak at 1 MHz is the desired sine wave signal and all of the other lines are quantization noise. By using a second order delta-sigma modulator with an oversampling ratio of 128, the quantization noise is about 100 dB below the signal frequency up to about 10 MHz.

If we could directly generate this perfect digital code, then there would be no need for the Josephson array because a perfect digital code is quantized by definition. However, the output S_D of typical digital code generators has both correlated and uncorrelated amplitude and phase noise. When we recompute the spectrum including realistic values for the amplitude and phase noise (normal distributions with σ = 2.5%) the result is the "real digital code" curve. This spectrum is far too noisy to be useful for metrology.

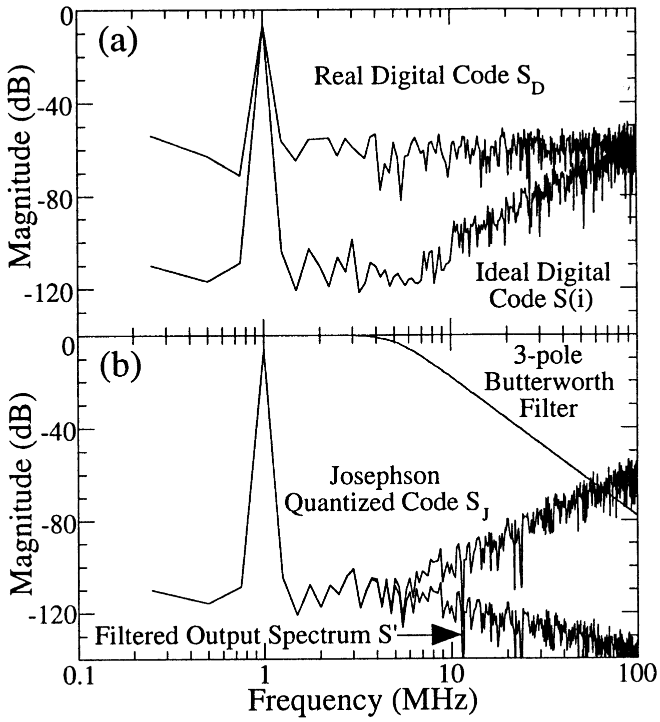

Fig. 3 The simulated Fourier spectra of the signals in the Josephson delta-sigma D/A converter. (a) spectra of input codes prior to Josephson quantizer. (b) spectra of Josephson quantized output waveforms.

Next we bias a Josephson junction (or array) with this real digital code. We use the optimum pulse width, $\tau = (2f_c)^{-1}$ [8]. The junction equations are integrated to find the periodic solution [11], resulting in the junction output voltage $S_J(t)$. The corresponding "Josephson quantized code" spectrum in Fig. 3(b) is almost identical to the ideal digital code spectrum. *Thus, even though the Josephson array does nothing to improve time jitter, and it does not quantize the pulse amplitudes, the fact that the time integral of the pulses is quantized is sufficient to produce a nearly ideal spectrum.* The effect of the amplitude and phase variations in the input digital code and random variations in junction parameters is to reduce the current bias range of the circuit and reduce the current that can be supplied to a load. $S_J(t)$ is filtered by a 3-pole Butterworth transfer function with a 5 MHz cutoff frequency and the resulting final output waveform $S'(t)$ is shown as the "filtered output spectrum." In another simulation, Josephson quantization also successfully removed the quantization noise from a digital code with voltage levels that were correlated to the bit pattern.

In a metrology application we might now ask what contribution the quantization noise makes to the rms value of the signal at f. This is readily computable by comparing the rms value of the line at f with the rms value of the total filtered output spectrum. In the example of Fig. 3, the quantization noise increases the rms value by only 1 part in 10^9. In practice, other effects, such as common mode rejection, will dominate the uncertainty of the output voltage.

III. EXPERIMENTAL RESULTS

In order to demonstrate these ideas, we have designed and fabricated a circuit consisting of a 7 mm long array of 1000 junctions along the center conductor of a 50 Ω superconducting coplanar stripline [3], [9]. The stripline is terminated by a 50 Ω resistor and connections to the ends of the array are made through low pass filters. Equation 1 shows that the maximum voltage range of this test circuit is only a few millivolts. This is sufficient for our feasibility study, but much larger circuits with up to 100 000 junctions will be required for practical D/A converters.

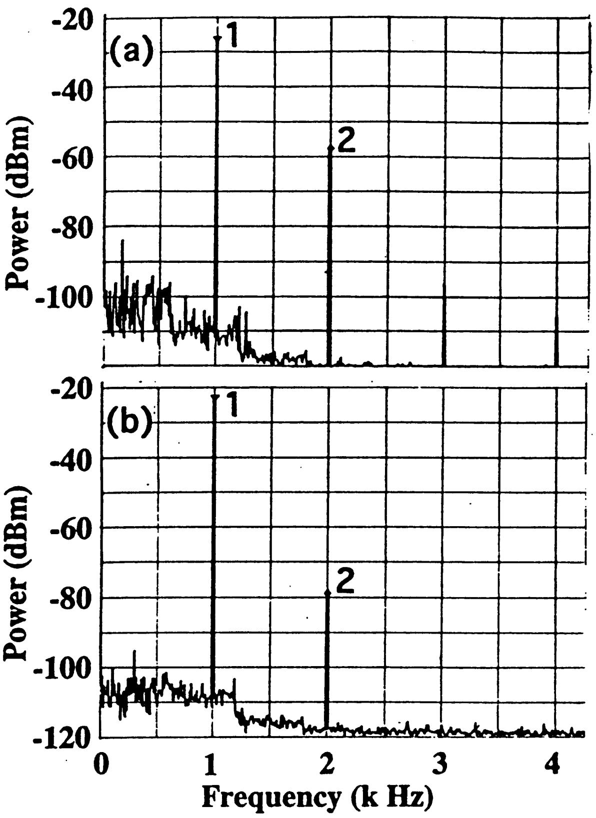

Fig. 4 Measured Fourier spectra showing how the quantizing effect of the array improves signal purity. (a) Digital code generator spectrum. (b) Amplified spectrum of Josephson array voltage.

Figure 4 is our first attempt to verify experimentally the theoretical results of Fig. 3. We synthesized one period of a 1 kHz sine wave using a 2.56 Mbit long code. The upper spectrum (a) shows the fundamental (peak 1) and the first few harmonics of the output of the digital code generator. These large harmonics are caused by voltage levels of the digital code that are correlated with the density of 1's. The spectrum of the corresponding amplified array output is shown on the bottom (b). Comparison of spectra (a) and (b) shows that the Josephson array circuit has reduced the unwanted harmonics by at least 22 dB.

This verifies the principle that the quantizing effect of the array will greatly reduce the effect of amplitude noise in the pulse input. However, Fig. 3 and our simulation on a correlated voltage pattern suggest that the improvement should be greater than 50 dB rather than just 22 dB. The reason for this discrepancy is believed to be insufficient rejection of the common mode voltage across Z_{term}.

IV. CONCLUSIONS

Implementations of Josephson voltage standards prior to this work use a fixed frequency to trigger quantized junction pulses that are averaged into a dc voltage. In this paper we demonstrate how a complex pattern of pulses driving an array of 1000 junctions produces a smooth sine wave output with high resolution and accuracy. Efforts are underway to increase the number of junctions and the clock frequency to achieve a voltage range of ±1 V. Success in this effort will lead to a new programmable Josephson voltage standard that can produce quantum mechanically accurate dc and ac voltages with a bandwidth of 0 to about 1 MHz.

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