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Brief Communication

In Vitro Exposure Parameters With Linearly and Circularly Polarized ELF Magnetic Fields

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A comparison is made of induced current densities, electric fields, and rates of energy deposition during in vitro studies with linearly and circularly polarized, extremely low frequency magnetic fields for a cylindrical volume of culture medium.

Key words: dosimetry, current density, electric field, exposure system

Because multiphase power lines will in general produce rotating magnetic fields, some in vitro biological experiments have been conducted with circularly polarized magnetic fields. Often, however, the magnetic fields are linearly polarized. This brief communication examines the differences in candidate exposure parameters that are induced in culture medium by either linearly or circularly polarized magnetic fields. An experimental geometry is chosen that is commonly used during in vitro studies and that simplifies some of the calculations of the induced quantities.

Figure 1a shows a cylindrical volume of culture medium of diameter 2a and depth 2h with extremely low frequency (ELF) magnetic fields applied along the x- and y-axes. Using equations developed by McLeod et al. [1983] for rectangular geometries and vertically slicing the cylinder into approximately rectangular sections, it is readily shown that for $2a >> 2h$, the induced current density and electric field at most locations on the top and bottom surfaces of the culture medium are uniform. Figure 1b shows normalized values of induced current density on the bottom surface of a liquid volume 6 cm in diameter and 0.2 cm deep, due to a magnetic field aligned with the x-axis.

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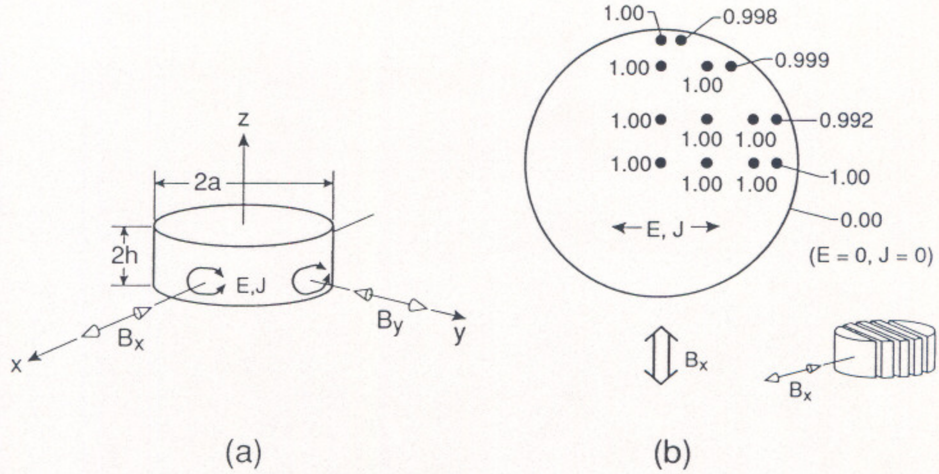

Fig. 1. Experimental configuration for in vitro studies: (**a**) Components of magnetic field applied parallel to x- and y-axes; (**b**) Normalized induced current density distribution along bottom (or top) surface of culture medium for uniform magnetic field parallel to x-axis for 2a = 6 cm and 2h = 0.2 cm.

In accord with the principle of superposition, the induced current density at the bottom surface of a rectangular section due to magnetic field components along the x- and y-axes is given by

$$\vec{J} = J_x\hat{i} + J_y\hat{j} \tag{1}$$

where [McLeod et al., 1983]

$$J_x \simeq \frac{\omega\sigma B_y 2w}{\pi^2}\sum_{n_{odd}}\frac{4}{n^2}\left(\pm\cos(\frac{n\pi x}{2w})\right)\left(\tanh(\frac{-n\pi h}{2w})\right)$$

and

$$J_y \simeq \frac{\omega\sigma B_x 2w}{\pi^2}\sum_{m_{odd}}\frac{4}{m^2}\left(\pm\cos(\frac{m\pi y}{2w})\right)\left(\tanh(\frac{-m\pi h}{2w})\right).$$

In equation (1), ω is 2π times the frequency f; σ is the conductivity of the culture medium; B_x and B_y are the ELF magnetic flux densities parallel to the x- and the y-axes respectively; 2w is the width of a rectangular section (2w >> 2h); $\hat{i}$ and $\hat{j}$ are unit vectors along the x- and y-axes; and, the $\pm$ sign indicates that the sign in the summations alternates. For most values of x and y on the bottom and top surfaces of the culture medium, the product of w and the summations in the expressions for J_x and J_y are constant and equal to one another. Therefore, the vector components of current density can be expressed as

$$J_x = CB_y, \tag{2}$$

and

$$J_y = CB_x, \tag{3}$$

where C is a constant given by

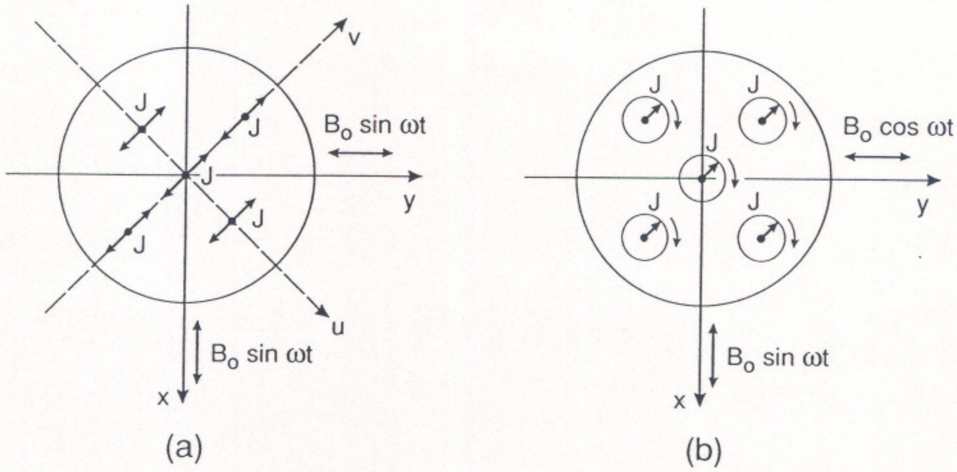

Fig. 2. **(a)** Induced current sheet oscillates along fixed direction for linearly polarized magnetic field. **(b)** Induced current sheet rotates with constant magnitude for circularly polarized magnetic field.

$$C = \frac{\omega\sigma 2w}{\pi^2} \times \text{(summation)}.$$

For a linearly polarized magnetic field where $B_x = B_o \sin \omega t$ and $B_y = B_o \sin \omega t$, the induced current density is also linearly polarized as shown at representative points (on bottom surface) in Figure 2a. Similarly, for a circularly polarized magnetic field where $B_x = B_o \sin \omega t$ and $B_y = B_o \cos \omega t$, the induced current density is circularly polarized as shown at some instant for several representative points in Figure 2b.

From equations (1)–(3), the magnitude of the current density induced by a linearly polarized magnetic field, J_l is

$$J_l = \sqrt{J_x^2 + J_y^2} = C\, B_o \sqrt{2} \sin \omega t. \tag{4}$$

Similarly, the magnitude of the current density induced by a circularly polarized magnetic field, J_o is

$$J_o = CB_o. \tag{5}$$

It is noted that the root mean square (rms) value of J_l is equal to the rms value of J_o.

Thus, when the applied magnetic field is linearly polarized, cells plated on the bottom surface of a culture vessel and which are away from the edges of the vessel will be exposed to a sheet of current that oscillates in magnitude [Eq. (4)] along a fixed direction indicated by the J vectors shown in Figure 2a. In contrast, when the applied magnetic field is circularly polarized, cells experience a current sheet, constant in magnitude, that never "turns off" [Eq. (5)] and that rotates 360 degrees about the z-axis for every cycle of the applied magnetic field. The rotating nature of the current sheet can be inferred from the rotating J vectors shown in Figure 2b. In addition, the peak value of the current induced by the linearly polarized magnetic field is 41% greater than for the circularly polarized case, i.e., J_l (peak)/J_o is equal to $\sqrt{2}$. Similar observations are true for the induced electric fields, $\vec{E}$, experienced by cells because $\vec{J}$ is equal to $\sigma\vec{E}$. That the induced electric field does not pass through zero for the circularly polarized case may have relevance if a resonance model is being tested.

An approximate expression for the rate of energy deposition near the bottom (or top) surface of the culture medium is developed next. The rate that energy is deposited in the culture medium, $\dot{\varepsilon}$ is given by the product, I^2R, where I is the current near the bottom surface and R is the electrical resistance offered by the culture medium. As for the induced current and electric field, $\dot{\varepsilon}$ will be very small compared with its naturally occurring counterpart in a biological system.

We consider first a current I_u through a small cross sectional area dA_u close to the bottom of the culture medium at some instant. This current can be expressed as

$$I_u = \vec{J} \cdot \overrightarrow{dA}_u \tag{6}$$

where $\overrightarrow{dA}_u$ is a vector parallel to $\vec{J}$ and has magnitude dA_u, i.e.,

$$\overrightarrow{dA}_u = dA_u \frac{J_x\hat{i} + J_y\hat{j}}{\sqrt{J_x^2 + J_y^2}} .$$

Thus, I_u is just

$$I_u = dA_u\sqrt{J_x^2 + J_y^2} . \tag{7}$$

The resistance, R_u, encountered by this current is approximately

$$R_u \simeq \frac{L_u}{\sigma dA_u} . \tag{8}$$

where L_u is the distance across the bottom surface (L_u does not actually extend completely across the surface because J vanishes at the perimeter as shown in Fig. 1b). Therefore, the rate that energy is deposited along a narrow path parallel to $\vec{J}$ is

$$d\dot{\varepsilon} \simeq I_u^2R_u = \frac{dA_u}{\sigma}(J_x^2 + J_y^2)L_u \tag{9}$$

To obtain an approximate expression for the total rate of energy deposition, we make use of the dashed coordinate system u, v shown in Figure 2a. The cross sectional area, dA_u, along the u-axis is taken to be z_0du where z_0 is a very small length in the z direction and du is an element of length along the u-axis. For a given value of u, L_u extends from $-\sqrt{a^2-u^2}$ to $\sqrt{a^2-u^2}$ or is equal to $2\sqrt{a^2-u^2}$. Therefore,

$$d\dot{\varepsilon}_u \simeq \frac{2z_0}{\sigma}(J_x^2 + J_y^2)\sqrt{a^2 - u^2}\, du \tag{10}$$

The total $\dot{\varepsilon}$ for $-a < u < a$ is then

$$\dot{\varepsilon} = \int_{-a}^{a} d\dot{\varepsilon} \simeq \frac{z_0\pi a^2}{\sigma}(J_x^2 + J_y^2) \tag{11}$$

From equations (4), (5), and (11), we have for a linearly polarized magnetic field that

$$\dot{\varepsilon}_l \simeq \frac{z_0 \pi a^2 C^2}{\sigma} 2B_0^2 \sin^2 \omega t = \frac{z_0 \pi a^2 C^2}{\sigma} B_0^2(1 - \cos 2\omega t). \tag{12}$$

and for a circularly polarized magnetic field,

$$\dot{\varepsilon}_o \simeq \frac{z_0 \pi a^2 C^2}{\sigma} B_0^2. \tag{13}$$

Equations (12) and (13) indicate that the rate at which energy is deposited by the linearly polarized magnetic field pulsates at twice the frequency of the applied field, and the peak rate is twice as large as the nonpulsating $\dot{\varepsilon}_o$ produced by the circularly polarized magnetic field. It is noted that the average value of $\dot{\varepsilon}_l$ is equal to $\dot{\varepsilon}_o$. Most of the above results extend to in vitro studies with culture vessels that are square or rectangular in geometry. However, the calculation of $\dot{\varepsilon}$ for a circularly polarized magnetic field is more difficult because R changes in a complicated way as $\vec{J}$ rotates.

In summary, when comparing the induction effects of circularly and linearly polarized magnetic fields for the experimental geometry chosen, the following observations can be made: the maximum J or E that most cells are exposed to will be 41% greater when the magnetic field is linearly polarized; J and E will oscillate along a fixed direction (passing through zero) when B is linearly polarized; J and E rotate with constant magnitude (never vanishing) when B is circularly polarized; the rms values of J (or E) are equal regardless of polarization; $\dot{\varepsilon}$ will pulsate twice every cycle and the maximum value of $\dot{\varepsilon}$ will be twice as large when B is linearly polarized; $\dot{\varepsilon}$ remains constant when B is circularly polarized but the average values of $\dot{\varepsilon}$ are equal regardless of polarization. The significance of the differences, if any, is unknown.

REFERENCES

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