Reprint Series
10 September 1999, Volume 285, pp. 1706–1709

Science

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A capacitance standard based directly on the definition of capacitance was built. Single-electron tunneling devices were used to place N electrons of charge e onto a cryogenic capacitor C, and the resulting voltage change ΔV was measured. Repeated measurements of $C = Ne/\Delta V$ with this method have a relative standard deviation of 0.3×10^{-6}. This standard offers a natural basis for capacitance analogous to the Josephson effect for voltage and the quantum Hall effect for resistance.

In the past four decades, there has been an accelerating trend in metrology toward standards based on fundamental quantum properties of nature. Until 1960, all units in what is now the International System of Units (SI) were based on carefully constructed artifacts and classical physics (*1*). Quantum physics first entered the SI in 1960, when the definition of the meter was based on the wavelength of radiation from a transition in the Kr atom. A voltage standard based on the Josephson effect was first adopted in 1972 and refined in 1990, and a resistance standard based on the quantum Hall effect was adopted in 1990 (*2, 3*). For capacitance, the best existing standards are known as "calculable capacitors" and rely on a special arrangement of several electrodes such that the capacitance per unit length is related to the permittivity of free space (a defined constant in the SI) (*4*). Realizing such a standard requires precise alignment of electrodes of order 1 m in length, one of which must be movable, and compensation of end effects in order to make a system of finite length behave like an infinite system over a limited range. With the development over the past decade of single-electron tunneling (SET) devices that can precisely manipulate and detect single electrons (*5*), it is now possible to create a capacitance standard based on the quantization of electric charge (*6*). Such a standard, which we describe here, places capacitance metrology on a quantum basis and is a natural complement to the voltage and resistance standards adopted in 1990 (*7*).

Our capacitance standard combines SET devices and a low-loss cryogenic capacitor. We explain the operation of the standard, demonstrate its repeatability and uncertainty, and consider the prospects for developing our prototype into a practical calibration system. This stan-

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dard is the result of several years of research and development at the National Institute of Standards and Technology (NIST), and components of the standard have been described in previous publications (*8–11*).

SET devices exploit the energy required to charge a capacitance C_0 with one electron, $e^2/2C_0$. For even the smallest value of capacitance in common electronic components, 1 pF, this charging energy corresponds to a temperature of order 1 mK and is negligible in comparison to thermal fluctuations. However, modern nanolithography allows the fabrication of ultrasmall tunnel junctions with $C_0 \approx 0.1$ fF and $e^2/2C_0 \approx 10$ K. When such junctions are cooled to of order 0.1 K, SET effects completely dominate thermal fluctuations. To illustrate how SET devices allow the precise manipulation of individual electrons, we briefly explain the electron pump (*12*) shown in Fig. 1A. It consists of a chain of tunnel junctions (≈40 nm by 40 nm), with a gate capacitor coupled to the island of metal between each pair of junctions (*13*). With no voltages applied to the gate capacitors, tunneling is suppressed by the charging energy barrier. When the gates are pulsed in sequence, this barrier is selectively lowered to allow tunneling at one junction after another, and a single quantum of charge is transferred through the pump. Using our seven-junction pumps (*14*), we can routinely transfer billions of electrons with one error in every 10^8 attempts (*15*).

In defining capacitance, we consider the transfer of a charge Q between two conductors. The charge transfer creates a potential difference ΔV, and the capacitance is simply $C = Q/\Delta V$. Figure 1B shows a simple implementation of this definition based on counting electrons. Our SET capacitance standard, shown in Fig. 2A, has three critical components: (i) a seven-junction electron pump (*8*, *9*), (ii) a two-junction SET transistor/electrometer (*5*) ("E" in Fig. 2A) that can detect a charge of order $e/100$ at its input capacitor, and (iii) a cryogenic vacuum-gap capacitor (*10*) having nearly ideal properties, in particular, extremely small leakage and frequency dependence. This capacitor has a three-terminal design with a well-defined value that is insensitive to stray capacitance.

Operation of the SET capacitance standard occurs in two phases, which we select by setting the mechanical cryogenic switches (*9*) N1 and N2. The configuration in Fig. 2A is used to determine C by counting electrons. As the pump transfers electrons, the voltage across it must be kept near zero to avoid errors. The electrometer accomplishes this by acting as a null detector for a feedback circuit that applies a voltage to the outside of C in order to keep the island between the pump and capacitor at virtual ground. This also ensures that all charge transferred through the pump appears at C and not at the 10-pF stray capacitance. After N electrons have passed through the pump, we stop the pump and measure ΔV. The configuration in Fig. 2B is used to compare C with another capacitor C_{ref} at room temperature using a conventional ac bridge. With voltages v_1 and v_2 adjusted to balance the bridge, $C/C_{\mathrm{ref}} = v_2/v_1$.

Figure 3 shows the voltage across the capacitor as the pump transfers electrons in one direction, stops for 20 s, transfers the same number in the other direction, and repeats. We averaged the voltage data on each 20-s plateau and calculated voltage differences between successive plateaus, thus obtaining many values of ΔV for each set of data like that in Fig. 3. The average $\langle \Delta V \rangle$ determines the capacitance

$$C = Ne/\langle \Delta V \rangle \qquad (1)$$

While the standard is kept at its operating temperature of 40 mK, the relative variations in the value of C are of order $1 \times 10^{-6} = 1$ part per million (ppm), as demonstrated in Fig. 4, A and B (*16*). The relative standard deviation σ for each set of data is 0.3 ppm for Fig. 4A and 0.7 ppm for Fig. 4B. The larger scatter in Fig. 4B is probably due to fluctuations in the dimensions of the vacuum-gap capacitor that did not occur during the shorter measurement period of Fig. 4A. We tested the voltage dependence of the value of C by pumping different numbers of electrons onto the capacitor, and Fig. 4C shows that there is no voltage dependence within the resolution of our measurements.

Before considering the accuracy of the SET capacitance standard, we must discuss the uncertainty (*17*) of each factor in Eq. 1. From independent tests of the pump immediately before and after operation of the standard, N has a relative uncertainty of 0.01 ppm. The uncertainties of e and $\langle \Delta V \rangle$ are subtle, and to explain them, we must briefly review the voltage standard adopted in 1990. An array of Josephson junctions excited at a frequency f produces a voltage nf/K_J, where n is an integer and $K_J = 2e/h$, where h is the Planck constant. In 1990, the following exact value was adopted by international convention: $K_{J\text{-}90} \equiv 483\,597.9$ GHz/V. This adoption of a defined value with no uncertainty established the 1990 volt, which is denoted by V_{90} and is related to the SI volt by $V_{90}/V = K_{J\text{-}90}/K_J$. The value of $K_{J\text{-}90}$ was chosen so that the 1990 volt is expected to be equivalent to the SI volt, but this equivalence has a relative uncertainty of 0.8 ppm because of uncertainties in our knowledge of various

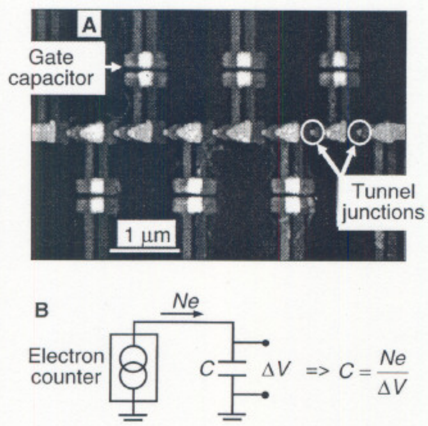

Fig. 1. (**A**) Scanning force microscope image of a seven-junction electron pump. The device consists of two layers of Al shifted horizontally by ≈0.2 μm to form tunnel junctions at the bright spots where the tip of each island overlaps its neighbor to the left. Pulsing the gates in sequence from left to right transfers electrons from left to right, and vice versa. After N cycles, the charge transferred through the pump is Ne, with an uncertainty of 1 part in 10^8. (**B**) Schematic implementation of the definition of capacitance by counting electrons.

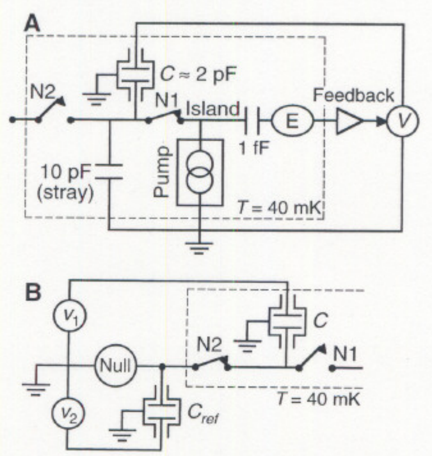

Fig. 2. Schematic diagram of the SET capacitance standard. (**A**) Configuration used to pump electrons onto C. The stray capacitance of 10 pF comes mostly from the third terminal of the vacuum-gap capacitor. (**B**) Configuration used to compare C with another capacitor at room temperature using an ac bridge.

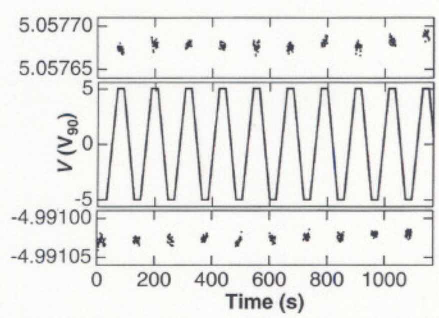

Fig. 3. Voltage applied to the capacitor by the feedback circuit while pumping electrons on and off C. Expanded views of the plateaus are shown above and below the main plot. For these data, N = 117 440 513 (= 7000001 hexadecimal) and $\langle \Delta V \rangle$ = 10.048 703 31 V_{90}, giving C = 1.872 484 77 pF from Eq. 2.

fundamental constants (2). We measured ΔV in terms of V_{90} (18), whereas for comparison with another fundamental capacitance standard we must express C in terms of the SI farad. If we naïvely use the recommended uncertainties (19) for the SI value of e (0.6 ppm) and for V_{90}/V (0.8 ppm), we find a combined uncertainty of $(0.6^2 + 0.8^2)^{1/2} = 1$ ppm. This implies that no matter how small the experimental uncertainty of the SET capacitance standard, the total uncertainty of C in SI units cannot be smaller than 1 ppm. However, this approach ignores the fact that the uncertainties in e and V_{90}/V are correlated. To account for this correlation, we express e in terms of the fine structure constant α and K_J as follows: $e = (4\alpha/\mu_0 c)(1/K_J)$, where $\mu_0 \equiv 4\pi \times 10^{-7}$ N/A^2 is the permeability of free space and $c \equiv 299\ 792\ 458$ m/s is the speed of light in vacuum (6). The expression for C then becomes

$$C = \frac{Ne}{\{\langle \Delta V \rangle\}_{SI} V} = \frac{N(4\alpha/\mu_0 c)(1/K_J)}{\{\langle \Delta V \rangle\}_{90} V_{90}} = \frac{N(4\alpha/\mu_0 c)}{\{\langle \Delta V \rangle\}_{90} K_{J\text{-}90} V} \qquad (2)$$

where $\{x\}_s$ denotes the dimensionless numerical value of x when it is measured in the system of units s. Because μ_0, c, and $K_{J\text{-}90}$ are defined constants, the only nonexperimental uncertainty in Eq. 2 is that of α. Currently, the recommended value (19) of $\alpha = 7.297\ 353\ 08 \times 10^{-3}$ has a relative uncertainty of 0.09 ppm, but this is expected to decrease by about a factor of 10 in the near future. Thus, the SET capacitance standard potentially offers a value of C in SI units with a total uncertainty of order 0.01 ppm if the experimental uncertainties in N and $\{\langle \Delta V \rangle\}_{90}$ can be made sufficiently small.

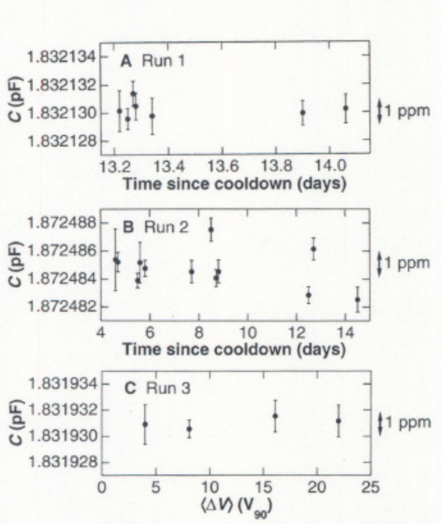

Fig. 4. Repeatability of the SET capacitance standard with $\langle \Delta V \rangle \approx 10$ V during periods of (**A**) 24 hours and (**B**) 10 days. (**C**) Measurements of C over a range of $\langle \Delta V \rangle$ show that the standard is independent of voltage. Uncertainty bars are ± 1 SD from the mean within each measurement.

The accuracy of the SET capacitance standard can be tested by comparing the value of C found by counting electrons with the value measured in terms of another fundamental standard, such as a calculable capacitor. We accomplished this by using a commercial capacitance bridge (20) operating at 1000 Hz and calibrated with a 10-pF silica-dielectric capacitor traceable to NIST's calculable capacitor at 1592 Hz. Figure 5 shows the values of C found from counting electrons and from the bridge for the same experimental runs as Fig. 4, A and B. The uncertainty bars on the electron counting values are $\pm U_{tot} C$, where $U_{tot} = [0.09^2 + 0.01^2 + 0.1^2 + (2\sigma)^2]^{1/2}$ is the combined relative uncertainty (in parts per million) from α, N, the voltmeter itself, and the statistical variations in each set of data. The uncertainty bars on the bridge values are ± 2.4 ppm owing to the uncertainty in the value of the 10-pF capacitor at 1000 Hz. Within these uncertainties, the measurement from counting electrons agrees with the measurement traceable to a calculable capacitor. We think that the actual systematic errors in the SET capacitance standard are smaller than the upper bound of ≈ 2 ppm demonstrated here, and we are pursuing a better comparison with a calculable capacitor.

The performance of our prototype is already impressive enough to suggest that a capacitance standard based on counting electrons will play an important role in electrical metrology. However, there are several issues that must be addressed before the SET capacitance standard can fulfill this promise (11). We briefly mention three important issues here, with further details to be presented elsewhere. (i) The frequency dependence of the cryogenic capacitor must be very small because the measurement of C by counting electrons occurs at an effective frequency much lower than that used for bridge comparisons. (ii) The input noise of the electrometer limits the ability of the feedback circuit to maintain virtual ground between the pump and the capacitor. Reducing this noise, which has a $1/f$ power spectrum and is caused by moving charged defects within or near the electrometer, will be important in achieving the full potential of the standard. (iii) A thorough analysis of the circuits in Fig. 2 must be performed to determine the magnitudes of all possible uncertainties in both phases of operation.

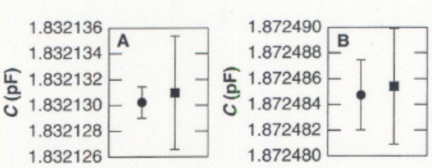

Fig. 5. Comparison of C from counting electrons (solid circles) and from a commercial ac bridge that is traceable to a calculable capacitor (solid squares). The electron counting values in (**A**) and (**B**) are the averages of the data in the corresponding parts of Fig. 4 and the bridge measurements were taken during the same conditions. Uncertainty bars are $\pm U_{tot} C$ for the electron counting values and ± 2.4 ppm for the bridge values.

The SET capacitance standard described here makes it possible to place capacitance metrology on a quantum basis, as was done previously for voltage and resistance. For this new standard to become a practical tool for metrologists, it must be developed into a system that is robust and easy to use, with a total relative uncertainty of order 0.1 ppm. Although the engineering challenges involved are substantial, our present understanding of the requirements for reliable operation of SET devices indicates that they can be met. We hope that ultimately the SET capacitance standard will join the Josephson voltage standard and the quantum Hall resistance standard as a widely adopted natural standard for electrical metrology.

References and Notes

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13. We fabricated these structures by evaporating Al through a suspended polymer mask that was patterned with electron beam lithography. We deposited Al from one angle, oxidized it to form an insulating layer, and then deposited a second layer of Al from a different angle so that it partially overlaps the first layer. This forms metal-insulator-metal sandwiches in the overlapping regions, and the insulator is thin enough to allow quantum tunneling.
14. Although an electron pump can be made with as few as three junctions, we used seven junctions in order to suppress undesired multijunction tunneling events that are rare but not negligible at temperatures well below the charging energy barrier.
15. We focus here on electron counting, but the pump is also an accurate current source. The maximum current of ≈ 10 pA is too small for conventional metrology, but it can be useful in situations where accurate measurements of extremely small currents are needed.
16. Thermal cycling to room temperature between these two experimental runs caused C to change by $\approx 2\%$ because of mechanical changes in the capacitor. Un-

like an artifact, the SET standard is not affected by this change because C is determined in situ each time the standard is cooled and operated.

17. Throughout this report, we use expanded uncertainty with coverage factor $k = 2$, which defines an interval that is expected to contain ≈95% of the reasonable values for the measured quantity.
18. We used a digital voltmeter that was calibrated with a Josephson voltage standard and has a relative uncertainty of 0.1 ppm for its range of 10 V. The finite input current of the voltmeter is supplied by the feedback circuit and does not affect the measurement of ΔV.
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20. We used commercial capacitance bridge model AH2500A (Andeen-Hagerling, Cleveland, OH). The identification of specific commercial instruments does not imply endorsement by NIST nor does it imply that the instruments identified are the best available for a particular purpose.
21. The authors gratefully acknowledge voltage calibration assistance from C. Hamilton, capacitance calibration assistance from A.-M. Jeffery, and discussions with E. Williams regarding Eq. 2.

19 May 1999; accepted 29 July 1999