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AC JOSEPHSON VOLTAGE STANDARD ERROR MEASUREMENTS AND ANALYSIS

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Abstract

The Josephson arbitrary waveform synthesizer can be used as a precision voltage source for both ac and dc signals. Recent improvements in circuit designs have resulted in output voltages greater than 100 mV so that we can investigate ac metrology measurements in the frequency range from 1 kHz to 50 kHz. In this paper we present experimental measurements of several significant error sources that must be considered in order to create an intrinsic ac voltage standard at these frequencies. These results and the corresponding theoretical analysis allow us to move toward creating a practical ac Josephson voltage standard.

Background

Research at NIST on pulse-driven Josephson arrays for ac and dc metrology has been ongoing since 1996. We have previously reported the details of various pulse-driven techniques including: bipolar waveform generation utilizing a combination of gigahertz frequency digital codes and sinusoidal rf-drive [1], referencing the Josephson output voltage to ground using various circuit configurations [2], and designing appropriate on-chip filters for the low-frequency output lines and bias connections [3]. The circuits described in this paper combine all these techniques, allowing us to quantitatively evaluate the performance of the Josephson arbitrary waveform synthesizer as a voltage source for ac metrology.

Our goal is to develop a quantum-based voltage source that delivers precisely calculable rms values for both ac (sinewaves and arbitrary waveforms) and dc voltage signals. The ac Josephson source uses a well-known digital-to-analog conversion technique called delta-sigma modulation [4], which allows any arbitrary waveform in our frequency range of interest (dc to 1 MHz) to be generated using a sequence of pulses at a much higher repetition frequency (10 Gbit/s presently). Josephson junctions are ideally suited for this work, because their voltage pulses are precisely quantized. Knowledge of the number of pulses and their position in time is sufficient to precisely determine the time-dependent voltage of any synthesized waveform and, in particular, its rms voltage.

Potential Sources of Error

Since we operate these devices with a wide output bandwidth (dc to MHz), there are many sources of error that need to be considered when computing the actual rms voltage delivered by the Josephson source to a device-under-test (DUT). We can summarize the most significant sources of error and uncertainty with the following equation:

$$V_{delivered} = V_{junctions} + V_{induc} + V_{io} + V_{harmonics} + V_{transmissionline} + V_{dctherm} \qquad (1)$$

This calculation gives the total rms voltage delivered to a DUT, which includes the Josephson array voltage and dominant sources of error. In performing this summation, both the magnitude and phase of each term must be considered. $V_{junctions}$ is the quantum-mechanically accurate voltage produced by the junctions. V_{induc} is the voltage generated by the on-chip inductance. V_{io} is the input/output coupling from the drive signal to the output leads. $V_{transmissionline}$ is the effect of the transmission line connecting the chip to the DUT. $V_{dctherm}$ is the dc thermal voltage. In order to calculate the exact rms voltage delivered to the DUT, all of the above terms must be either precisely calculable or directly measurable.

To analyze these sources of error more fully, we first consider the on-chip effects. The drive signal is comprised of two basic components – the low-frequency drive (which is a current of many milliamps at exactly the fundamental frequency), and the high-frequency drive (which is coupled through a 10 MHz high-pass filter). The sum of these two currents flows through the Josephson junctions and generates the term $V_{junctions}$, the perfectly quantized desired voltage waveform. However, this same current creates one of the main challenges for pulse-driven arrays in that it also flows through the on-chip inductance and generates the voltage of the second term in Eq.1, V_{induc}. Fortunately, V_{induc} has a phase angle of 90° with respect to $V_{junctions}$, so it combines in quadrature to the final rms value. However the next term, the input/output coupling voltage V_{io}, causes more difficulty because it can have a phase angle of 0° or 180° with respect to $V_{junctions}$, and as such adds directly (not in quadrature) to the array voltage. Thus V_{io} must be orders of magnitude smaller than the other error terms in order to make a useful voltage standard. $V_{transmissionline}$ and $V_{dctherm}$ are not dominant errors at this time, and will not be discussed in this paper since they have been theoretically analyzed previously. [5]

Since the two terms V_{induc} and V_{io} are due primarily to the low-frequency drive current, we can directly measure them by simply turning off the high-frequency drive. In this case, the junctions don't pulse ($V_{junctions} = 0$) and the measured output from the chip is primarily $V_{induc} + V_{io}$. Table 1 shows

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some measured values for these terms, and compares them to theoretical values calculated from estimates of the on-chip inductance and the drive current. Since V_{induc} should be much larger than V_{io} at these frequencies, V_{induc} dominates because the phase difference between them is 90° and they combine in quadrature. Notice that since the two frequencies differ by a factor of 20, the value of the error term is 20 times greater at higher frequency as expected.

Table 1. Measured and calculated values of the dominant error terms V_{induc} and V_{io}, for a 62 mVrms sinewave produced by a single array of 3750 Josephson junctions.

Frequency	Measured Error ($V_{induc} + V_{io}$)	Calculated Error (V_{induc} only)
2.67 kHz	8.2 µV rms	5.6 µV rms
53.4 kHz	169 µV rms	112 µV rms

If V_{io} were zero, these measured values of V_{induc} would increase the rms value of $V_{delivered}$ by only approximately 5 parts in 10^7 at 2.67 kHz, and by about 4 parts in 10^6 at 53.4 kHz. At the present time, it is difficult for us to measure V_{io} independently since it is much smaller than V_{induc}. However, V_{io} presents a significant challenge since it must be less than 300 nV to keep the corresponding error under 5 parts in 10^6 when $V_{junctions} = 62$ mV.

Table 2. Measured and calculated values of the error term $V_{harmonics}$ for a 62 mVrms sinewave with 990 intentionally added harmonic tones at two different amplitudes.

Power level for each of the 990 tones	Measured $V_{harmonics}$ (rms contribution)	Calculated $V_{harmonics}$ (rms contribution)
-77.34 dBc	+12.2 parts in 10^6	+10.0 parts in 10^6
-100 dBc	below noise floor	+0.05 parts in 10^6

The 4th term in Eq.1, $V_{harmonics}$, represents the contribution to the total rms voltage made by higher harmonics of the fundamental frequency, f. Since the high-frequency drive signal is 4 to 6 orders of magnitude higher than f, the fundamental is generated with a very high degree of spectral purity. However, to see the effect of the term $V_{harmonics}$, we made a special waveform in which we deliberately included 990 harmonics with amplitudes at 77.34 dB below the primary sinewave signal [i.e. -77.34 dBc (carrier)]. This value was chosen so that $V_{harmonics}$ would contribute exactly 10 parts in 10^6. The results of these measurements are summarized in Table 2. Notice that for typical Josephson array performance, these harmonics are −100 dBc or lower which places them well below the noise floor of our rms voltage-measuring instrument.

Error Correction Circuit

To increase our understanding of the error terms V_{induc} and V_{io}, we used a correction circuit (a 10 Ω series resistor driven by a separate synchronized sinewave signal generator) to remove the measured error signals. With the low-frequency drive on (and the high-frequency drive off) we directly measured V_{induc} and V_{io} using a spectrum analyzer, and then applied a sinewave of the correct magnitude and phase to exactly cancel these terms to the noise floor of the spectrum analyzer, which was about -120 dBc. When the high-frequency drive is turned back on, the Josephson array operates normally and the errors due to V_{induc} and V_{io} are greatly reduced. This is illustrated in Table 3, where we compare sinewaves at several frequencies with and without the error correction signal. Of course the most ideal way to reduce these error terms is to improve the circuit designs. Nevertheless, this error correction technique is useful because it always enables an increased operating bandwidth by providing a way to decrease errors at higher frequencies by measuring and canceling them.

Table 3. Measurement of ac-ac difference comparing the rms value of a sinewave at 2.67 kHz to the same waveform at higher frequencies, with and without the error correction signal (for V_{induc} and V_{io}).

Frequency	Uncorrected ac-ac difference	Corrected ac-ac difference
8.1 kHz	+23 parts in 10^6	+21 parts in 10^6
53.4 kHz	-176 parts in 10^6	-32 parts in 10^6

Conclusion

We have made direct measurements of the dominant sources of error and uncertainty for the pulse-driven Josephson voltage standard (for ac and dc), and compared those results with theoretical predictions. We measured and characterized the dominant error signals and demonstrated a correction circuit that decreases the uncertainty in the rms voltage delivered to the device under test.

References

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