Diversity Performance of a Practical Non-Coherent Detect-and-Forward Receiver

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***Abstract*—We propose a non-coherent receiver for the fixed detect-and-forward relay channel and derive a closed-form tight upper bound on its bit error probability in Rayleigh fading channels. Using this upper bound, we show that the receiver achieves nearly full, second-order diversity, and the gap from full diversity is quantified. While the general receiver depends on the source-relay channel statistics, we show that similar diversity performance can be achieved when only first-order statistics of the source-*destination* channel are known. The receiver is derived using the generalized likelihood ratio test to eliminate dependence on the channel gains, applies to M-ary orthogonal signal sets, and is independent of the fading distribution. The results demonstrate that low-complexity transceivers, such as those used in some sensor networks, can benefit from cooperative diversity.**

I. Introduction

The relay channel, over which a destination receives information directly from a source as well as a relay, has garnered renewed interest as a means to obtain spatial diversity in wireless communications over fading channels.

Previous work has shown that certain relaying schemes obtain diversity benefits while others do not. For example, when the relay amplifies what it receives (i.e., amplify-and-forward), diversity can be achieved provided the destination has channel state information (CSI) of all three links [1], [2]. However, when the relay fully decodes the source's message (i.e., fixed decode-and-forward), there is no diversity gain [2]. More sophisticated decode-and-forward protocols can recover the diversity gain by adapting to the channel state, such as selection relaying in which the relay is only utilized when the source-relay channel is of sufficient quality [2]. It was also observed in the case of coherently-detected binary signals that when the relay performs symbol-by-symbol detection of the source's transmission, what we refer to as fixed detect-and-forward, diversity gain can be achieved provided the destination knows the relay's probability of error [1].

Motivated by the desire to place the benefits of cooperative diversity within reach of low-complexity wireless devices, we investigate the potential for achieving spatial diversity gain with fixed relaying protocols and very limited CSI. In this paper, we consider fixed detect-and-forward relaying with non-coherent reception at the relay and destination.

In related work, a non-coherent detect-and-forward receiver derived for Rayleigh fading channels was shown to provide diversity gains [3]. This receiver requires knowledge of the average relay-destination and source-destination signal-to-noise ratios (SNRs) and the error probability at the relay. Non-coherent decode-and-forward space-time processing is proposed in [4], [5] where a cyclic redundancy check is required at the relays. Other work has considered non-coherent amplify-and-forward reception using on-off keying and binary frequency shift keying [6] as well as differential modulation [7]. However, detect-and-forward imposes at the relay neither the memory requirements of amplify-and-forward nor the complexity of decode-and-forward.

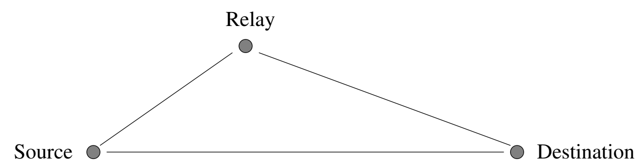

Fig. 1. Sample topology of source, destination and relay

This work differs from previous work, first, in that our proposed detect-and-forward receiver requires knowledge of only the destination's average noise energy and the source-relay transition probabilities. Second, the general form of the receiver is not specific to any fading distribution. Third, for the special case of Rayleigh fading and binary signals, we derive a tight upper bound on the bit error probability in closed form. Using this bound, we precisely quantify the diversity order of the receiver, showing that it approaches full, second-order diversity with increasing SNR. Finally, we show that even if the destination only has the average source-destination SNR instead of the average source-relay SNR (a significant complexity reduction), similar diversity performance is achieved.

Section II describes the system model for the analysis. Section III derives the detector using the generalized likelihood ratio test. The bit error probability is analyzed in Section IV, and numerical results are presented in Section V.

II. System Model

The source transmits a message which is received by a relay and a destination (see Fig. 1) using a constellation of M orthogonal signals (e.g., M-ary frequency shift keying). The relay detects the transmitted signal using an optimal non-coherent receiver and forwards its hard decision to the destination. All transmissions are assumed to be orthogonal to one another so that there is no multiple access interference.

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The destination employs non-coherent reception and combines the received signals from the source and relay to make a decision using the decision rule derived in Section III.

The channel attenuates each transmission with a random gain and phase rotation and adds noise. Using a discrete-time model representing the sampled outputs of the non-coherent quadrature correlators, the received signals at the relay and destination are

$$\begin{aligned}\mathbf{y}_{sd} &= \mathbf{A}_{sd}\mathbf{x}_m + \mathbf{z}_{sd}\\ \mathbf{y}_{sr} &= \mathbf{A}_{sr}\mathbf{x}_m + \mathbf{z}_{sr}\\ \mathbf{y}_{rd} &= \mathbf{A}_{rd}\mathbf{x}_{\hat{m}_r} + \mathbf{z}_{rd}\end{aligned}$$

where the subscripts sd, sr, and rd refer to the source-destination, source-relay and relay-destination links, respectively. The elements of the $M \times 1$ vectors $\mathbf{y}_{ij}$ are complex numbers corresponding to the M non-coherent correlators—one for each signal—and whose real and imaginary parts represent the phase quadrature outputs of the correlators. The transmitted symbol with phase offset ϕ is represented by the $M \times 1$ vector $\mathbf{x}_m$, $m = 1, \ldots, M$, having $\sqrt{E_s}e^{j\phi}$ as its mth element and 0 as its other elements. The relay's transmitted signal is denoted by $\mathbf{x}_{\hat{m}_r}$, where the $\hat{m}_r$th signal is detected by the relay based on the source-relay channel output, $\mathbf{y}_{sr}$.

The channel gain of link ij, $\mathbf{A}_{ij}$, is an $M \times M$ matrix containing random complex channel gains on its main diagonal and 0 at its other elements. Modeling additive white Gaussian noise at the output of each correlator, the elements of $\mathbf{z}_{ij}$ are independent zero-mean, circularly symmetric complex Gaussian random variables with variance σ_j^2. Furthermore, the channel gains $\mathbf{A}_{ij}$ and noise vectors $\mathbf{z}_{ij}$ are assumed mutually independent over all three links.

III. GLR Test Receiver

A. General Case

The destination uses channel outputs $\mathbf{y}_{sd}$ and $\mathbf{y}_{rd}$ and a decision rule to decide which signal the source transmitted. The maximum-likelihood decision rule would be

$$\hat{m}_d = \arg \max_{m=1,\ldots,M} f\left(\mathbf{y}_{sd}|\mathbf{x}_m\right) f\left(\mathbf{y}_{rd}|\mathbf{x}_m\right) \tag{1}$$

where $f\left(\mathbf{y}_{ij}|\mathbf{x}_m\right)$ is the conditional density of channel output $\mathbf{y}_{ij}$ given that the source transmitted $\mathbf{x}_m$. Since, conditioned on $\mathbf{x}_m$, the channel outputs are independent, the joint conditional density $f\left(\mathbf{y}_{sd}, \mathbf{y}_{rd}|\mathbf{x}_m\right)$ can be expressed as the product of the marginal conditional densities in (1).

The conventional approach to obtaining these conditional densities requires knowledge of the distribution of the channel gain. In some cases, such as when the fading is Rayleigh (i.e., the elements of $\mathbf{A}_{ij}$ are complex Gaussian), these densities can be written in closed form. Even then, however, they will depend on some statistical information (e.g., the variance of the complex Gaussian channel gains).

To avoid these dependencies and to generalize the detector beyond a given fading distribution, we consider the use of the generalized likelihood ratio test to derive the decision rule. In a GLR test, the likelihood function for each hypothesis is evaluated using the maximum-likelihood estimate of the unknown parameter [8, p. 92] (the channel gain, in this case). In other words, the likelihood function is maximized over the unknown parameter.

To apply the GLR test to our problem, we begin with the density of the channel output conditioned on both the transmitted signal and the channel gain. Since the noise is Gaussian, the conditional density of the source-destination channel output, $\mathbf{y}_{sd}$, is

$$f\left(\mathbf{y}_{sd}|\mathbf{x}_m, \mathbf{A}_{sd}\right) = \frac{1}{\left(\pi\sigma_d^2\right)^M} \times \exp\left(-\frac{|y_{sd,m} - \alpha_{sd,m}\sqrt{E_s}e^{j\phi}|^2 + \sum_{k\neq m}|y_{sd,k}|^2}{\sigma_d^2}\right) \tag{2}$$

where $\alpha_{sd,m}$ is the mth element of the diagonal of $\mathbf{A}_{sd}$. Maximizing (2) over $\alpha_{sd,m}$ gives

$$\begin{aligned}\hat{f}\left(\mathbf{y}_{sd}|\mathbf{x}_m\right) &= \frac{1}{\left(\pi\sigma_d^2\right)^M}\exp\left(-\frac{\sum_{k\neq m} w_{sd,k}^2}{\sigma_d^2}\right)\\ &= \frac{1}{\left(\pi\sigma_d^2\right)^M}\exp\left(-\frac{\|\mathbf{y}_{sd}\|^2 - w_{sd,m}^2}{\sigma_d^2}\right)\end{aligned} \tag{3}$$

where $w_{sd,k} = |y_{sd,k}|$ is the magnitude output of the kth non-coherent correlator. The notation $\hat{f}(\cdot)$ is used to refer to the maximized likelihood and is sometimes referred to as the *profile* or *concentrated* likelihood.

Following a similar procedure for the relay-destination channel output, the profile likelihood of $\mathbf{y}_{rd}$ conditioned on the relay's transmitted signal is

$$\hat{f}\left(\mathbf{y}_{rd}|\mathbf{x}_{\hat{m}_r}\right) = \frac{1}{\left(\pi\sigma_d^2\right)^M}\exp\left(-\frac{\|\mathbf{y}_{rd}\|^2 - w_{rd,\hat{m}_r}^2}{\sigma_d^2}\right). \tag{4}$$

To obtain the profile likelihood of $\mathbf{y}_{rd}$ conditioned on the *source's* transmitted signal, we must average (4) over the probability distribution of the relay's transmitted signal:

$$\hat{f}\left(\mathbf{y}_{rd}|\mathbf{x}_m\right) = \sum_{\hat{m}_r=1}^{M}\hat{f}\left(\mathbf{y}_{rd}|\mathbf{x}_{\hat{m}_r}\right)p\left(\mathbf{x}_{\hat{m}_r}|\mathbf{x}_m\right). \tag{5}$$

Using the product of (3) and (5) in place of the conditional densities in (1) and eliminating non-contributing factors yields the following GLR test receiver for non-coherent detect-and-forward:

$$\hat{m}_d = \arg\max_{m=1,\ldots,M} e^{w_{sd,m}^2/\sigma_d^2}\sum_{\hat{m}_r=1}^{M} e^{w_{rd,\hat{m}_r}^2/\sigma_d^2}p\left(\mathbf{x}_{\hat{m}_r}|\mathbf{x}_m\right).$$

The above GLR decision rule is in a general form that is not specific to any particular channel fading distribution. It requires knowledge of the noise variance at the destination as well as the transition probability matrix of the equivalent discrete-input, discrete-output source-relay channel. Typically, these probabilities depend on the channel statistics of the source-relay link. In the following, a special case is considered that is amenable to further analysis.

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B. Special Case: Rayleigh Fading, Binary Signals

We now apply the GLR test receiver to the special case of Rayleigh fading channels and binary signals ($M = 2$). We consider the receiver's performance in the diversity-poor scenario of flat Rayleigh fading channels, or when each channel's gain matrix can be expressed as $\mathbf{A}_{ij} = \alpha_{ij}\mathbf{I}_M$, where $\mathbf{I}_M$ is the $M \times M$ identity matrix.

In the case of Rayleigh fading, channel gain α_{ij}, like the noise, is zero-mean complex Gaussian, and the squared-magnitude correlator outputs are therefore exponentially distributed. Let T_1 and T_0 be the signal-bearing and non-signal bearing squared-magnitude correlator outputs at the relay with means λ_1^{-1} and λ_0^{-1}, respectively. Then, the probability that the relay makes a correct (incorrect) decision is the probability that $T_1 > T_0$ ($T_1 < T_0$), and the transition probabilities of the source-relay channel are easily shown to be

$$\begin{aligned} p(\mathbf{x}_1|\mathbf{x}_1) \;=\; p(\mathbf{x}_2|\mathbf{x}_2) &= \frac{\lambda_0}{\lambda_0+\lambda_1} \\ p(\mathbf{x}_1|\mathbf{x}_2) \;=\; p(\mathbf{x}_2|\mathbf{x}_1) &= \frac{\lambda_1}{\lambda_0+\lambda_1}. \end{aligned}$$

Consequently, the GLR decision rule for the special case of binary signals and flat Rayleigh fading is to decide $\hat{m}_d = 1$ if

$$\begin{aligned} & e^{w_{sd,1}^2/\sigma_d^2}\left(\lambda_0 e^{w_{rd,1}^2/\sigma_d^2} + \lambda_1 e^{w_{rd,2}^2/\sigma_d^2}\right) \\ & \quad > \; e^{w_{sd,2}^2/\sigma_d^2}\left(\lambda_1 e^{w_{rd,1}^2/\sigma_d^2} + \lambda_0 e^{w_{rd,2}^2/\sigma_d^2}\right) \end{aligned} \tag{6}$$

and decide $\hat{m}_d = 2$ otherwise. If we normalize λ_0^{-1} and λ_1^{-1} by the noise energy of the source-relay channel, we have

$$\lambda_0 = 1 \tag{7}$$

$$\lambda_1^{-1} = \gamma_{sr} + 1 \tag{8}$$

where γ_{sr} is the average SNR of the source-relay channel. Thus, the side information required by the destination for decision rule (6) is the average noise energy at the destination (σ_d^2) and the average source-relay SNR. We shall observe in the performance analysis only a small penalty when the more readily available source-*destination* average SNR is used in place of the source-*relay* SNR.

IV. Bit Error Probability

In this section, we derive an expression for the exact bit error probability of the non-coherent binary GLR detect-and-forward receiver in Rayleigh fading channels. We also derive a tight upper bound of the bit error probability in closed form and use it to examine the asymptotic performance of the receiver for large SNR.

A. Exact Bit Error Probability

The bit error probability given that the source transmitted $\mathbf{x}_1$ can be expressed in terms of the error probabilities conditioned on the relay's transmission as

$$\begin{aligned} P_2(\mathcal{E}|\mathbf{x}_1) &= p(\mathbf{x}_1|\mathbf{x}_1) P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1) \\ &\quad + p(\mathbf{x}_2|\mathbf{x}_1) P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 2) \\ &= \frac{\lambda_0}{\lambda_0+\lambda_1} P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1) \\ &\quad + \frac{\lambda_1}{\lambda_0+\lambda_1} P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 2). \end{aligned} \tag{9}$$

For equal energy signals, (9) is also the error probability given $\mathbf{x}_2$ was sent, and for equiprobable signals, it is equal to the overall bit error probability.

Beginning with the error probability conditioned on $\hat{m}_r = 1$ (i.e., that the relay correctly detects and forwards $\mathbf{x}_1$), we have from (6)

$$P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1) = \Pr\left[U_1 - U_2 < \log \frac{\lambda_0 + \lambda_1 e^{(V_1 - V_2)}}{\lambda_1 + \lambda_0 e^{(V_1 - V_2)}}\right] \tag{10}$$

where we have used the substitutions $U_i = w_{sd,i}^2/\sigma_d^2$ and $V_i = w_{rd,i}^2/\sigma_d^2$ to simplify notation. Due to the Rayleigh fading and Gaussian noise assumptions, the channel outputs $\mathbf{y}_{sd}$ and $\mathbf{y}_{rd}$ are complex Gaussian, and the variates in (10) are independent and exponentially distributed with means

$$\mu_1^{-1} \triangleq \mathrm{E}[U_1] = \gamma_{sd} + 1 \tag{11}$$

$$\mu_2^{-1} \triangleq \mathrm{E}[U_2] = 1 \tag{12}$$

$$\nu_1^{-1} \triangleq \mathrm{E}[V_1] = \gamma_{rd} + 1 \tag{13}$$

$$\nu_2^{-1} \triangleq \mathrm{E}[V_2] = 1 \tag{14}$$

where γ_{sd} and γ_{rd} are the average source-destination and relay-destination SNRs, respectively. Equalities (11) and (12) are a result of the condition that the source transmitted $\mathbf{x}_1$, and (13) and (14) are a result of the condition that the relay detected $\hat{m}_r = 1$ (i.e., the relay made a correct decision).

To assist with evaluating (10), we first present the following lemma.

Lemma: If Z_1 and Z_2 are independent and exponentially distributed with means ζ_1^{-1} and ζ_2^{-1}, respectively, then the density of the difference $Z_1 - Z_2$ is

$$f(x) = \begin{cases} \frac{\zeta_1\zeta_2}{\zeta_1+\zeta_2} e^{-\zeta_1 x} & ; \quad x \geq 0 \\ \frac{\zeta_1\zeta_2}{\zeta_1+\zeta_2} e^{\zeta_2 x} & ; \quad x < 0 \end{cases} \tag{15}$$

and its distribution function is

$$F(x) = \begin{cases} 1 - \frac{\zeta_2}{\zeta_1+\zeta_2} e^{-\zeta_1 x} & ; \quad x \geq 0 \\ \frac{\zeta_1}{\zeta_1+\zeta_2} e^{\zeta_2 x} & ; \quad x < 0 \end{cases}. \tag{16}$$

The proof is omitted for brevity but can be easily shown by convolving the densities of Z_1 and $-Z_2$.

Applying (16) to (10) and conditioning on $W = V_1 - V_2$, we have

$$\begin{aligned} & P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1, W = w) \\ & \quad = \begin{cases} 1 - \frac{\mu_2}{\mu_1+\mu_2} e^{-\mu_1 \log\rho} & ; \quad \rho \geq 1 \\ \frac{\mu_1}{\mu_1+\mu_2} e^{\mu_2 \log\rho} & ; \quad 0 < \rho < 1 \end{cases} \\ & \quad = \begin{cases} 1 - \frac{\mu_2}{\mu_1+\mu_2} \left(\frac{\lambda_1+\lambda_0 e^w}{\lambda_0+\lambda_1 e^w}\right)^{\mu_1} & ; \quad w \leq 0 \\ \frac{\mu_1}{\mu_1+\mu_2} \left(\frac{\lambda_0+\lambda_1 e^w}{\lambda_1+\lambda_0 e^w}\right)^{\mu_2} & ; \quad w > 0 \end{cases} \end{aligned} \tag{17}$$

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where $\rho \triangleq (\lambda_0 + \lambda_1 e^w) / (\lambda_1 + \lambda_0 e^w)$. Since $\lambda_0 > \lambda_1$, negative w implies $\rho > 1$.

Next, using (15) to obtain the density of W, we take the expectation of (17) with respect to W, giving the probability of error conditioned on a correct relay decision as

$$
\begin{aligned}
&P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1) \qquad (18)\\
&= \int_{-\infty}^{\infty} P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1, W = w) f_W(w)\, dw \\
&= \int_{-\infty}^{0} \left[1 - \frac{\mu_2}{\mu_1+\mu_2}\left(\frac{\lambda_1+\lambda_0 e^w}{\lambda_0+\lambda_1 e^w}\right)^{\mu_1}\right] \frac{\nu_1\nu_2}{\nu_1+\nu_2} e^{\nu_2 w}\, dw \\
&\quad + \int_{0}^{\infty} \frac{\mu_1}{\mu_1+\mu_2}\left(\frac{\lambda_0+\lambda_1 e^w}{\lambda_1+\lambda_0 e^w}\right)^{\mu_2} \frac{\nu_1\nu_2}{\nu_1+\nu_2} e^{-\nu_1 w}\, dw.
\end{aligned}
$$

When the relay makes an error and transmits $\mathbf{x}_2$, given that the source transmitted $\mathbf{x}_1$, the values of ν_1 and ν_2 in (13) and (14) are reversed. Thus, the error probability conditioned on a relay error, $P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 2)$, can be obtained by swapping ν_1 and ν_2 in (18). The overall bit error probability, then, is evaluated by using these two conditional probability expressions in (9).

B. Upper Bound

Because evaluation of the exact bit error probability requires numerical integration, we develop below a closed-form upper bound. Numerical examples in Section V demonstrate this bound to be very tight and a good approximation for the exact error probability.

Beginning with the error probability conditioned on a correct relay decision, let us denote the integrals over negative and positive values of w in (18) as $\mathcal{A}$ and $\mathcal{B}$, respectively. Then, the first of these is equivalent to

$$
\mathcal{A} = \frac{\nu_1\nu_2}{\nu_1+\nu_2}\left[\frac{1}{\nu_2} - \frac{\mu_2}{\mu_1+\mu_2}\int_{-\infty}^{0}\left(\frac{\lambda_1+\lambda_0 e^w}{\lambda_0+\lambda_1 e^w}\right)^{\mu_1} e^{\nu_2 w}\, dw\right] \qquad (19)
$$

By substituting the following lower bound for the expression in parentheses in (19)

$$
\frac{\lambda_1+\lambda_0 e^w}{\lambda_0+\lambda_1 e^w} > \frac{\lambda_1 e^w+\lambda_0 e^w}{\lambda_0+\lambda_1} \quad ; \quad w < 0
$$

then an upper bound for (19) can be easily evaluated as

$$
\mathcal{A} < \frac{\nu_1\nu_2}{\nu_1+\nu_2}\left(\frac{1}{\nu_2} - \frac{\mu_2}{\mu_1+\mu_2}\cdot\frac{1}{\mu_1+\nu_2}\right). \qquad (20)
$$

Next, after recalling from (12) that $\mu_2 = 1$, the integral over positive values of w in (18) can be expressed as

$$
\mathcal{B} = \frac{\nu_1\nu_2}{\nu_1+\nu_2}\cdot\frac{\mu_1}{\mu_1+1}\int_0^{\infty}\left(\frac{\lambda_0 e^{-w}+\lambda_1}{\lambda_1 e^{-w}+\lambda_0}\right) e^{-\nu_1 w}\, dw. \qquad (21)
$$

Upper-bounding the quantity in parentheses with

$$
\frac{\lambda_0 e^{-w}+\lambda_1}{\lambda_1 e^{-w}+\lambda_0} < \frac{\lambda_0 e^{-w}+\lambda_1}{\lambda_0}
$$

yields the following upper bound for (21):

$$
\mathcal{B} < \frac{\nu_1\nu_2}{\nu_1+\nu_2}\cdot\frac{\mu_1}{\mu_1+1}\left(\frac{1}{\nu_1+1} + \frac{\lambda_1}{\nu_1\lambda_0}\right). \qquad (22)
$$

Summing (20) and (22) provides the upper bound for the error probability conditioned on a correct relay decision as

$$
\begin{aligned}
&P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1) \\
&< \frac{\nu_1\nu_2}{\nu_1+\nu_2}\left[\frac{1}{\nu_2} - \frac{1}{\mu_1+1}\cdot\frac{1}{\mu_1+\nu_2}\right. \qquad (23)\\
&\quad \left. + \frac{\mu_1}{\mu_1+1}\left(\frac{1}{\nu_1+1} + \frac{\lambda_1}{\nu_1\lambda_0}\right)\right] \quad ; \quad \begin{matrix}\nu_1^{-1} = \gamma_{rd}+1 \\ \nu_2 = 1\end{matrix}.
\end{aligned}
$$

Recalling that the error probability conditioned on a relay error can be obtained by swapping the values of ν_1 and ν_2, this conditional error probability can be upper bounded by (23) with $\nu_1 = 1$ and $\nu_2^{-1} = \gamma_{rd} + 1$. However, for this value of ν_2, a tighter upper bound than (20) can be obtained by using the lower bound

$$
\frac{\lambda_1+\lambda_0 e^w}{\lambda_0+\lambda_1 e^w} > \frac{\lambda_1}{\lambda_0+\lambda_1} \quad ; \quad w < 0
$$

in place of the expression in parentheses in (19), giving

$$
\mathcal{A} < \frac{\nu_1\nu_2}{\nu_1+\nu_2}\left[\frac{1}{\nu_2} - \frac{1}{\mu_1+1}\left(\frac{\lambda_1}{\lambda_0+\lambda_1}\right)^{\mu_1}\frac{1}{\nu_2}\right]. \qquad (24)
$$

Summing (22) and (24) provides the following upper bound for the error probability conditioned on a relay error:

$$
\begin{aligned}
&P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 2) \\
&< \frac{\nu_1}{\nu_1+\nu_2}\left\{1 - \frac{1}{\mu_1+1}\left(\frac{\lambda_1}{\lambda_0+\lambda_1}\right)^{\mu_1}\right. \qquad (25)\\
&\quad \left. + \frac{\nu_2\mu_1}{\mu_1+1}\left(\frac{1}{\nu_1+1} + \frac{\lambda_1}{\nu_1\lambda_0}\right)\right\} \quad ; \quad \begin{matrix}\nu_1 = 1 \\ \nu_2^{-1} = \gamma_{rd}+1\end{matrix}.
\end{aligned}
$$

Using (7), (8), and (11) to express these upper bounds directly in terms of the average channel SNRs, (23) and (25) can be written, respectively, as

$$
\begin{aligned}
&P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1) \\
&< \frac{1}{\gamma_{rd}+2}\left[1 - \left(\frac{\gamma_{sd}+1}{\gamma_{sd}+2}\right)^2\right. \\
&\quad \left. + \frac{1}{\gamma_{sd}+2}\left(\frac{\gamma_{rd}+1}{\gamma_{rd}+2} + \frac{\gamma_{rd}+1}{\gamma_{sr}+1}\right)\right] \qquad (26)\\
&P_2(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 2) \\
&< \frac{\gamma_{rd}+1}{\gamma_{rd}+2}\left\{1 - \frac{\gamma_{sd}+1}{\gamma_{sd}+2}\left(\frac{1}{\gamma_{sr}+2}\right)^{\frac{1}{\gamma_{sd}+1}}\right. \\
&\quad \left. + \frac{1}{\gamma_{rd}+1}\cdot\frac{1}{\gamma_{sd}+2}\left(\frac{1}{2} + \frac{1}{\gamma_{sr}+1}\right)\right\}. \qquad (27)
\end{aligned}
$$

Finally, using these upper bounds in (9) gives an upper bound for the overall bit error probability.

C. Asymptotic Performance

The previous subsection derived a tight upper bound for the bit error probability of the binary decode-and-forward receiver in Rayleigh fading channels. We now investigate the asymptotic performance of this receiver by analyzing the behavior of the upper bound with increasing SNR.

Let $\gamma = E_s/\sigma_d^2$ and $\gamma_{ij} = \gamma/\eta_{ij}$, so that the average SNR of channel ij is a scaled version of γ. The deterministic scaling

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factor η_{ij} could, for example, be an increasing function of the distance of link $ij \in \{sd, sr, rd\}$. For large γ, the upper bound of the error probability conditioned on a correct relay decision (26) can be approximated as

$$P_{2,\text{ub}}\left(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1\right) \approx \frac{\eta_{rd}}{\gamma}\left[\epsilon_1(\gamma) + \frac{\eta_{sd}}{\gamma}\left(1 + \frac{\eta_{sr}}{\eta_{rd}}\right)\right] \tag{28}$$

where

$$\epsilon_1(\gamma) \triangleq 1 - \left(\frac{\gamma_{sd}+1}{\gamma_{sd}+2}\right)^2.$$

Using the approximation

$$\frac{1+\delta_1}{1+\delta_2} \approx 1 + \delta_1 - \delta_2 \quad ; \quad \begin{array}{l} |\delta_1| \ll 1 \\ |\delta_2| \ll 1 \end{array}$$

$\epsilon_1(\gamma)$ can be approximated for large γ as

$$\epsilon_1(\gamma) = 1 - \left(\frac{1+1/\gamma_{sd}}{1+2/\gamma_{sd}}\right)^2 \approx \frac{2}{\gamma_{sd}} - \frac{1}{\gamma_{sd}^2}$$

which is dominated by the term $2/\gamma_{sd}$. Using this dominant term for $\epsilon_1(\gamma)$ in (28) gives the high-SNR approximation for the error probability conditioned on a correct relay decision as

$$P_{2,\text{ub}}\left(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1\right) \approx \frac{\eta_{sd}\left(\eta_{sr} + 3\eta_{rd}\right)}{\gamma^2}. \tag{29}$$

As for the error probability conditioned on a relay error, upper bound (27) can be approximated for large γ as

$$P_{2,\text{ub}}\left(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 2\right) \approx \epsilon_2(\gamma) + \frac{\eta_{rd}\eta_{sd}}{\gamma^2}\left(\frac{1}{2} + \frac{\eta_{sr}}{\gamma}\right) \tag{30}$$

where

$$\epsilon_2(\gamma) \triangleq 1 - \left(\frac{\eta_{sr}}{\gamma}\right)^{\eta_{sd}/\gamma}. \tag{31}$$

When (29) and (30) are used in (9), the upper bound of the overall bit error probability is approximated as

$$\begin{aligned} P_{2,\text{ub}}\left(\mathcal{E}|\mathbf{x}_1\right) &= \frac{\gamma_{sr}+1}{\gamma_{sr}+2} P_{2,\text{ub}}\left(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 1\right) \\ &\quad + \frac{1}{\gamma_{sr}+2} P_{2,\text{ub}}\left(\mathcal{E}|\mathbf{x}_1, \hat{m}_r = 2\right) \qquad (32) \\ &\approx \frac{\eta_{sd}\left(\eta_{sr}+3\eta_{rd}\right)}{\gamma^2} + \frac{\eta_{sr}}{\gamma}\left[\epsilon_2(\gamma) + \frac{\eta_{sd}\eta_{rd}}{2\gamma^2} + \frac{\eta_{sd}\eta_{sr}\eta_{rd}}{\gamma^3}\right] \\ &\approx \frac{\eta_{sd}\left(\eta_{sr}+3\eta_{rd}\right)}{\gamma^2} + \frac{\eta_{sr}}{\gamma}\epsilon_2(\gamma). \qquad (33) \end{aligned}$$

where in the last line we neglect the third- and fourth-order terms as the lower-order terms dominate when γ is large.

Clearly, the first term of (33) is second-order. To examine the behavior of the second term, we evaluate the slope of $\epsilon_2(\gamma)$ with γ in the log-log domain. Denoting the negative of this slope with Ω_{ϵ_2}, we have

$$\begin{aligned} \Omega_{\epsilon_2}(\gamma) &\triangleq -\frac{d\left\{\log \epsilon_2(\gamma)\right\}}{d\left\{\log \gamma\right\}} = -\gamma \frac{d\left\{\log \epsilon_2(\gamma)\right\}}{d\gamma} \\ &= -\frac{\eta_{sd}\left(1+\log\frac{\eta_{sr}}{\gamma}\right)\left(\frac{\eta_{sr}}{\gamma}\right)^{\eta_{sd}/\gamma}}{\gamma\left[1-\left(\frac{\eta_{sr}}{\gamma}\right)^{\eta_{sd}/\gamma}\right]}. \qquad (34) \end{aligned}$$

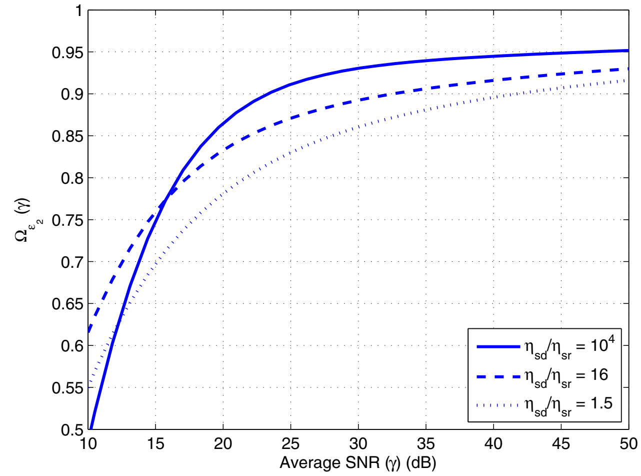

Fig. 2. Diversity order of $\epsilon_2(\gamma)$ (31) vs. average source-destination SNR

where the last line was obtained after differentiating $\log \epsilon_2(\gamma)$.

Fig. 2 plots (34) as a function of γ for various η_{sd}/η_{sr}. The selected values of η_{sd}/η_{sr} correspond to relay positions located along the source-destination line at 10%, 50%, and 90%, respectively, of the source-destination distance, and a scaling of the average SNR with the fourth-power of distance (i.e., $\eta_{ij} = d_{ij}^4$). In general, we observe that $\Omega_{\epsilon_2}(\gamma)$ increases towards unity with increasing SNR, more rapidly for relay locations closer to the source than the destination at moderate SNRs. The intuition behind the faster convergence is that as the relay moves farther from the destination, the harmful impact of a relay error is reduced.

We observe, then, that the second term of (33) decreases as $\gamma^{1+\Omega_{\epsilon_2}}$, where Ω_{ϵ_2} approaches one, and therefore the proposed receiver achieves near-second-order diversity in Rayleigh fading channels.

D. Bit Error Probability Without Source-Relay CSI

A drawback of the proposed receiver is that it depends on the destination knowing the average source-relay SNR, γ_{sr}, which is needed to compute λ_1 for the binary decision rule (6). In practice, the destination does not have direct access to this channel from which to estimate the average SNR. However, the destination can directly measure the average SNR of the source-destination channel, γ_{sd}, which is related to γ_{sr} through the transmitted signal energy. In many uniform channel models, the two differ by a constant scaling factor; for example, in our channel model, we have $\gamma_{sd} = (\eta_{sr}/\eta_{sd})\,\gamma_{sr}$.

Inspection of the bit error probability upper bound reveals that using γ_{sd} in place of γ_{sr} as side information in the decision rule does not alter the diversity order, but only shifts the error probability approximation by a constant factor. Specifically, making this change replaces η_{sr} with η_{sd} in (28), (30), and (31), but the near-second-order diversity behavior remains the same. Thus, provided the average source-destination SNR is proportional to the average source-relay SNR, near-second-order diversity can be achieved by the proposed receiver with only locally measurable first-order statistics at the destination,

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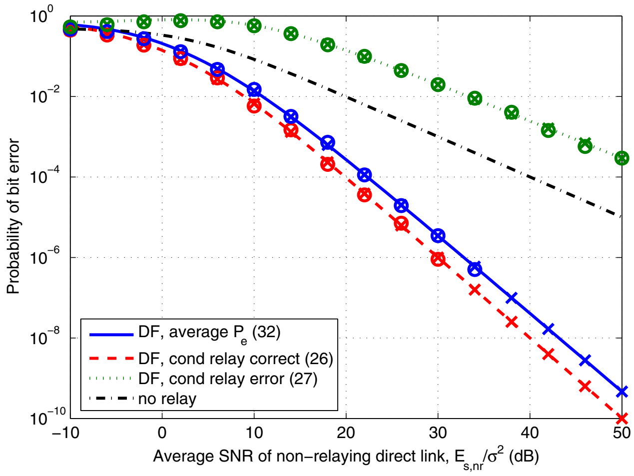

Fig. 3. Bit error probability vs. average direct-link SNR, relay at midpoint; ×'s exact, ○'s simulation

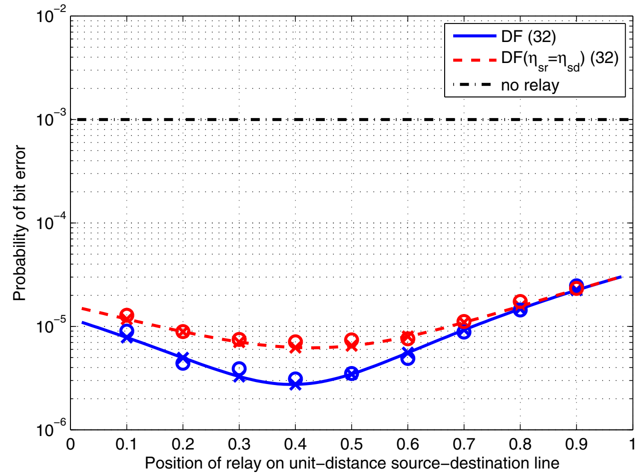

Fig. 4. Bit error probability vs. relay position, $E_{s,\mathrm{nr}}/\sigma_d^2 = 30$ dB; ×'s exact, ○'s simulation

namely the average noise energy at the receiver, σ_d^2, and the average source-destination SNR, γ_{sd}. The gap from full second-order diversity decreases with SNR and is quantified as $1 - \Omega_{\epsilon_2}(\gamma)$ with $\eta_{sr} = \eta_{sd}$ in (34).

V. Numerical Results

We present examples of the performance of the proposed receiver using the probability of error expressions developed in the previous section as well as results obtained by Monte Carlo simulation. The channel gain is assumed to be a combination of flat Rayleigh fading and path loss with the fourth power of distance, so that $\eta_{ij} = d_{ij}^4$ where d_{ij} is the distance of link $ij \in \{sd, sr, rd\}$. Results are given in terms of the average SNR of the non-relaying system using the same total energy as the relaying system; that is, the energy transmitted per symbol by the source and relay is $E_s = E_{s,\mathrm{nr}}/2$, where $E_{s,\mathrm{nr}}$ is the symbol energy in a non-relaying system.

Fig. 3 illustrates the bit error probability versus the average SNR with binary signaling for the non-relayed direct link as well as for a relaying system in which the relay is placed at the midpoint between the source and destination. Shown are the upper bounds of the error probabilities conditioned on a correct relay decision (26), on relay error (27), and the overall unconditional error probability (32). Comparison with the exact bit error probabilities evaluated by numerical integration of (18) and plotted with × markers show the bounds to be very tight. Results obtained by Monte Carlo simulation are plotted with ○ markers.

Next, the position of the relay is varied along the line between the source and destination. Fig. 4 plots the bit error probability as a function of the source-relay distance when the average direct-link SNR is 30 dB. The upper bound is observed to be tight over the entire range, and the optimum position of the relay is observed to be near 0.4 of the source-destination distance. Results are also shown for the case when the average source-destination SNR is used as side information to the detector instead of the average source-relay SNR, as discussed in Section IV-D. The two detectors differ by a factor of only 2.3 in bit error rate near the optimum relay position, and less elsewhere.

VI. Conclusion

We have found a non-coherent receiver for the relay channel that obtains nearly full diversity gain when a fixed detect-and-forward protocol is used and only locally available first-order statistics of the channel are available. Specifically, the destination needs the average noise energy at its receiver and the average source-destination SNR. The proposed receiver was developed for orthogonal signal sets (e.g., M-ary FSK), using the generalized likelihood ratio test to eliminate dependence on the channel gains. A tight upper bound on the bit error probability of the binary receiver was derived for the case of Rayleigh fading channels, and this upper bound was used to show that the receiver achieves near-second-order diversity. The gap from full second-order diversity is quantified and is shown to diminish with increasing SNR.

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