Cumulative Acknowledgement Multicast Repetition Policy For Wireless LANs or Ad Hoc Network Clusters

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Abstract **For a wireless LAN or a cluster of nodes in an ad hoc wireless network, multicast transmissions from a controlling station must be acknowledged by the nodes that are addressed. To ensure reliable reception at all the nodes, the multicast packet may be repeated on different transmissions, or on the same transmission as a form of diversity, or both. In this paper, policies for repeat transmissions of multicast packets are analyzed in terms of the average number of repeat transmissions required and the corresponding efficiency of the signaling on the multicast channel. The analytical results are parametric in packet/ACK error probabililty, the number of nodes, and the number of required ACKs. It is shown that a policy of accumulating ACKs and addressing only unacknowledged nodes on repeated non-diversity transmissions is more efficient than simply using a higher order of diversity on the multicast transmissions. Methods are shown for avoiding unnecessary repetitions in the case of a missing node.**

I. INTRODUCTION

In a wireless LAN (WLAN) that is controlled by an access point, or in a cluster of nodes in a wireless ad hoc network that is operating under the control of a "leader" node, typically there is an uplink and a downlink, each functioning much like those in one cell of a cellular communications system. On the uplink, the transmission resource (frequency and/or time slots) is shared by the mobile wireless terminals (nodes), which forward all communications to the access point or leader node for relaying to other nodes or to a larger network with which the network or cluster is associated. The medium access on the uplink can be with or without contention. On the downlink, the access point broadcasts all messages whether the messages are intended for a single node or for multiple nodes. For both downlink and uplink, acknowledgement messages (ACKs) are used to ensure accurate delivery of packets; if a packet is received correctly (for example, as determined by a packet quality check code), the receiving node sends an ACK.

Let the packet error probability be denoted α. When the wireless channel quality is poor, the effective probability of error can be reduced using forward error control coding (for example, as in [1]) or simply by repeating the packet L times as a form of time diversity, making the effective packet error rate α^L. In either case the efficiency of the transmissions in terms of the ratio of the number of transmitted symbols to information bits is reduced in order to increase the reliabililty of the communications. If the probability of error for an ACK is β, the probability that a packet is *not* both successfully received and successfully acknowledged is

$$\gamma = 1 - (1 - \alpha^L)(1 - \beta) \tag{1}$$

When the downlink transmissions are intended for multiple destination nodes (*i.e.*, for multicasting), there are many possible policies for controlling repeat downlink transmissions, assuming that repetitions are performed until a certain number of the intended receiving nodes have received the packet correctly, as indicated by the ACKs received at the access point. In what follows, for an adaptive "cumulative acknowledgement" (CACK) multicast policy we analyze the average numbers of repeats that are required when the downlink packet is intended for K mobile nodes, parametric in the packet and ACK error probabilities. Provision is made for requiring fewer than K ACKs because some nodes may have left the network. The concept embodied in the CACK policy is the accumulation of ACKs from successive transmissions, rather than requiring all the ACKs to occur in response to a particular transmission. The adaptation that enables this concept consists of removing from the list of destination nodes, prior to repeating the multicast, those nodes for which a successful ACK was received.

II. DIVERSITY REPETITION

For reference, we first consider a diversity repetition policy. If success for the multicast message is defined as achieving correct reception and acknowledgement at $M \le K$ nodes on the same transmission, the probability of success on any given transmission is

$$p(\gamma;\, K,\, M) = \sum_{k=M}^{K} \binom{K}{k} (1-\gamma)^k \gamma^{K-k} \tag{2}$$

Let $p_1 = p(\gamma; K, K)$. To use an example given in [2], let $K = 200$, $\alpha = 0.01$, and $\beta = 0$. The probability that the packet will be received correctly by all $M = K = 200$ mobile nodes on the same transmission is $p_1 = (0.99)^{200} = 0.134$ for $L = 1$ and $p_1 = (0.9999)^{200} = 0.980$ for $L = 2$. Assuming now that the transmission is repeated as necessary to achieve success, the probability that this successful event occurs on the nth transmission is

$$\mathrm{P}_n = (1-p)^{n-1} \times p = p\, q^{n-1} \tag{3}$$

using $q = 1 - p$. Thus the expected number of *packet* repetitions that will be required to implement this simple repeat policy is

$$\overline{n} = L\sum_{n=1}^{\infty} np\,q^{n-1} = Lp\cdot\frac{1}{(1-q)^2} = \frac{L}{p} \tag{4a}$$

using the formula for the summation in [3, §0.231]. For $M = K$, the result is exponential in the value of K:

$$\overline{n} = \frac{L}{p_1} = \frac{L}{(1-\gamma)^K} = L\,\mathrm{e}^{K|\ln(1-\gamma)|} \tag{4b}$$

For the example, for $L = 1$ it would take an average of $p_1^{-1} = (0.134)^{-1} = 7.46$ transmissions to achieve correct reception of the packet at all of the mobile nodes on the same downlink transmission and for $L = 2$, an average of $(0.980)^{-1} = 1.02$ transmissions, with two repeats on each, or an average of 2.04 repeats.

Graphs of the average number of repeats using simple repetition and requiring all K nodes to acknowledge are shown in Figure 1 as functions of K parametric in α for negligible ACK error. It is obvious from Figure 1 that there are combinations of α and K for which diversity transmissions are more efficient than repeating single packets. It is easy to show that, despite having a minimum number of two repeats, $L = 2$ has this advantage over $L = 1$ when

$$K > \frac{\ln(2)}{\ln(1+\alpha)} \approx \frac{0.693}{\alpha} \qquad \text{when } \alpha \ll 1 \tag{5}$$

One method to guard against needless repetition when one or more nodes are not working or have left the network is to require $M < K$ nodes to acknowledge on any transmission. For example, if $M = K - 1$ the success probability on any repetition equals $p_2 = p(\gamma;\, K,\, K-1)$ and the average number of repetitions equals L/p_2. Figure 2 illustrates that relaxing the requirement that all intended nodes successfully receive the packet greatly reduces the average number of repeats, at the risk of terminating the repetition while "ignoring" the up to $K - M$ nodes that have not successfully received and acknowledged the packet.

Another method for controlling the number of repeat transmissions is to require K acknowledgements for the first transmission and, say, $K - 1$ on subsequent transmissions, if they are needed. As illustrated in Figure 3, the average number of repetitions for this policy is a compromise between using just p_1 or p_2:

$$\overline{n} = \frac{L}{p_2}(1 + p_2 - p_1) \tag{6}$$

III. CACK REPETITION POLICY

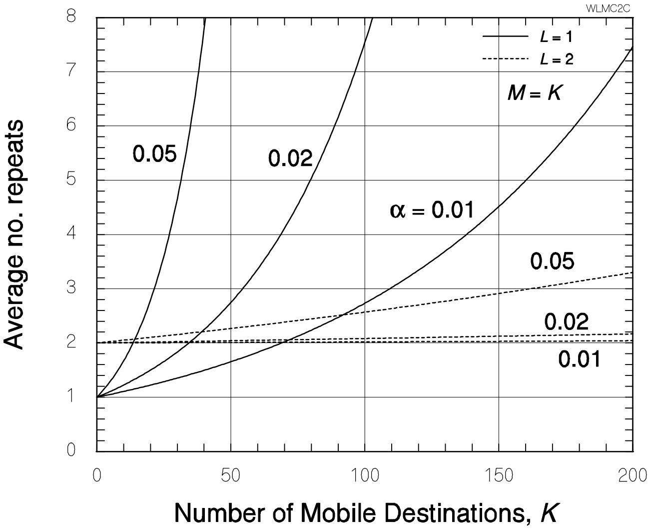

Fig. 1. $\overline{n}$ vs. K for diversity repetition with $M = K$ and $L = 1$ and $L = 2$, parametric in the packet error rate p_e.

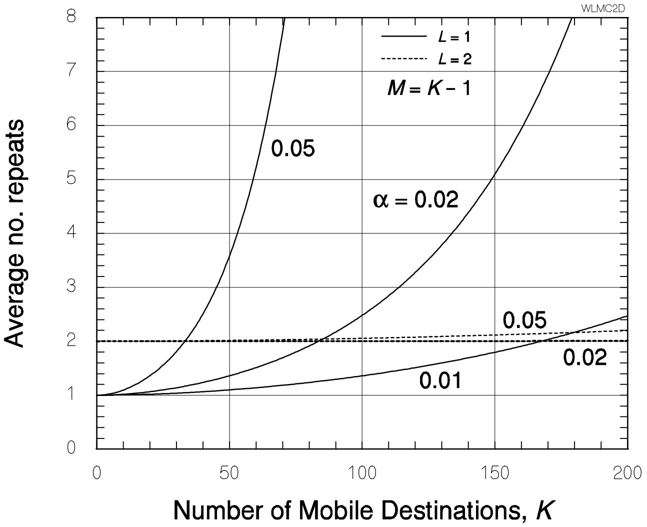

Fig. 2. $\overline{n}$ vs. K for diversity repetition with $M = K - 1$ and $L = 1$ and $L = 2$, parametric in the packet error rate p_e.

Now we consider the CACK adaptive multicast repeat transmission policy that assumes that the access point not only accumulates ACKs from different transmissions but keeps track of the identities of the nodes from which ACKs have been received. We further assume, as in [4], that the multicast packet lists the destination nodes that so far have not successfully acknowledged reception of a previous version of the packet. In this manner, the transmission is adaptively repeated until the number of successful ACKs equals the required number. To prevent needless repetitions, we may require $M < K$ successful ACKs to accumulate.

To derive the average number of packet repetitions under this policy, we define the binomial random variables (RVs) $\{x_{ij},\quad i = 1, 2, \ldots, K;\ j = 1,\, 2,\, \ldots, n\}$, where $x_{ij} = 1$ $(x_{ij} = 0)$ denotes a correct (incorrect) reception at receiver i on transmission j, followed by a successful ACK. On the

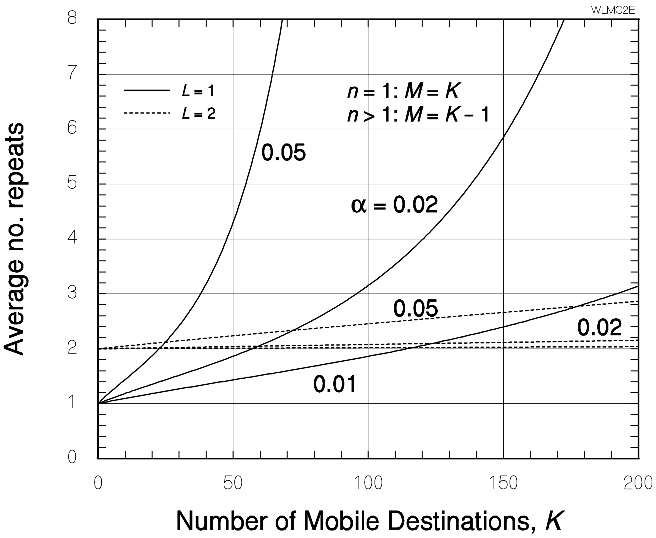

Fig. 3. $\overline{n}$ vs. K for diversity repetition but requiring $M = K$ on the first transmission, $M = K - 1$ subsequently.

first transmission, there are K binomial RVs $\{x_{i1},\ i = 1, 2, \ldots, K\}$; after the second transmission, there are $2K$ binomial RVs $\{x_{i1}$ and $x_{i2},\ i = 1, 2, \ldots, K\}$. If $x_{i1} + x_{i2} + \cdots + x_{in} = 0$, with probability γ^n, there were incorrect receptions and/or ACKs on each of the first n repetitions; if this sum is not zero, then the node was successful on at least one of the repetitions. The probability that at least M nodes succeeded at least once in n transmissions is

$$\mathrm{P}_{\leq n} = p(\gamma^n;\ K,\ M) \tag{7}$$

and exactly n transmissions are required with probability

$$\mathrm{P}_n = \mathrm{P}_{\leq n} - \mathrm{P}_{\leq (n-1)} = \mathrm{P}_{>(n-1)} - \mathrm{P}_{>n} \tag{8}$$

where $\mathrm{P}_{>n} = 1 - \mathrm{P}_{\leq n}$. The average number of transmissions required (also the number of repetitions required since we assume $L = 1$) is derived from (8) as follows:

$$\begin{aligned} \overline{n} &= \sum_{n=1}^{\infty} n\, \mathrm{P}_n = \sum_{n=1}^{\infty} n\, \mathrm{P}_{>(n-1)} - \sum_{n=1}^{\infty} n\, \mathrm{P}_{>n} \\ &= \sum_{n=0}^{\infty} (n+1)\, \mathrm{P}_{>n} - \sum_{n=0}^{\infty} n\, \mathrm{P}_{>n} = \sum_{n=0}^{\infty} \mathrm{P}_{>n} \\ &= \sum_{n=0}^{\infty} (1 - \mathrm{P}_{\leq n}) = \sum_{n=0}^{\infty} \left[1 - p(\gamma^n;\ K,\ M)\right] \end{aligned} \tag{9}$$

This series tends to converge quickly for small γ values, and can be manipulated to yield the following finite summation:

$$\overline{n} = \sum_{k=M}^{K} \binom{K}{k} \sum_{l=0}^{k} \binom{k}{l} \frac{(-1)^{l+1}}{1 - \gamma^{K-k+l}} \left[1 - \delta(K - k + l)\right] \tag{10}$$

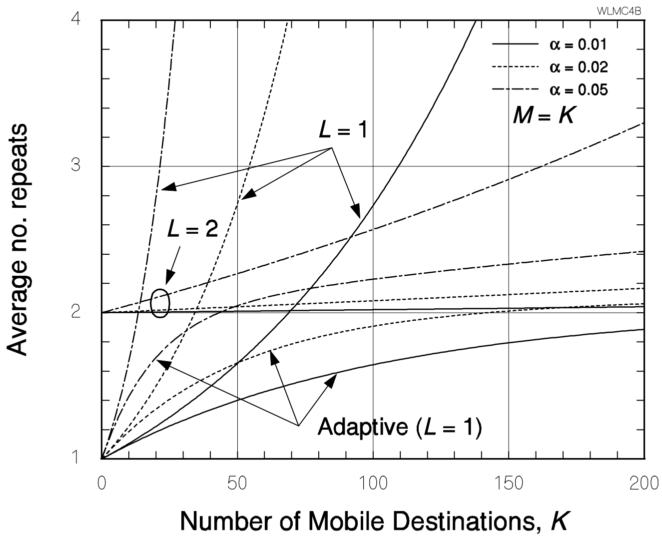

Fig. 4. Comparison of the average number of transmissions for diversity and adaptive (CACK) multicast repetition policies.

Calculations of $\overline{n}$ for the CACK repetition policy are compared in Figure 4 with those for a diversity one (for $L = 1$ and $L = 2$) when $M = K$. The figure suggests that the adaptive policy is always better. A proof that such is the case: (1) For any event (pattern of individual receiver successes and failures) resulting in success on the nth repetition for the diversity repetition policy, the adaptive policy would have been successful in n or fewer repetitions; (2) events can be found for which the number of repetitions needed for the adaptive policy is smaller than that for diversity repetitions; (3) therefore, the average number of repetitions required for success is smaller for the adaptive policy.

Calculations of (9) are shown in Figure 5 for $M = K$, $M = K - 1$, and $M = K - 2$ and indicate that a significant reduction in $\overline{n}$ is achieved just by relaxing the requirement

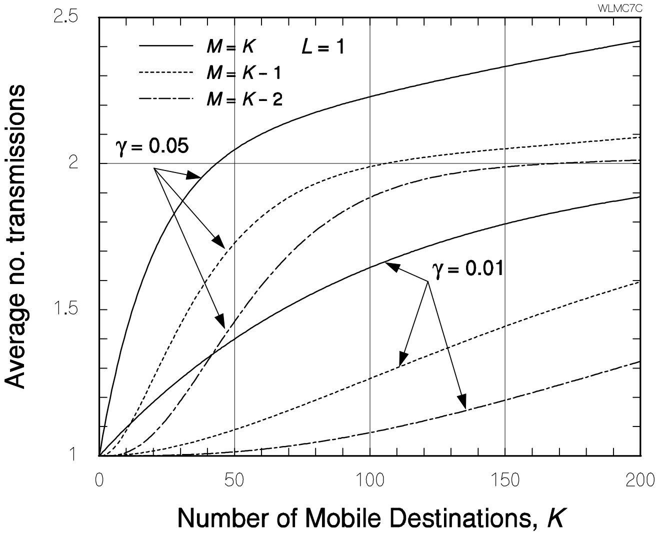

Fig. 5. Comparison of $\overline{n}$ values for the CACK repetition policy when $M = K$, $M = K - 1$, and $M = K - 2$

for valid ACKs to be received from all K mobile nodes. To hedge against the risk of ignoring nodes we show in Figure 6 the resulting $\overline{n}$ when $M = K$ on the first transmission, and $M = K - 1$ subsequently.

IV. EFFICIENCY OF REPETITION POLICIES

A concern in multicast operations for multihop networks is the possibility that, in addition to the repetitions of the downlink packet, the generation of a large number of ACKs will affect the system throughput significantly [5, 6]. Here, although a variable number of nodes respond to each transmission, we assume that the overhead for ACKs is fixed by reserving K uplink slots, one N_a-bit slot for each destination, following each downlink transmission. The uplink overhead for each transmission then is equivalent to KN_a bits.

For the diversity repetition policy, the downlink overhead in this case consists of the number of bits in the repeated N_d-bit data messages (including $L-1$ copies on the first transmission). Thus the expected overhead for the simple repetition policy is

$$\overline{\omega}_1 = N_d(\overline{n}-1) + KN_a\,\overline{n} \tag{11a}$$

with $\overline{n}$ given by (4a). The corresponding transmission efficiency, denoted η_1, is

$$\eta_1 = \frac{N_d}{N_d + \overline{\omega}_1} = \frac{1}{1 + \overline{\omega}_1/N_d} = \left\{\overline{n}\left(1 + \frac{K}{N_d}N_a\right)\right\}^{-1} \tag{11b}$$

For the CACK policy the expected overhead depends on the scheme used to specify the nodes from which successful ACKs were not received on the previous transmission. If the second and subsequent downlink packets each use K bits for this purpose, the overhead is

$$\overline{\omega}_2 = (N_d + K)(\overline{n}-1) + KN_a\,\overline{n} \tag{12a}$$

with the substitution of (9) for $\overline{n}$. The corresponding efficiency is

$$\eta_2 = \frac{N_d}{N_d + \overline{\omega}_2} = \left\{\overline{n}\left[1 + \frac{K}{N_d}(N_a+1)\right]\right\}^{-1} \tag{12b}$$

Since the average number of unsuccessful responses to the first downlink transmission equals $K\gamma$ and $\log_2 K$ bits are required to specify one of K nodes, a smaller downlink overhead theoretically can be achieved by simply listing the nodes when $K\gamma\log_2 K < K$ or $K < 2^{1/\gamma}$, which is almost always satisfied. Under this approach, we estimate the number of destination nodes on the rth transmission as $K\gamma^r$, which suggests the total number of overhead bits that is approximated by

$$\overline{\omega}_3 \approx N_d(\overline{n}-1) + \frac{K\gamma}{1-\gamma}\log_2 K + KN_a\,\overline{n} \tag{13a}$$

With this approximation for the number of overhead bits, the transmission efficiency for the CACK policy becomes

$$\eta_3 \approx \left\{\overline{n}\left[1 + \frac{K}{N_d}\left(N_a + \frac{\gamma\log_2 K}{1-\gamma}\right)\right]\right\}^{-1} \tag{13b}$$

Given the fixed structure of the assumed ACK procedure, the number of ACK bits per node can be rather small, but still perhaps much larger than 1. In that case the ratios of efficiencies η_2/η_1 or η_3/η_1 both are practically equal to a ratio of the average numbers of packet repeats:

$$\frac{\eta_2}{\eta_1} \approx \frac{\eta_3}{\eta_1} \approx \frac{\overline{n}(\text{diversity repeat policy})}{\overline{n}(\text{CACK repeat policy})} \tag{14}$$

Calculations of (14) for different repetition control mehods and different error probability values are shown in Figures 7–9. The CACK policy is uniformly more efficient than diversity repetition, to varying degrees. For example,

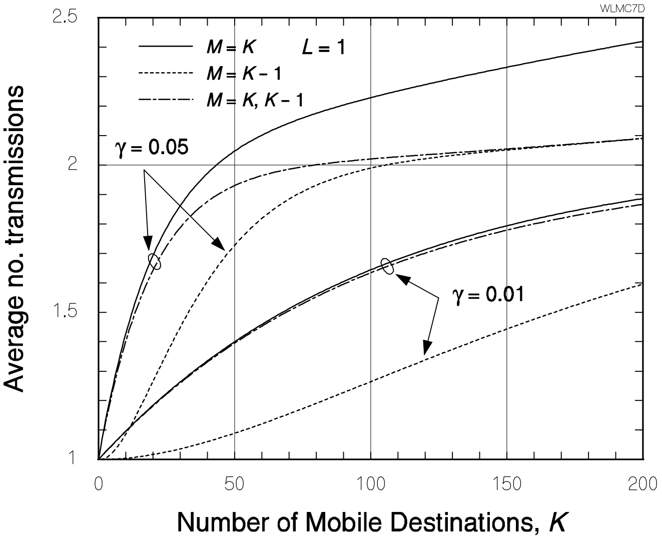

Fig. 6. Comparison of CACK variations.

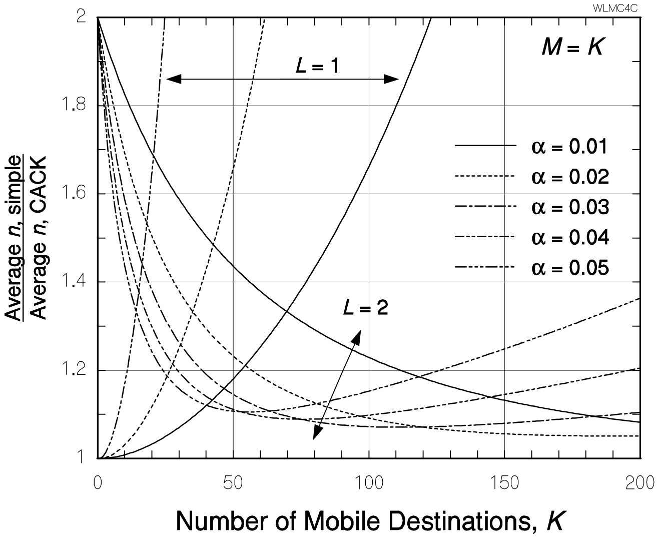

Fig. 7. Relative efficiency of CACK policy for $M = K$.

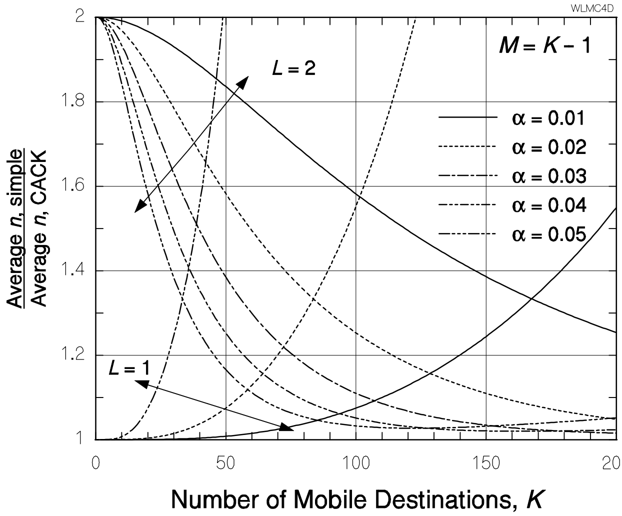

Fig. 8. Relative efficiency of CACK policy for $M = K - 1$.

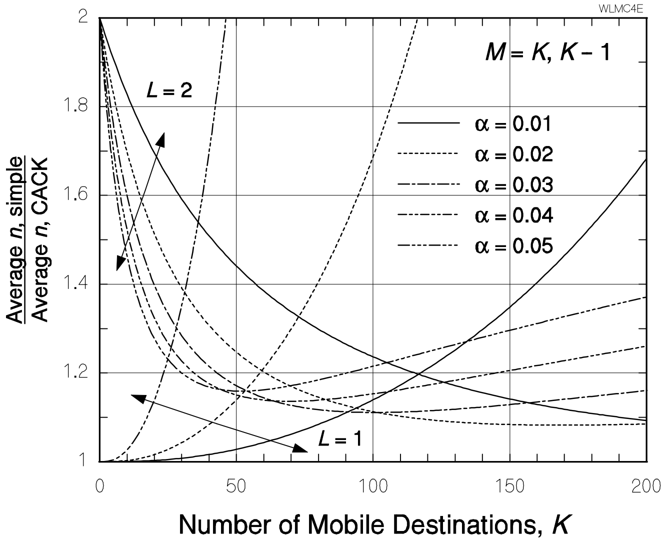

Fig. 9. Relative efficiency when $M = K$ on first transmission and $M = K - 1$ subsequently.

the CACK policy's advantage over $L = 1$ diversity repetition is a factor of about 1.65 in Figure 7 for $K = 100$ nodes and $\alpha = 0.01$, but is reduced to about 1.08 in Figure 8 and to about 1.14 in Figure 9. Thus for smaller numbers of nodes, ignoring persistently nonresponsive nodes in ACK requirements is an effective way to increase the efficiency of a simple repetition policy. When $L = 2$ is used by the diversity repetition policy, for the same K and α the advantage of the CACK policy is about 1.23 in Figure 7, 1.57 in Figure 8, and 1.24 in Figure 9. These results highlight the tradeoff between the fact that minimum number of repeated packets is two for the diversity repetition policy and the improvement gained from using diversity. Note from Figures 7–9 that if the packet error probability is relatively high, the $L = 2$ diversity repetition policy can be nearly as efficient as the CACK policy. However, when there are $K = 50$ or more nodes and the packet error rate is high, the CACK policy's advantage can grow rapidly as a function of K.

V. IGNORED NODES

When $M < K$, there is the risk of stopping the repetitions before some or all of $K - M$ "nonresponsive" nodes, if present, have succeeded in receiving and acknowledging the packet. For the CACK repetition policy, the probability of one such unsuccessful node when $M = K - 1$ is

$$\pi_1 = K \sum_{n=1}^{\infty} \gamma^n \left[\left(1 - \gamma^n\right)^{K-1} - \left(1 - \gamma^{n-1}\right)^{K-1} \right] \tag{15}$$

and when $M = K$ on the first transmission and $M = K - 1$ thereafter, it is $\pi_2 = \pi_1 - \gamma(1 - \gamma)^{K-1}$. The corresponding probabilities for diversity repetition are, respectively

$$\pi_3 = \frac{(1 - p_1)(p_2 - p_1)}{1 - p_1 + p_2} \quad \text{and} \quad \pi_4 = 1 - \frac{p_1}{p_2} \tag{16}$$

Typically, the probability of an ignored node is high for $M = K - 1$, except for $L = 2$, but is quite small for the policy of reducing M on the second and later repeats.

VI. CONCLUSION

An adaptive, cumulative acknowledgement (CACK) policy for multicasting in WLANs and ad hoc network clusters was introduced and analyzed in comparison with diversity repetition. The CACK policy requires a relatively small number of repetitions, even for large numbers of nodes, and therefore is very efficient. A modification to prevent missing nodes from prolonging the repetitions was shown to preserve efficiency with low risk of ignoring a node that is present.

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