# On the Scalability of Ad Hoc Networks: a traffic analysis at the center of a network

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***Abstract*— We investigate the inherent scalability problem of ad hoc networks originated from the nature of multi-hop networks. First, the expected packet traffic at the center of a network is analyzed. The result shows that the expected packet traffic at the center of a network is linearly related with the network size, that is, the expected packet traffic at the center of a network is $O(k)$, where $k$ is the radius of a network. From the result, the upper bound of the diameter of a network $D = 2k$, that guarantees the network is scalable, is obtained. The upper bound is given by $C/r - 1$, where $C$ is the channel capacity available to each node and $r$ is the packet arrival rate at each node.**

## I. Introduction

An ad hoc network is an autonomous system of nodes connected by wireless links, where the communications between nodes are often achieved by multi-hop links. With the increased interest in the mobile communications in the wireless communication community and the promise of convenient infrastructure-free communication of ad hoc networks, the development of *large-scale* ad hoc networks has drawn a lot of attention and the scalability of ad hoc networks has been the subject of extensive research. Recently, a research group has also been formed to address the problems involved in the development of large-scale ad hoc networks [1].

Because of the multi-hop nature of ad hoc networks, the scalability of ad hoc networks is directly related to the routing protocol. For example, a mobile ad hoc network can be made more scalable by reducing the overhead of the routing protocol [2]. A comparison study of the scalability of various routing protocols by Santiváñez et al. is available in [3]. Huang and Lai showed that the scalability of an ad hoc network is also affected by the underlying physical layer [4].

While the routing protocol is a prominent factor of the scalability of ad hoc networks, the scalability is subject to the fundamental limitation imposed by the multi-hop nature of ad hoc networks. Even with an *ideal* routing protocol that can handle constantly changing topology of the mobile nodes in the network, the network will not scale indefinitely due to the physical constraint such as the bandwidth of the channel. In a multi-hop network environment, the problems caused by the physical constraint will be exacerbated as the network size grows. In a typical route the number of hops is of order $\sqrt{N}$, where $N$ is the number of nodes in the network [5]. Thus, for a network with large number of nodes, much of the traffic carried by the nodes are relayed traffic and the proportion of the actual useful throughput diminishes as $N$ grows.

In this paper, we investigate the inherent scalability problem of ad hoc networks. This inherent scalability problem is originated from the nature of multi-hop networks. In our analysis, we recognize that the center of the network is the "hot spot" of the network in the sense that most of the relayed traffic goes through the center of the network. Thus, we first analyze the expected packet traffic at the center of a network, where the expected packet traffic includes the relayed packets. We find that the expected packet traffic at the center of the network is $O(k)$, where $k$ is the radius of the network in the number of hops. From this result, the upper bound of the diameter of a network $D = 2k$ is obtained to guarantee the network is scalable.

The paper is organized as follows. In Section II, the analysis model of the network is described. In Section III, the scalability of the network is investigated; in Section III-A, the expected packet traffic at the center of a network is analyzed, and it is used to obtained the upper bound of the network size in Section III-B. Section IV concludes the paper.

## II. Analysis Model of the Network

In this paper, we investigate the inherent scalability problem of ad hoc networks which is originated from the nature of multi-hop networks. This is accomplished by analyzing the relationship between the expected packet traffic at the center of a network and the network size. For the analysis, we make the following assumptions on the network.

1) *Uniform geometric distribution of the nodes:* We consider a network structured in honey comb shape as shown in Fig. 1. The transmission power of a node should be high enough to reach the neighbor nodes while causing minimal interference at other nodes. Thus, we assume that each node has 6 neighboring nodes (except for the nodes at the boundary), where a neighbor node means a node with a single-hop wireless link.
2) *Stationary nodes:* Nodes in the network are assumed to be stationary. Even though the topology may constantly change in ad hoc networks, by freezing the topology of

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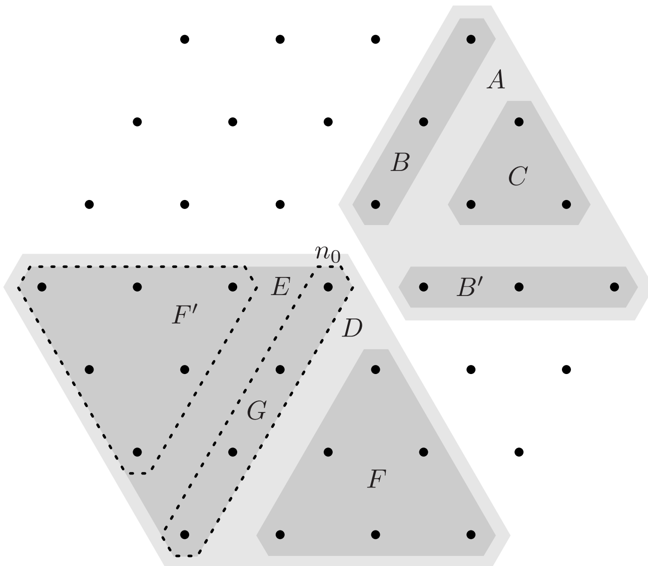


Fig. 1. A network model with radius $k = 3$.

the network, we can analyze the expected packet traffic at each node of the *snapshot* of the network, and it will make the analysis much easier. Furthermore, in most mobility scenarios, the relative movement of the nodes to the packet transit time is insignificant [6].

3) *Identical node property:* We assume that all nodes in the network will act equally in terms of the demand on the physical resources, and of the needs to communicate with one another. Thus, it is assumed that all nodes generate packets at the same rate $r$.
4) *Uniform distribution of destination:* It is assumed that the distribution of the destination nodes is uniform over the entire network of interest. Here, "uniform" means that the probability of transmitting a packet from a source node to any of other $N-1$ nodes is the same as $\frac{1}{N-1}$.
5) *Shortest path:* We assume each packet is relayed through the shortest path available. If there are more than one paths of the same length to the destination, the probability of each path being chosen is identical.

In this paper, it is assumed that the nodes are stationary. Grossglauser and Tse showed that the mobility of nodes increases the capacity of the wireless ad hoc networks [7]. They used a quite unique routing strategy with loose delay constraints which has only two hops from a source node to a destination node. However, their routing strategy depends heavily on the movement of the nodes and the long time delay limits the applicability of the result in many situations.

## III. The Scalability of a Network

### A. Packet Traffic at the Center of a Network

A network model we consider is illustrated in Fig. 1. The size of the network is given by the radius $k$, and the radius is defined as the number of hops from the center node to the boundary node. The number of nodes $N$ and the radius $k$ have the following relationship:

$$\begin{aligned} N &= 1 + (1 + 2 + \cdots + k) \cdot 6 \\ &= 1 + 3k(k+1) \end{aligned} \tag{1}$$

The expected packet traffic at the center node $n_0$ can be calculated by exploiting the symmetry of the network. First, we calculate the expected packet traffic generated by the nodes in the shaded areas $B$ and $C$ that pass or destined to $n_0$, then it is multiplied by 6 to obtain the total amount of the expected packet traffic at $n_0$. Note that, because of the shortest path assumption, the packets generated by nodes in $B$ *may* pass $n_0$ only when the packets' destination nodes are in $D$. Similarly, packets generated by nodes in $C$ *may* pass $n_0$ only when the destination nodes are in $E$. Let $n_s$ and $n_d$ represent the source and the destination nodes, respectively. Then, assuming a uniform distribution of destination nodes, that is the probability of a packet from a source node $n_s$ is transmitted to a destination node $n_d$ is $1/(N-1)$ for all nodes $n_d \neq n_s$, the expected packet traffic at node $n_0$, $\mathrm{PT}_{n_0}$, can be calculated as follows:

$$\begin{aligned} \mathrm{PT}_{n_0} = \Bigg\{ &\sum_{n_s \in B} \sum_{n_d \in D} \frac{p_{n_0}(n_s, n_d) \cdot r}{N-1} \\ &+ \sum_{n_s \in C} \sum_{n_d \in E} \frac{p_{n_0}(n_s, n_d) \cdot r}{N-1} \Bigg\} \times 6, \end{aligned} \tag{2}$$

where $r$ is the number of packets a node generates in a unit time (the packet arrival rate), and $p_{n_0}(n_s, n_d)$ is the probability that $n_s$ will send a packet to $n_d$ through $n_0$.

Define

$$\begin{aligned} B \to D &= \sum_{n_s \in B} \sum_{n_d \in D} p_{n_0}(n_s, n_d)\,; \\ C \to E &= \sum_{n_s \in C} \sum_{n_d \in E} p_{n_0}(n_s, n_d). \end{aligned}$$

Then, (2) can be written as

$$\begin{aligned} \mathrm{PT}_{n_0} &= \big\{(B \to D) + (C \to E)\big\} \times \frac{6r}{N-1} \\ &= \big\{(B \to E) + (B \to F) + (C \to E) \\ &\quad + (B' \to E) - (B' \to E)\big\} \times \frac{6r}{N-1}. \end{aligned} \tag{3}$$

Since $B$, $C$, and $B'$ are mutually exclusive areas, (3) can be reduced as follows:

$$\begin{aligned} \mathrm{PT}_{n_0} = \big\{ &\big[(B \cup C \cup B') \to E\big] \\ &- \big[(B' \to E) - (B \to F)\big]\big\} \times \frac{6r}{N-1}. \end{aligned}$$

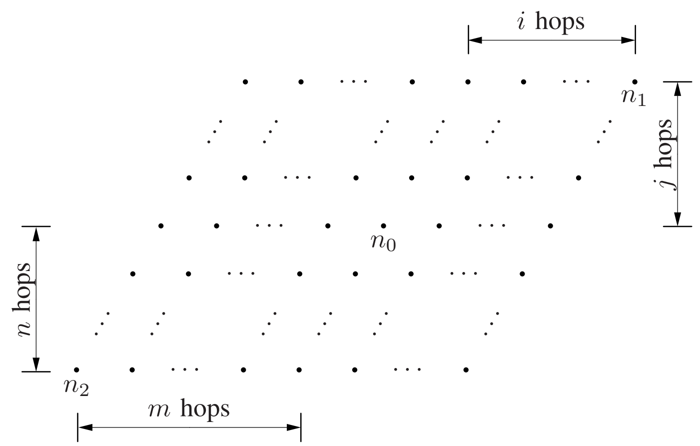


Fig. 2. The paths from the source node ($n_1$) to the destination node ($n_2$).

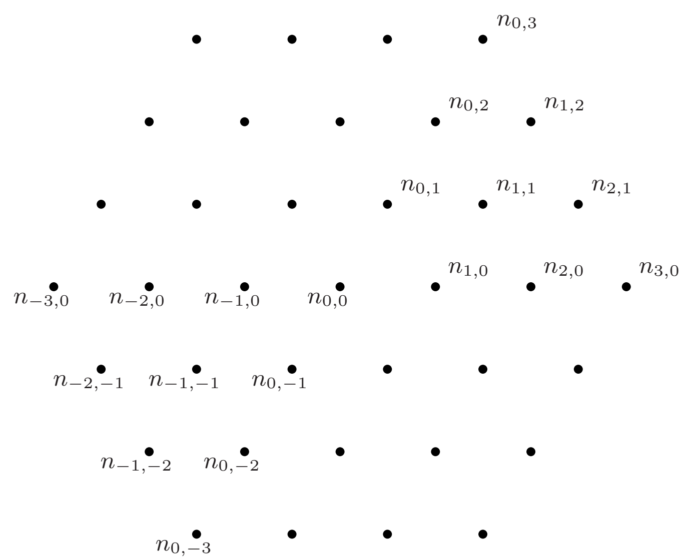


Fig. 3. A network model with node indices ($k = 3$).

Since $B \cup C \cup B' = A$ and

$$B' \to E = B \to E\,;$$
$$B \to F = B \to F',$$

we have

$$\begin{aligned}\mathrm{PT}_{n_0} &= \big\{(A \to E) \\ &\qquad - \big[(B \to E) - (B \to F')\big]\big\} \times \frac{6r}{N-1} \\ &= \big\{(A \to E) - (B \to G)\big\} \times \frac{6r}{N-1}. \end{aligned} \tag{4}$$

Thus, the expected packet traffic at $n_0$ can be expressed as follows:

$$\begin{aligned}\mathrm{PT}_{n_0} = \Bigg\{ &\sum_{n_s \in A} \sum_{n_d \in E} \frac{p_{n_0}(n_s, n_d) \cdot r}{N-1} \\ &- \sum_{n_s \in B} \sum_{n_d \in G} \frac{p_{n_0}(n_s, n_d) \cdot r}{N-1} \Bigg\} \times 6, \end{aligned} \tag{5}$$

The probability $p_{n_0}(n_s, n_d)$ can be calculated from the shortest path with equal probability assumption (see assumption 5) in Section II). Consider Fig. 2, where the number of hops from $n_1$ to $n_0$ is $i + j$, and the number of hops from $n_0$ to $n_2$ is $m+n$. Then, the probability $f(i, j, m, n)$ that a packet from $n_1$ to $n_2$ will pass $n_0$ is calculated by

$$f(i, j, m, n) = \frac{{}_{i+j}C_i \cdot {}_{m+n}C_n}{{}_{i+j+m+n}C_{i+m}}, \tag{6}$$

where ${}_xC_y$ is defined as $\frac{x!}{(x-y)!\,y!}$. Note that ${}_{i+j+m+n}C_{i+m}$ is the total number of the shortest paths from $n_1$ to $n_2$, and ${}_{i+j}C_i \cdot {}_{m+n}C_n$ is the number of the shortest paths from $n_1$ to $n_2$ that include node $n_0$. Let $n_{i,j}$ be a node whose location is indicated by the indices $i$ and $j$ as shown in the Fig. 3. Then, for $i, j > 0$ and $m, n \geq 0$, it can be shown that

$$p_{n_0}(n_{i,j}, n_{-m,-n}) = f(i, j, m, n). \tag{7}$$

By substituting (1) and (7) into (5), $\mathrm{PT}_{n_0}$ can be written as

$$\begin{aligned}\mathrm{PT}_{n_0} = \frac{2r}{k(k+1)} \Bigg( &\sum_{i=0}^{k} \sum_{j=0}^{k-i} \sum_{m=0}^{k} \sum_{n=0}^{k-m} f(i, j, m, n) \\ &- \sum_{m=0}^{k} \sum_{n=0}^{k-m} f(0, 0, m, n) \Bigg) - 2r \end{aligned} \tag{8}$$

From (6), the term with the double summation in (8) can be readily calculated as follows:

$$\begin{aligned}\sum_{m=0}^{k} \sum_{n=0}^{k-m} f(0, 0, m, n) &= \sum_{m=0}^{k} \sum_{n=0}^{k-m} \frac{m!\,n!}{(m+n)!} \, \frac{(m+n)!}{m!\,n!} \\ &= (k+1)\big(\frac{k}{2} + 1\big) \end{aligned} \tag{9}$$

The term with the quadruple summation is also reduced to a simple form as follow

$$\begin{aligned}&\sum_{i=0}^{k} \sum_{j=0}^{k-i} \sum_{m=0}^{k} \sum_{n=0}^{k-m} f(i, j, m, n) \\ &= \sum_{i=0}^{k} \sum_{j=0}^{k-i} \sum_{m=0}^{k} \sum_{n=0}^{k-m} \frac{(i+j)!\,(m+n)!\,(i+m)!\,(j+n)!}{(i+j+m+n)!\,i!\,j!\,m!\,n!} \\ &= (k+1)^3. \end{aligned} \tag{10}$$

Equation (10) can be easily verified by substituting a positive integer into $k$. We leave (10) as a conjecture without proof.

Finally, substituting (9) and (10) into (8), we have

$$\mathrm{PT}_{n_0} = (2k+1) \times r, \qquad (11)$$

which is an unexpectedly simple form. Equation (11) indicates that the expected packet traffic at the center of a network is linearly related to the radius $k$ of the network. Note that $\mathrm{PT}_{n_0}$ is $O(k)$ as opposed to the number of nodes $N$ which is $O(k^2)$.

### *B. The Upper Bound of a Network Size*

Let $C$ be the channel capacity available to each node, that is the maximum achievable throughput determined by the physical layer and medium access control layer. If the expected packet traffic at a node is greater than $C$, the node is not able to handle the traffic load. Thus, to make a network scalable, we must guarantee that $\mathrm{PT}_{n_0}$ is smaller than $C$. That is, from (11),

$$(2k+1) \times r = \mathrm{PT}_{n_0} \; < \; C. \qquad (12)$$

Let $D = 2k$ be the diameter of a network, then (12) can be written as

$$D < \frac{C}{r} - 1. \qquad (13)$$

Equation (13) gives an upper bound of the diameter of a *scalable* ad hoc network with an ideal shortest path routing protocol. Note that the upper bound is inversely proportional to $r$; the network is more scalable when the packet arrival rate is small.

### *C. Discussion*

In the analysis, $r$ represents the packet arrival rate at each node and we do not take into account the overhead caused by the control packets of the routing protocol. The amount of the routing overhead depends not only on the mobility of the nodes but also on the network size. As the network size grows, the routing overhead is expected to grow. Consequently, $\mathrm{PT}_{n_0}$ may be larger than that given in (11).

As shown in (1), the number of node $N$ is $O(k^2)$ as same as in the most of networks. Even though our result in (11) and (13) are obtained under assumption of symmetric topology of the network, the results are also applicable to general networks. Thus, we can presume that the expected packet traffic at the center of the network will be $O(k)$.

## IV. Conclusion

An ad hoc network is an autonomous system of nodes connected by wireless links, where the communications between nodes are often achieved by multi-hop links. In this paper, we have investigated the inherent scalability problem of ad hoc networks which is originated from the nature of multi-hop networks. The scalability of ad hoc networks depends not only on the routing protocol, but also on the traffic patterns, physical layer and medium access control layer. In the analysis, we recognized that the center of the network is the "hot spot" of the network in the sense that most of the relayed traffic goes through the center of the network. Thus, the expected packet traffic at the center of a network was first analyzed and the ideal shortest path routing protocol was used. The result shows that the expected packet traffic at the center of a network is linearly related with the radius of a network $k$, that is, the expected packet traffic at the center of a network is $O(k)$. From the result, the upper bound of the diameter of a network $D = 2k$ is obtained to guarantee the network scalable. The upper bound is given by $C/r - 1$, where $C$ is the channel capacity available to each node and $r$ is the packet arrival rate at each node.

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