On Competition between Greedy Autonomous Systems in Providing Internet Services: Emergent Behavior and Stability

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Abstract — **We consider the effect of competition between two greedy autonomous systems on pricing and availability of Internet services with elastic demand. The problem is formalized in two different ways: (1) as a non-cooperative game, where the autonomous systems attempt to maximize their profits by adjusting prices and bandwidths, and (2) as a natural evolutionary algorithm, where the behaviors of the autonomous systems are modeled by a system of nonlinear differential equations. To investigate the stability of this evolutionary system, we also consider a system of fixed-point equations describing the global behavior. Necessary conditions are presented for equilibria of the game and the evolutionary algorithm, and analytical solutions are found that give the optimal capacities and the corresponding utilities. The economic implications of competition on both the users and the autonomous systems are quantified.**

Keywords – **Pricing, Capacity Optimization, Internet, Game Theory.**

I. INTRODUCTION

Incorporating economic concepts into models of providing Internet services allows one to develop better understanding of real-life situations. In a social welfare maximization framework, pricing may allow the system to achieve the global optimum by a decentralized algorithm without assuming knowledge of the user's utilities by the central planner [1]. In profit maximization models, autonomous systems (ASs) attempt to maximize their profit by adjusting prices and capacities [2]. Since the Internet is comprised from a large number of ASs, cooperation and competition among ASs is an important factor in understanding the evolution of Internet services.

The competitive pricing can be described [3] as a non-cooperative game of the ASs, where each AS_i owns a set of links and attempts to maximize its revenue by adjusting prices:

$$V_i = \sum_{l \in E_i} \sum_{r \in R_l} p_{lr} d_r \left(\sum_{k \in L_r \setminus l} p_{kr} + p_{lr} \right), \qquad (1)$$

where E_i is the set of external links owned by AS_i, R_l is the set of routes traversing link l, and L_r is the set of links for route r; p_{lr} is the price for route r on link l, and d_r is the demand for route r.

There are two ways to incorporate capacity constraints into this model. One way is to assume capacity fixed and view them as constraints [3]

$$\sum_{r \in R_l} d_r \left(\sum_{k \in L_r \setminus l} p_{kr} + p_{lr} \right) \leq C_l, \quad \forall l \in E_i, \qquad (2)$$

where C_l is the capacity of link l. Another way is to introduce capacity cost [4] and assume that provider utility (profit) is

$$U_i = V_i - \phi_i(c_i), \qquad (3)$$

where V_i is the revenue for AS_i, and $\phi_i(c_i)$ is the capacity cost for capacity c_i. In this paper we assume utilities given by Eq. (3), since we are interested in joint pricing and dimensioning ASs policies.

A single route shared by two providers has been considered by He and Walrand [3]. They observed that a bottleneck provider may not have incentive to expand its capacity; instead, it may prefer to create congestion and raise its price for service. To address this problem, they [3] proposed a limited form of regulation by fixing the portion of the total revenue to be allocated to each provider. However, this market inefficiency may be a result of lack of competition among providers.

In this paper we investigate the effect of competition in a simple case of two alternative routes owned by different providers. This situation has been considered in [4] within a game-theoretic framework assuming linear capacity cost. We, however, approach this problem both from a game-theoretic framework and from the view of evolutionary algorithms. Additionally, we model the effect of barriers to entering the market by assuming that the bandwidth cost is a concave function of the bandwidth. The main justification for adopting an evolutionary point of view of this competitive situation is that Nash equilibria based on the best responses requires complete knowledge of user utilities or, equivalently, price/demand curves, which may not be practically realistic. On the other hand, evolutionary algorithms, based on local information (gradients), can be estimated from historical observations.

We assume that the competition between the two ASs proceeds on two timescales. At the fast timescale, the capacity is assumed fixed and the prices are allowed to reach the corresponding Nash equilibrium. Under natural elasticity assumptions, the prices charged by the ASs are the same and such that the capacity is filled. On the longer timescale, the ASs attempt to adjust their capacities in order maximize their profits. We consider two approaches: (1) a local gradient-based evolutionary algorithm, modeled as a system of nonlinear differential equations, that does not assume complete knowledge of the utility functions, and (2) a global optimization approach requiring complete knowledge of the utility functions. Both approaches follow the concept of Nash equilibrium: no player has an incentive to deviate from the equilibrium. The evolutionary approach considers only small changes, while the global approach uses fixed-point equations and allows all possible capacity deviations. These two models correspond to two extreme cases, allowing for capacity adjustment by ASs.

Our investigation reveals that depending on the capacity cost and the elasticity, there may be either one or three equilibrium capacities achieved in the evolutionary process. For a
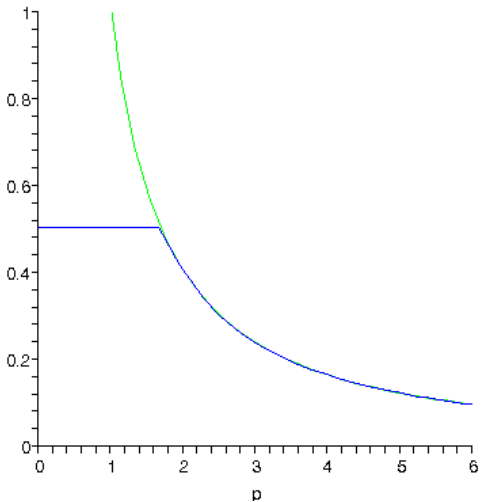
Figure 1: Demand as a function of price (green), and demand limited by a finite capacity of $c = 0.5$ (blue). $d = p^{-1.3}$.

linear capacity cost, there is a single equilibrium capacity vector, while for nonlinear costs, there are three equilibria. We investigate the stability of these equilibria from the perspective of the global optimization approach. In the case were one of these three equilibria is characterized by positive capacity and profit for each AS, then this point is the unique Nash equilibrium, which can be found as the unique globally-stable point of the corresponding fixed-point equations. Otherwise, the solution to the fixed-point equations oscillates.

The paper is organized as followed. In Section II, the models for demand and capacities are presented, and the dynamics of the evolutionary algorithm are described. Section III considers the emergent behavior of the system, by computing analytical solutions for the Nash equilibria of the non-cooperative game and equilibria of the evolutionary algorithm. Section IV discusses the results in terms of the economic benefits to the users and the autonomous systems. Finally, Section V provides a few conclusions and addresses future work.

II. MODEL

DEMANDS

We consider elastic demand, characterized by a concave user utility function $u(d)$ [5]. The user chooses demand d by maximizing its net utility

$$\max_{d \geq 0} u(d) - pd, \qquad (4)$$

where p is the fixed unit price for carrying a unit demand. Maximizing Eq. (4) for all values of p leads to a curve giving demand as a function of price.

A common model for a demand-price curve is of the form

$$d(p) = Ap^{-\alpha}, \qquad (5)$$

where A is the demand potential and α is the elasticity [2]; for telecommunications data traffic, values of α are often in the range 1.3 to 1.7 [2]. Further, we assume that in all cases $\alpha > 1$. Fig. 1 shows a typical demand curve for $\alpha = 1.3$. In the

this section, we assume that each provider has a fixed limited capacity; the resulting demand curve is also shown in the figure. For the demand curve given by Eq. (5), it is also interesting to consider the user's utility. Taking the derivative of Eq. (4) and setting it equal to zero gives $u'(d) = p = (\frac{A}{d})^{1/\alpha}$. Integrating with respect to d gives

$$u(d) = \frac{A^{1/\alpha}}{1 - 1/\alpha} d^{1 - 1/\alpha}. \qquad (6)$$

Denote the price charged by providers 1 and 2 by p_1 and p_2, respectively. Now, the demand seen by each provider depends on the prices charged by the two providers; that is $(p_1, p_2) \mapsto d_1(p_1, p_2)$ and $(p_1, p_2) \mapsto d_2(p_1, p_2)$. Each provider will continue to adjust the price in response to the price set by its competitor. The main questions here are whether an equilibrium point exists and if it does, what is it. There are three cases to consider: (1) $p_1 < p_2$, (2) $p_2 < p_1$, and (3) $p_1 = p_2$.

For case 1 ($p_1 < p_2$),

$$d_1(p_1, p_2) = \begin{cases} d(p_1) & \text{if } d(p_1) < c_1 \\ c_1 & \text{otherwise,} \end{cases} \qquad (7)$$

where c_1 is the fixed capacity of provider 1. Defining the revenue obtained by provider one as $V_1(p_1) = p_1 d_1(p_1, p_2)$ and using Eq. (7) for the demand, one gets the revenue as a function only of p_1. Hence, it is straight-forward to find the price that maximizes the revenue; it turns out that for the demand curve given in Fig. 1, the maximum occurs when provider 1 sets its price such that the demand is equal to its capacity. This result is generally true for a number of related demand curves. Since it is assumed that provider 2 has a higher price, there is no excess demand, and provider 2 ends up with zero revenue. The implication is that when viewed as a game of setting prices, $p_1 < p_2$ is unstable. By symmetry, the case where $p_2 < p_1$ is also unstable.

Therefore, case three where $p_1 = p_2 = p$ leads to a Nash equilibrium, which is found by maximizing the total revenue $V_1(p) + V_2(p) = pd(p)$. Again, setting $\frac{d(V_1(p) + V_2(p))}{dp} = 0$ yields p^* to be the point where the demand is equal to the total capacity $c_1 + c_2$. In comparison, Hajek and Gopal [4] consider a somewhat different game where middle agents acting in parallel buy capacity from a common provider and then resell it. They show that in this case the Nash equilibrium occurs when each agent gets zero profit. A difference is that in their game they simultaneously optimize over prices and capacities.

CAPACITIES

Now, we consider the process of capacity adjustment, assuming the prices already have reached equilibrium, i.e. both ASs charge the same price $p_1 = p_2 = p$.

Given the capacity of each provider, one can invert the demand-price curve to get the price as a function of the two capacities c_1 and c_2:

$$p(d) = p(c_1 + c_2) = (\frac{A}{c_1 + c_2})^{1/\alpha}. \qquad (8)$$

The revenue obtained by the two providers is then given by

$$V_i(c_1, c_2) = c_i p(c_1 + c_2), \quad i = 1, 2. \qquad (9)$$

In general, each provider's capacity cost function may be different, denoted by $\phi_1(c_1)$ and $\phi_2(c_2)$, respectively. In the
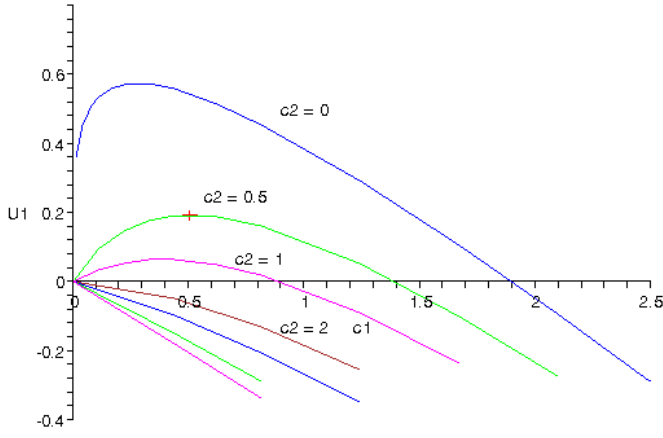
Figure 2: $U_1(c_1, c_2)$ for fixed values of c_2. $A = 1$, $\alpha = 1.3$, $B = 0.61$, and $\gamma = 1$.

rest of this paper, it is assumed that each provider has the same cost for adding capacity $\phi(c) := \phi_1(c) = \phi_2(c)$. The simplest analytical formula is a linear function $\phi(c) = Bc$, which is used by [4] and others. However, other factors such as economies of scale and fixed costs of entering the market suggest the use of a nonlinear concave capacity cost. Kleinrock [6, p. 336] considers the function

$$\phi(c) = Bc^\gamma, \quad 0 \leq \gamma \leq 1. \tag{10}$$

In this paper, we are interested in relatively small deviations from linearity, and so we assume $\gamma \geq 0.5$. Thus, the utility (profit) functions for each provider are given by $U_i(c_1, c_2) = V_i(c_1, c_2) - \phi(c_i), \quad i = 1, 2$. Combining this equation with Eqs. (8)-(10) gives

$$U_i(c_1, c_2) = c_i \left(\frac{A}{c_1 + c_2}\right)^{1/\alpha} - Bc_i^\gamma, \quad i = 1, 2, \tag{11}$$

where $\alpha > 1$ and $0.5 \leq \gamma \leq 1$. Fig. 2 shows $U_1(c_1, c_2)$ as a function of c_1 for various values of c_2, for $A = 1$, $\alpha = 1.3$, $B = 0.61$ and $\gamma = 1$ (*i.e.* linear capacity cost). Because of symmetry, U_2 is the same as U_1 with c_1 and c_2 reversed.

Stability, according to the theory of Nash equilibrium, is when no player has an incentive to deviate from this equilibrium; this implies that one needs to determine maxima. So, equilibrium capacities for $c_i^* > 0$, $i = 1, 2$, are determined by the following system of equations

$$\frac{\partial U_i(c_1, c_2)}{\partial c_i} = 0, \quad i = 1, 2. \tag{12}$$

Depending on the values of the parameters A, α, B, and γ, there may be up to three equilibria. One equilibrium corresponds to the solution of Eqs. (12). If the other two equilibria occur, they are on the c_1 axis and the c_2 axis, respectively. The equilibrium on the c_1 axis simultaneously satisfies the following conditions

$$\frac{\partial U_1(c_1, c_2)}{\partial c_1}\bigg|_{c_2=0} = 0 \tag{13}$$

$$\frac{\partial U_2(c_1, c_2)}{\partial c_2}\bigg|_{c_1=c_1^{**}, \, c_2=0} < 0 \tag{14}$$

where c_1^{**} is the value of c_1 that results from solving Eq. (13) [1]. By symmetry, Eqs. (13) and (14) with c_1 and c_2 reversed give the conditions for the equilibrium on the c_1 axis.

DYNAMICS

In the case where the system has a unique, non-negative solution, this solution (12) represents unique Nash equilibrium capacities. However, when there are several equilibria, including solutions of Eqs. (13) and (14), one needs to further examine their stability properties. We approach this issue in two ways, through both local and global stability analysis. To investigate local stability, we view the process of capacity adjustment as an evolutionary algorithm, where small changes are made from a given point in order to improve utility. For global stability analysis, we use fixed point equations to optimize over the entire range of the capacities.

The first dynamic model associated with the game played by the two ASs [7, pp. 148-150] allows local adjustments to the capacity according to the following system of two ordinary differential equations

$$\frac{dc_i}{dt} = \eta \frac{\partial U_i(c_1, c_2)}{\partial c_i} \quad i = 1, 2, \tag{15}$$

where $\eta > 0$ is some constant, and $c_1 > 0$ and $c_2 > 0$. If $c_1 = 0$, $c_2 > 0$ the capacity adjustment is described by the following system of equations

$$\frac{dc_1}{dt} = \max\{0, \quad \eta \frac{\partial U_1(c_1, c_2)}{\partial c_1}\} \tag{16}$$

$$\frac{dc_2}{dt} = \eta \frac{\partial U_2(c_1, c_2)}{\partial c_2}. \tag{17}$$

The case $c_2 = 0$, $c_1 > 0$ can be obtained from Eqs. (16)-(17) by symmetry. Lastly, for $c_1 = 0$, $c_2 = 0$

$$\frac{dc_i}{dt} = \max\{0, \quad \eta \frac{\partial U_i(c_1, c_2)}{\partial c_i}\} \quad i = 1, 2. \tag{18}$$

The second model considers the global behavior, by constructing an algorithm consisting of the fixed point equations

$$c_1(n+1) = \arg\max_{c_1 \geq 0} U_1(c_1, c_2(n)) \tag{19}$$

$$c_2(n+1) = \arg\max_{c_2 \geq 0} U_2(c_1(n), c_2), \tag{20}$$

where n is an integer time step. Convergence of this algorithm ensures the existence of a unique capacity vector, which is both a Nash equilibrium and the global solution to the game.

III. EMERGENT BEHAVIOR

In this section, we first present analytical solutions to the Nash equilibrium conditions given by Eqs. (12) and Eqs. (13) and (14). Next, we examine the equilibria of the evolutionary algorithm, relating them to the Nash equilibria.

NASH EQUILIBRIA

Numerical solution of Eqs. (12) for $c_1 > 0$ and $c_2 > 0$ suggest that any solutions occur when $c_1 = c_2 = c^*$, which satisfy

$$\frac{\partial U_i(c_1, c_2)}{\partial c_i}\bigg|_{c_1=c_2=c} = 0, \quad i = 1, 2, \tag{21}$$

[1]The second derivatives also need to be checked to ensure a maximum.

or using Eq. (11),

$$\left(\frac{A}{2c}\right)^{1/\alpha} - \frac{1}{2\alpha}\left(\frac{A}{2c}\right)^{1/\alpha} - \frac{Bc^\gamma\gamma}{c} = 0. \qquad (22)$$

It is straight-forward to verify that the solution of Eq. (22) is

$$c^* = \exp\left(-\frac{\ln(\frac{2}{A}) + \alpha\ln(\frac{2B\gamma\alpha}{2\alpha-1})}{\gamma\alpha - \alpha + 1}\right). \qquad (23)$$

Substituting $c_1 = c_2 = c^*$ into Eqs. (11) then gives the resulting equilibrium utilities $U^* = U_i(c^*, c^*)$, $i = 1, 2$.

Now, let us turn to the equilibria on the c_1 and c_2 axes. Eq. (13) can be written as

$$\left(\frac{A}{c_1}\right)^{1/\alpha} - \frac{1}{\alpha}\left(\frac{A}{c_1}\right)^{1/\alpha} - \frac{Bc_1^\gamma\gamma}{c_1} = 0, \qquad (24)$$

which has the solution

$$c_1^{**} = \exp\left(-\frac{\ln(\frac{1}{A}) + \alpha\ln(\frac{B\gamma\alpha}{\alpha-1})}{\gamma\alpha - \alpha + 1}\right). \qquad (25)$$

To evaluate Eq. (14), we take the partial derivative of Eq. (11) giving

$$\frac{\partial U_2(c_1, c_2)}{\partial c_2} = \left(\frac{A}{c_1 + c_2}\right)^{1/\alpha} - \frac{c_2\left(\frac{A}{c_1+c_2}\right)^{1/\alpha}}{\alpha(c_1 + c_2)} - \frac{Bc_2^\gamma\gamma}{c_2}, \quad (26)$$

which needs to be compared to zero. Making the substitutions $c_1 = c_1^{**}$ and $c_2 = 0$, one sees that the first term is a positive constant, the second term is zero, and the third term depends on γ. When $\gamma = 1$, $\frac{\partial U_2(c_1, c_2)}{\partial c_2}\big|_{c_1=c_1^{**}, c_2=0} = \frac{B}{\alpha-1}$, which is greater than zero. Hence, there are no equilibria on the c_1 axis. For $0.5 \leq \gamma < 1$, $-Bc_2^{\gamma-1}\gamma \to -\infty$ as $c_2 \to 0$, so there is a stable equilibrium on the c_1 axis. Again by symmetry, the same results hold for the c_2 axis.

Evolutionary Equilibria

Let us now look at the equilibria of the evolutionary algorithm given by Eqs. (15)-(18). It is clear that the equilibrium of Eq. (15) is the same as the Nash equilibrium given by Eq. (12). Moreover, the equilibria of Eqs. (16)-(17) are equivalent to Eqs. (13) and (14). From an evolutionary perspective, what is interesting is how the system evolves given an initial starting point.

As an example, consider the system (15)-(18) with the parameters $A = 1$, $\alpha = 1.3$, $B = 0.32$, $\gamma = 0.63$. Fig. 3 shows the resulting phase portrait, where the trajectories were obtained by numerically solving this system. The three equilibrium points are denoted by the squares in the figure. To confirm that the equilibrium point at $\mathbf{c}^* = (c_1^*, c_2^*) \approx (4.305, 4.305)$ is locally stable, we linearize the system (15)-(17) about this point. The resulting linearized system is $\frac{d\xi}{dt} = \mathbf{A}\xi$, where $\xi = \mathbf{c} - \mathbf{c}^*$. Since the eigenvalues of $\mathbf{A}$ are negative, the equilibrium $\mathbf{c}^*$ is locally stable. This equilibrium is reached by system (15)-(18) if the initial point $(c_1(0), c_2(0))$ lies far enough from the axes $c_1 = 0$ and $c_2 = 0$.

If the initial point $(c_1(0), c_2(0))$ lies close enough to one of the axes, for example the c_2 axis, then $c_1(t)$ decreases with time until $c_1(t^\dagger) = 0$, for $t^\dagger < \infty$. For all $t > t^\dagger$, the trajectory stays on the c_2 axes, i.e., $c_1(t) = 0$. The evolution is now described by sliding mode behavior, which is modeled by the single ordinary differential equation

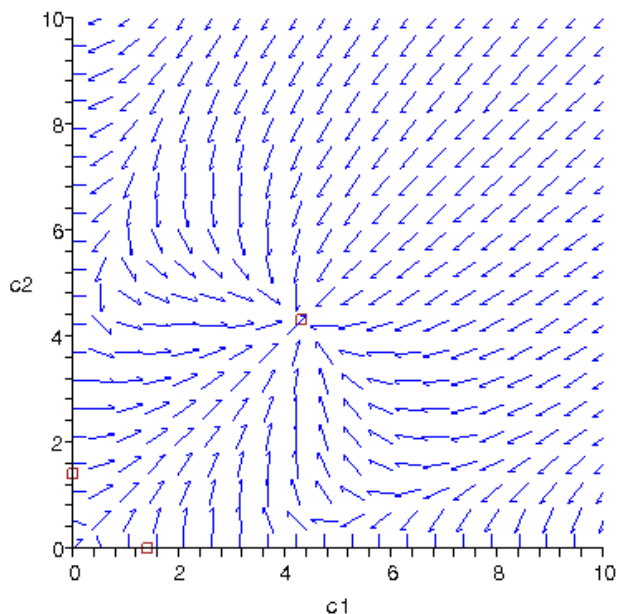$$\frac{dc_2}{dt} = \eta\frac{dU_2(0, c_2)}{dc_2}. \qquad (27)$$

Figure 3: Phase portrait of system (15)-(18). $A = 1$, $\alpha = 1.3$, $B = 0.32$, $\gamma = 0.63$.

The resulting solution has $c_2(t)$ moving along the c_2 axis until it reaches the equilibrium at $c_2^{**} = 1.403$. By symmetry, trajectories in the lower right-hand corner move down to the c_1 axis, and then while $c_2(t)$ stays at zero, $c_1(t)$ moves along the c_1 axis to its equilibrium point $c_1^{**} = 1.403$.

So, one can see that the initial conditions determine which evolutionary equilibrium is reached. Since these equilibria are equivalent to the Nash equilibria in the corresponding game, it is also useful to consider the behavior from a more global perspective. For the parameters used in the previous example, the equilibria are approximately at $(4.305, 4.305)$, $(0, 1.403)$, and $(1.403, 0)$, with corresponding utilities $(0.019, 0.019)$, $(0, 0.685)$, and $(0.685, 0)$. Starting at various initial conditions, the fixed-point algorithm given by Eqs. (19) and (20) converges to the point $(4.305, 4.305)$, where the utilities are small but positive. Hence, this is the single global optimum. The other two equilibria are not global optima, since the autonomous system with zero capacity gets zero utility, and so it has an incentive to increase its capacity.

If the utilities at the equilibrium c^* are negative, the global behavior is quite different. To see this, consider the parameter set $A = 1$, $\alpha = 1.3$, $B = 0.4$, and $\gamma = 0.5$. The equilibria are at $(8.973, 8.973)$, $(0, 1.702)$, and $(1.702, 0)$, with corresponding utilities $(-0.225, -0.225)$. $(0, 0.609)$, and $(0.609, 0)$. Using the algorithm given by Eqs. (19) and (20), one sees some interesting behavior, as depicted in Fig. 4. Staring at the equilibrium point $(8.973, 8.973)$ leads to both c_1 and c_2 following the same trajectory. Starting at other initial conditions, e.g. $(0, 1)$ leads to slightly more complicated behavior. Now, c_1 and c_2 again oscillate among the same three points, but the two trajectories are out of phase.

IV. Discussion

First, consider the case of linear capacity cost, i.e. $\gamma = 1$. Fig. 5(A) shows the optimal capacity, c^*, as a function of the cost parameter B for four values of α. When there is only
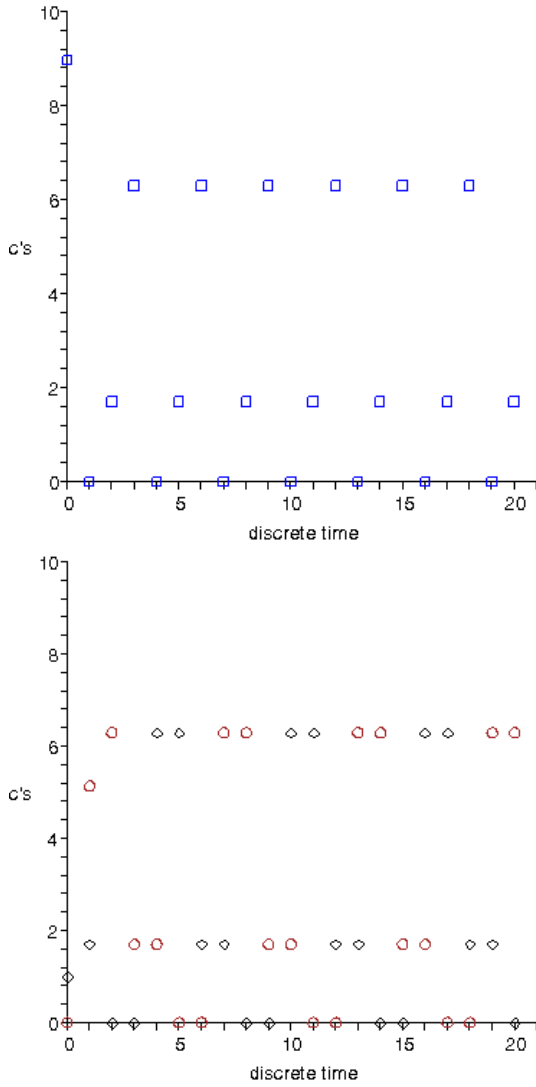
Figure 4: $\boxed{\frac{(A)}{(B)}}$ c_1 and c_2 as solutions of the fixed-point equations (19) and (20) for initial conditions (A) $\mathbf{c} = (8.973, 8.973)$ and (B) $\mathbf{c} = (0, 1)$: circles c_1 and diamonds c_2.

Figure 5: $\boxed{\frac{(A)}{(B)}}$ (A) c^* and (B) c_1^{**} vs. B for $A = 1$ and $\gamma = 1$. $\alpha = 1.3$, 1.5, 1.7, and 1.9.

a single provider, *e.g.* provider 1, Eq. (13) gives a necessary condition for finding the optimal capacity c_1^{**}, which is shown in Fig. 5(B) for the same parameters. Varying A scales c^* and c_1^{**} by the same constant, and so does not change their ratio. Looking at Eqs. (23) and (25), it is clear that $\lim_{B\to\infty} c^* = 0$ and $\lim_{B\to\infty} c_1^{**} = 0$, so there is less incentive to provide capacity as the unit capacity cost increases.

More interesting are the corresponding utilities, U^* and U_1^{**}, which are always positive and monotonically decreasing with B. Since these utilities are positive, the autonomous system(s) have an economic incentive to provide service. Fig. 6(A) shows the ratio $\frac{U^*}{U_1^{**}}$. Note that for all values of α and B, the provider's utility is higher when there is no competition; also, this ratio does not depend on A. As the cost B increases, the profits U^* and U_1^{**} decrease, with $\lim_{B\to\infty} U_i(c^*, c^*) = 0$, $i = 1, 2$, and $\lim_{B\to\infty} U_1(c_1^{**}, 0) = 0$.
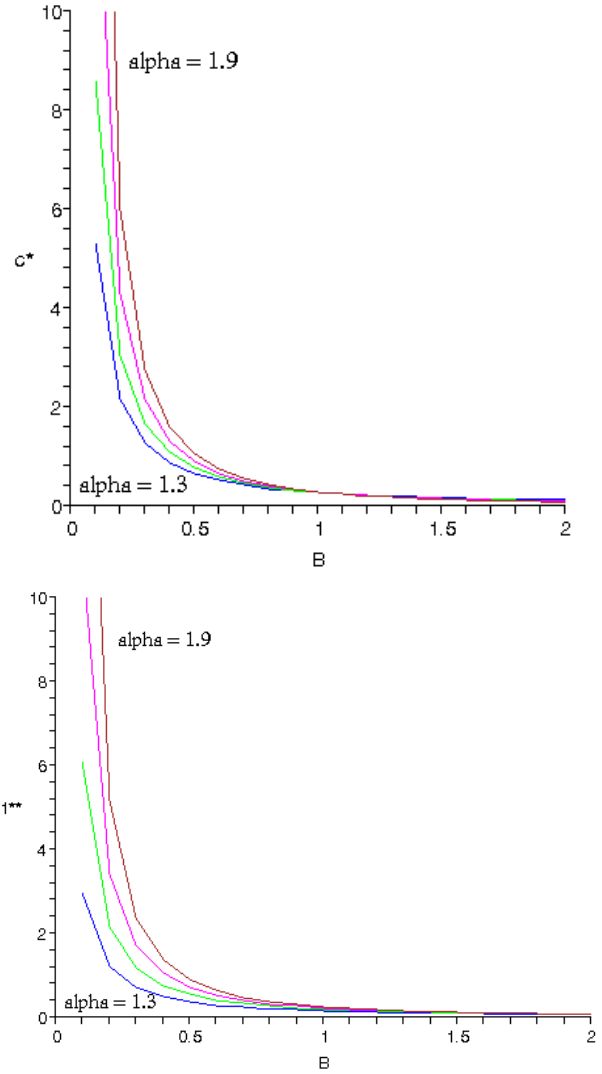
Another interesting comparison is that between the user's utility when there are two providers compared to when there is only one. Fig. 6(B) shows the ratio $\frac{u^*}{u^{**}}$, where u^* and u^{**} are the resulting user's utility for the two equilibria c^*, and c_1^{**}, respectively. The competition between the two autonomous systems results in more total capacity (sum of the capacities provided by each AS), which leads to lower prices, and hence higher utility for the user. Again, the parameter A does not change the ratio.

To investigate the effects of economies of scale and barriers to market entry, we consider non-linear capacity costs, *i.e.* $\gamma < 1$. Fig. 7(A) shows the behavior of the equilibria as a function of B and γ, when $A = 1$ and $\alpha = 1.3$. As discussed above, there are three locally stable equilibria. Below approximately $\gamma = 0.62$, the equilibrium at (c^*, c^*) results in negative values for U^*, which means that neither autonomous system will have an economic incentive to remain at this point; hence, unstable behavior may occur as illustrated in Fig. 4. Above this line (and below $\gamma = 1$), U^* is positive, and so (c^*, c^*) is the Nash equilibrium that is the global solution to the capacity game. Since the equilibria on the c_1 and c_2 axes give zero
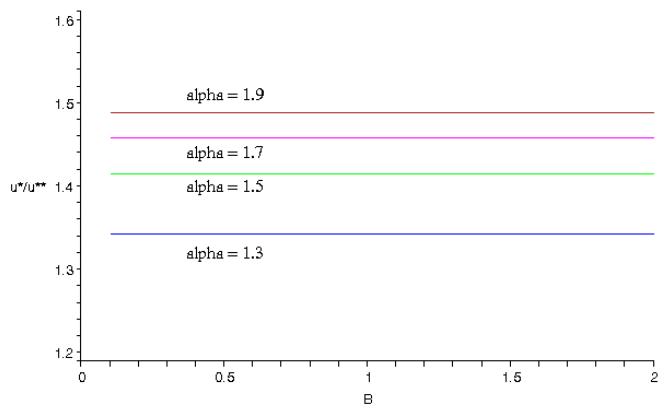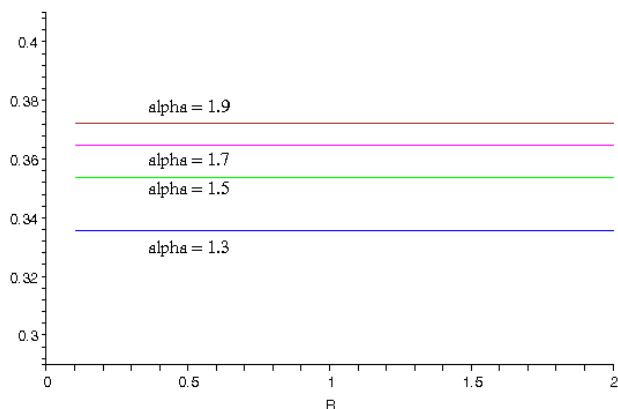
Figure 6: $\boxed{\frac{(A)}{(B)}}$ (A) $U^*/U_1(c_1^{**}, 0)$ and (B) u^*/u^{**} vs. B for $A = 1$ and $\gamma = 1$. $\alpha = 1.3$, 1.5, 1.7, and 1.9.

Figure 7: $\boxed{\frac{(A)}{(B)}}$ Bifurcation diagram. (A) γ vs. B, with $\alpha = 1.3$. (B) γ vs. α.

utility to one of the autonomous systems, they are globally unstable. If each AS makes local decisions, these equilibria may be reached, perhaps indicating that a barrier to market entry exists. Over a wide range, the parameter B has negligible effect. Similarly, varying A changes the value of the utilities, but not the sign; hence, Fig. 7(A) is valid for $A > 0$.

Since the parameters α and γ are the most important, Fig. 7(B) shows the behavior of the equilibria as a function of them. As α increases over the range $1.3 - 1.9$, the value of γ needed to ensure $U^* > 0$ increases slightly.

V. CONCLUSIONS

The analysis above suggests that for the case of two autonomous systems providing Internet service where there is elastic demand, competition is economically beneficial for the users. In contrast, He and Walrand [3] suggest the use of regulation, perhaps because the system they are analyzing allows each AS to have monopoly power. For our model with linear capacity cost, competition is achieved because the only stable equilibrium results in both ASs providing positive capacity. For non-linear capacity costs, local (evolutionary) behavior may result in only one AS providing service, while more global behavior may result in oscillations in the provided capacities; operating in the region $U^* > 0$ ensures a globally stable solution. Future research will investigate extending these results
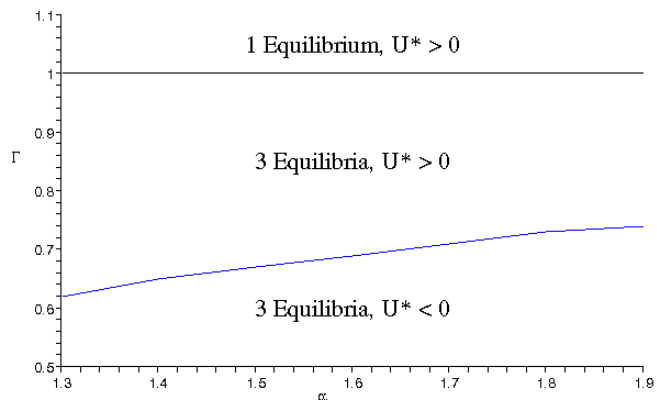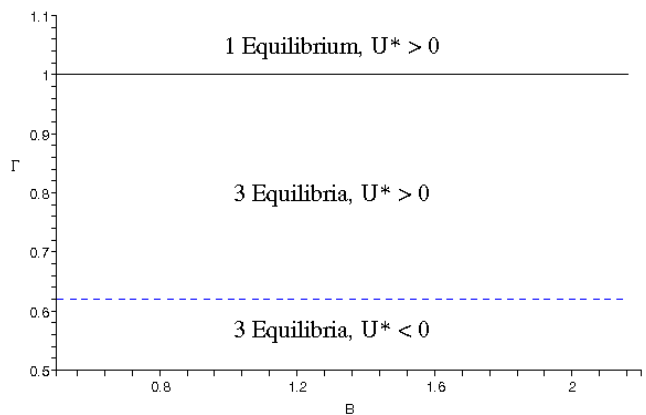
to N greedy autonomous systems, and it will also consider unequal capacity costs.

REFERENCES

[1] F. P. Kelly, A. K. Maulloo, and D. H. K. Tan, "The rate control for communication networks: shadow prices, proportional fairness and stability," *Journal of the Operational Research Society*, pp. 237-252, vol. 49, 1998.

[2] D. Mitra, K. G. Ramakrishnan, and Q. Wang, "Combined economic modeling and traffic engineering: joint optimization of pricing and routing in multi-service networks," *Proc. ITC*, Brazil 2001.

[3] L. He and J. Walrand, "Pricing Internet services with multiple providers," *Proc. Allerton Conference on Communications, Control, and Computing*, Oct. 2003.

[4] B. Hajek and G. Gopal, "Do greedy autonomous systems make for a sensible Internet?", *Proc. Conference on Stochastic Networks*, June, 2002.

[5] S. Shenker, "Fundamental design issues for the future Internet," *IEEE JSAC*, Vol. 13, pp. 1176-1188, Sept. 1995.

[6] L. Kleinrock, *Queueing Systems Volume II: Computer Applications*, John Wiley & Sons, New York, 1976.

[7] A. Rapoport, *Two-Person Game Theory The Essential Ideas*, The University of Michigan Press, Ann Arbor, MI, 1966.