Survivability of SONET/WDM Ring Networks with Optimum Number of Spare Components

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Abstract

This paper addresses the optimization and performability of survivability strategies used in the next generation of high-capacity optical transport networks based on Wavelength Division Multiplexing (WDM) technology. The framework of survivability presented is based on the self-healing mechanism used in SONET/SDH. The key issue addressed is how to determine the number of redundant components and wavelength units necessary to achieve a guaranteed level of survivability. A multi-modal failure probability model is developed and used in the reliability model of optical components. The multi-modal reliability analysis is used in conjunction with a reliability optimization technique to determine the optimum number of redundant components and wavelengths necessary in a SONET/WDM network to achieve a given degree of survivability.

1. Introduction

The increasing demand for guaranteed Quality of Service (QoS) and network availability has made network survivability an area of significant importance in today's network design and management. Users are demanding and are willing to pay for guaranteed service availability. Efficient and robust survivability must therefore be engineered into emerging high-capacity networks such as WDM networks which carry traffic aggregated over a wide geographical area. Fault-tolerant features have been successfully implemented in SONET/SDH to address network survivability concerns [1]. Significant efforts are being made at ANSI/T1 and at ITU-T [2],[4] to include survivability features in the emerging draft recommendation for optical networks. As network survivability increasingly becomes an important design and management consideration, there is an increasing need to develop qualitative measures of network survivability to guide designers to build cost effective fault-tolerant optical networks. Fault-tolerant capabilities provide networks with automatic traffic restoration features activated during critical network component failures. In a typical network being considered, the degree of fault-tolerance depends on the number of redundant units allocated to each component to provide failure transparency. In this paper, reliability modeling and integer programming techniques are used to estimate the number of redundant units required to achieve a desired degree of fault tolerance.

2. Survivability Features of SONET/WDM Networks

The traffic carrying capacity of WDM networks is projected to be on the order of magnitude several times higher than that of SONET/SDH. At this traffic carrying capacity, the failure of a WDM component be can catastrophic, resulting in the loss of a large volume of data aggregated over a wide geographical area. This potential loss of high volume of data heightens the need for robust fault-tolerant capabilities in SONET over WDM networks. SONET/SDH protection strategies are examples of fault-tolerant capabilities desired in WDM networks. In its simple form, the SONET/SDH protection mechanism uses $(1:1)$, $(1+1)$, and $(1:n)$ Automatic Protection Switching (APS) schemes. These schemes, when automated to detect faults and reroute traffic automatically without operator interventions in SONET/SDH ring networks are referred to as self-healing mechanisms. SONET/SDH's self-healing uses an efficient traffic restoration algorithm to reroute traffic over redundant or spare components in the event of network component failures. A fully redundant unidirectional self-healing SONET ring (USHR) network is depicted in Figure 1. It shows the weakness of USHR in a multiple and simultaneous component failures scenario since most of the traffic cannot be recovered using loop-back. This is because redundant components are distributed uniformly. The allocation of spare units uniformly in the network assumes that all network components have the same failure rates. This assumption may not be true in networks in which some components are deployed in hostile environments such as undersea installations where the failure rate could be higher. To achieve a reasonably uniform degree of survivability in this type of networks, components with higher failure rates must have more redundant units than those with lower failure rates. The exact number of redundant units allocated to the components in the hostile environments can be obtained through a reliability optimization technique presented in this paper.

3. Multi-Modal Failure Reliability Modeling of Components

In this section, we develop the reliability analysis of WDM components by assuming that a typical component can fail in one or more modes. We further assume that each WDM component consists of interconnected parts such as lasers and receivers, each of which fails indepen-

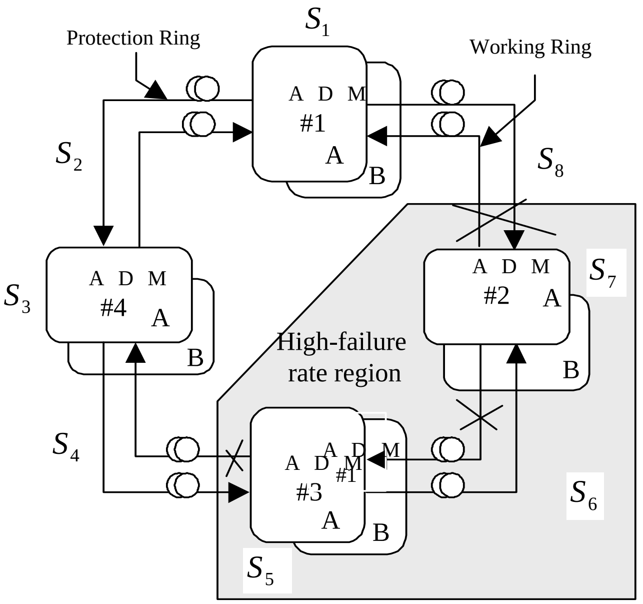

Figure 1: USHR network with concurrent faults and components deployed in a hostile environmen

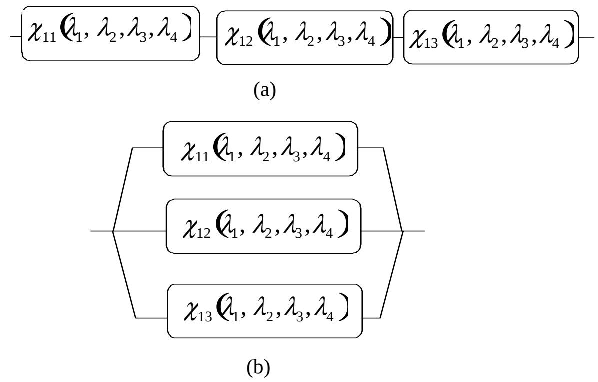

Figure 2: Logic Circuits: (a) α class failures, (b) β class failures

dently. We shall also consider only two general classes of a component failure, the α-class which describes all partial failures, and the β-class which describes total failures. Several modes of failure are further defined within each class of failure. All redundant components are subject to the same classes and modes of failure. All components are subject to the α and β classes of failure defined as follows:

- Class α failures: This is a severe class of faults which affects the primary components and redundant units. In particular, the entire subsystem fails if one component fails.
- Class β failures: This is a class of failures which requires all the components in the subsystem to fail before the entire subsystem fails.

Each class of failure may include one or more modes of failure, each with an assigned probability of failure. To capture the definition of these failure modes, we use logic diagrams, shown in Figure 2. The logic circuit with series components models the α class of failure. This is because circuit opens if at least one component fails. Similarly, the logic circuit with parallel components models the β class, because the circuit partially function if one or two components fail. The analysis assumes that the failure probabilities of all components, including redundant units, are statistically independent. Redundant units are connected in parallel. The classes and modes of failure for this component are defined as follows:

- Class α failures

  1. Mode 1: Operational temperature exceeds the set limit.
  2. Mode 2: Power failure

- Class β failures

  1. Mode 3: Failure of two or three receivers.
  2. Mode 4: Failure of two or three transmitters

Now, let p_{ij} be the probability of the i^{th} mode failure in the j^{th} component, where $i = 1, 2$ designates the mode of failures in class α, and $i = 3, 4$ designates the mode of failure in class β failures. The logic diagrams shown in Figure 2 are parallel and series configurations of three components interconnected to form a subsystem. The probability that the j^{th} component will fail is given by

$$p_j = \sum_{i=1}^{4} p_{ij} \tag{3.1}$$

Thus, the probability that the subsystem will fail is the sum of the probabilities of its failure in each of the four modes. For β modes, using 2.a, the failure probabilities for modes 3 and 4 are given by

$$q_3 = p_{31}p_{32}p_{33}, \tag{3.2}$$

$$q_4 = p_{41}p_{42}p_{43}p.$$

Similarly, for mode α failures, using Figure 2.a, the probabilities for mode 1 and 2 failures are given by

$$q_1 = 1 - (1 - p_{11})(1 - p_{12})(1 - p_{13}), \tag{3.3}$$

$$q_2 = 1 - (1 - p_{21})(1 - p_{22})(1 - p_{23}).$$

The probability of a complete or total failure is the sum of the probabilities of failure of each component in each of the four modes of failure.

$$Q = 2-\sum_{i=1}^{2}(1-p_{i1})(1-p_{i2})(1-p_{i3}) + \sum_{i=3}^{4} p_{i1}p_{i2}p_{i3}. \quad (3.4)$$

This result can be easily extended to a network with m redundant components and s modes of failure, h of which are in $\alpha-$mode and the remaining $(m-h)$ in β-mode. Under this generalization, the component failure probability becomes

$$Q = h - \sum_{i=1}^{h}\prod_{j=1}^{m+1}(1-p_{ij}) + \sum_{i=h+1}^{s}\prod_{j=1}^{m+1} p_{ij}. \quad (3.5)$$

If the components are alike, the probability of failure given by (3.5) becomes

$$Q = h - \sum_{i=1}^{h}(1-p_i)^{m+1} + \sum_{i=h+1}^{s} p_i^{m+1}. \quad (3.6)$$

where $i = 1, 2, 3. \ldots, h$ denotes the α-node failures and $i = h+1, h+2, h+3, \ldots, s$ denote the β-mode failures. Both the working components and the redundant units are subject to the same modes, as described in section 2. In the next section, we will use the results in (3.5) and (3.6) to obtain the optimum number of redundant units in a SONET/WDM ring network with N components connected in series.

4. Reliability Optimization of WDM Ring Networks

There is an increasing need to render WDM networks fault-tolerant with features such as the SONET/SDH's self-healing capabilities. A key component of fault-tolerant networks is the automatic allocation of redundant network components in the event of failures. SONET/SDH self-healing mechanisms use (1:1),(1+1), and $(1:n)$ schemes for fiber failure protection. Dual-homing is also used to protect other critical components such as Add/Drop Multiplexers (ADMs) and Optical Cross Connect (OCX). The (1:1) and (1+1) protection schemes in which each component is protected by a redundant unit provides a high degree of fault tolerance but can be cost prohibitive in large scale networks. On the other hand, the $(1:n)$ scheme, in which one redundant component provides protection to n components offers a cost effective option at the expense of a lesser degree of fault-tolerance. In this section, we use a reliability optimization technique to obtain the optimum number of redundant units in $(1:n)$, protection schemes. The objective is to optimize the reliability of network components given by (3.5) and (3.6), subject to some design and cost constraints. The formulation is based on (0/1) integer programming techniques using the approach proposed in [6].

Stage	Failure Mode	p_{ij}	g_j
1	α	$p_{11} = 0.1230$	8
	β	$p_{12} = 0.0245$	
	β	$p_{13} = 0.0909$	
	β	$p_{14} = 0.2990$	
2	α	$p_{21} = 0.0918$	9
	β	$p_{22} = 0.0156$	
	β	$p_{23} = 0.0823$	
	β	$p_{24} = 0.0917$	
3	α	$p_{31} = 0.0382$	6
	β	$p_{32} = 0.0551$	
	β	$p_{33} = 0.1000$	
	β	$p_{34} = 0.1401$	
4	α	$p_{41} = 0.0497$	7
	β	$p_{42} = 0.0619$	
	β	$p_{43} = 0.0691$	
	β	$p_{44} = 0.2722$	
5	α	$p_{51} = 0.0497$	8
	β	$p_{52} = 0.0617$	
	α	$p_{53} = 0.0691$	
	β	$p_{54} = 0.2722$	
6	α	$p_{61} = 0.0235$	4
	β	$p_{62} = 0.0262$	
	β	$p_{63} = 0.0164$	
	β	$p_{64} = 0.1902$	
7	α	$p_{71} = 0.0324$	6
	β	$p_{72} = 0.0591$	
	β	$p_{73} = 0.0721$	
	β	$p_{74} = 0.1929$	
8	α	$p_{81} = 0.0163$	3
	β	$p_{82} = 0.0545$	
	β	$p_{83} = 0.0156$	
	β	$p_{84} = 0.0879$	

Table 1: Component Failure Probability

4.1. Problem Formulation

Redundant units optimization is a good example of a problem that requires an integer value decision variable. If the problem objective and cost functions are of the form $W = \sum_{j=1}^{N} f_j(m_j)$ then the problem is a separable linear optimization problem with N stages. The stages are connected in series and the redundant units are connected in series. The reliability optimization of a SONET/SDH over WDM ring network can be formulated as an N-stage optimization problem by considering each network component as a stage. For simplicity, we assume that the maximum number of components that can be used in the network at each stage is limited to 4. Finding the optimum number of redundant components required to achieve a desired survivability level becomes a constrained reliability optimization problem. Each stage requires an initial or primary component and m_j redundant units. The objective is to determine m_j, where $1 \leq j \leq 4$. The problem is formulated and solved with an integer-programming approach as follows:

optimize:

$$W = \sum_{j=1}^{N} f_j(m_j), \tag{4.1}$$

subject to:

$$\sum_{j=1}^{N} g_{ij}(m_j) \leq b_i \quad i = 1, 2, 3, \ldots, r, \tag{4.2}$$

$$\sum_{j=1}^{M} R_j \geq \ln\ M$$

and

$$m_j = 0, 1, 2, 3, , \ldots, \tilde{m}_j \quad j = 1, 2, 3, \ldots, N\ ,$$

where W and m_j are the unknowns and the remaining terms in (4.1) are defined as follows:

W : the objective function of the system to be minimized

N : the number of stages in the system

$f_j(m_j)$: the objective function of the i^{th} component as a function of m_j, the number of redundant units

$g_j(m_j)$: the amount of resources consumed at stage j as a function of m_j, the number of redundant units

b_i : amount of resources available at the i^{th} stage

r : the number of components

R_j : reliability at the j^{th} stage with $m_j + 1$ (redundant + initial) units

M : the minimum acceptable reliability of the system

m_j : the number of redundant units used at stage j

$\tilde{m}_j$: the maximum number of redundant units allowed at stage j

Letting m_j be the number of components used in stage j, the optimization problem given by equation (4.3) can be solved as the following integer programming problem:

Optimize

$$f = \sum_{j=1}^{N} \sum_{k=1}^{m_j} \Delta f_{jk} m_{jk},$$

subject to:

$$\sum_{j=1}^{M} \sum_{k=1}^{m_j} \Delta g_{jk} m_{jk} \leq b_i \quad i = 1, 2, 3, \ldots, r\ , \tag{4.3}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{m_j} \Delta \ln R_{jk} m_{jk} \geq \ln\ M \tag{4.4}$$

and

$$m_{jk} = 1 \quad k = 0 \tag{4.5}$$

$$m_{jk} - m_{jk-1} \leq 0, \quad \begin{array}{l} k = 1, 2, 3, \ldots, r \\ j = 1, 2, 3, \ldots, N \end{array} \tag{4.6}$$

$$m_{jk} \geq 0 \quad j \text{ and } k\ , \tag{4.7}$$

where W and m_j are unknown as in (4.2) and the new terms are defined as follows:

k : index used to denote a given redundant unit at stage j

m_{jk} : the k^{th} redundant unit at stage j, where $m_{jk} = \begin{cases} 1 & k \leq m_j \\ 0 & m_j \leq k \leq \tilde{m}_j \end{cases}$

$\triangle f_{jk}$: change in the objective function $f_j(m_j)$ by adding the k^{th} redundant unit, where $\triangle f_{jk} = \begin{cases} f_{jk} & k = 0 \\ f_{jk} - f_{jk-1} & k = 1, 2, \ldots, \tilde{m}_j \end{cases}$

$\triangle g_{ijk}$: change in $g_{ijk}(m_j)$ by addition of the k^{th} redundant unit at stage j, where $\triangle g_{jk} = \begin{cases} g_{ijk} & k = 0 \\ g_{ijk} - g_{ijk-1} & k = 1, 2, \ldots, \tilde{m}_j \end{cases}$

$\triangle \ln R_{ijk}$: change in $R_{ijk}(m_j)$ by addition of the k^{th} redundant unit at stage j, where $\triangle \ln R_{jk} = \begin{cases} R_{ijk} & k = 0. \\ R_{ijk} - R_{ijk-1} & k = 1, 2, \ldots, \tilde{m}_j \end{cases}$

where R_j, is given by the reliability at the j^{th} stage which is:

$$R_j = 1 - (h - \sum_{i=1}^{h} (1 - p_i)^{m_j+1} + \sum_{i=h+1}^{s} p_i^{m_j+1}) \tag{4.8}$$

The reliability of failure given by $(1 - Q_j)$ in (4.8) assumes that WDM components in our ring topology and the redundant units at each j^{th} stage are identical.

5. Numerical Example

Consider the USHR network shown in Figure 1. It consists of a working and a protection ring, each supporting four wavelengths. The network be viewed as a system with a redundant unit connected in parallel. The network components consist of four ADMs and four fiber links, which together make up eight stages denoted by $(S_1, S_2, \cdots, S_8)$. The cost of a component at the 8 stages is $(8, 9, 6, 7, 7, 4, 3)$, and the total cost of components in the network should not exceed 104. Each stage has two primary components ($m_{j0} = 1$, $m_{j1} = 1, 1 \leq j \leq 8$) such that

$$\begin{array}{ll} m_1 = 2 & m_5 = 2 \\ m_2 = 2 & m_6 = 2 \\ m_3 = 2 & m_7 = 2 \\ m_4 = 2 & m_8 = 2 \end{array} \tag{5.1}$$

and $m_j = m_{j0} + m_{j1}$, $1 \le j \le 8$. The WDM components used in the network are assumed to be alike at each stage and are subject to four modes of failure. The failure probabilities of each component at various modes are given in Table 1. The total number of failure modes, s, for all the components in the systems is assumed to be 4 ($s = 4$). Each component has a single α mode of failure ($h = 1$), and three β modes of failure. As a result, equation (4.8) reduces to

$$\begin{aligned} R_{ji} &= 1 - (h - Q_j) \quad (5.2) \\ &= 1 - \left(1 - (1 - p_i)^{m_j+1} + \sum_{i=2}^{4} (p_i)^{m_j+1} \right), \end{aligned}$$

and the change in the reliability of the j^{th} component obtained by adding a redundant component is given by

$$\triangle \ln R_{jk} = \begin{cases} \ln R_{jk} & k = 0 \\ \ln R_{jk} - \ln R_{jk-1} & k = 1, 2, \ldots, \tilde{m}_j \end{cases}. \quad (5.3)$$

For $k = 0$

$$\begin{aligned} \triangle \ln R_{j0} &= \ln R_{jk} \quad (5.4) \\ &= \ln \left(1 - \left(1 - (1 - p_i) + \sum_{i=2}^{4} p_i \right) \right) \quad (5.5) \end{aligned}$$

For $k = 1$

$$\begin{aligned} \triangle \ln R_{j1} &= \ln R_{j1} - \ln R_{j0} \quad (5.6) \\ &= \ln \left(1 - \left(1 - (1 - p_i)^2 + \sum_{i=2}^{5} p_i^2 \right) \right). \\ &\quad - \ln R_{j0} \end{aligned}$$

This process is repeated for $k = 2, 3, 4$ to obtain

m_{jk}	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
$j=1$	1	1	0	0	0
$j=2$	1	1	0	0	0
$j=3$	1	1	0	0	0
$j=4$	1	1	1	0	0
$j=5$	1	1	1	0	0
$j=6$	1	1	1	0	0
$j=7$	1	1	1	0	0
$j=8$	1	1	0	0	0

Table 2: Integer Solution

$\triangle R_{j0}, \triangle R_{j1}, \triangle R_{j2}, \triangle R_{j3}$, and $\triangle R_{j4}$ for $j = 1, 2, 3, ..., 8$ stages. Thus, the objective function $f(m_{jk})$ given by (4.2) at stage $j, 1 \le j \le 8$, becomes $f(\triangle \ln R_{jk} m_{jk})$. The second constraint $\sum_{j=1}^{8} \sum_{k=0}^{8} \triangle g_{jk} m_{jk}$ is obtained using a similar procedure. The remaining constraints are interpreted as follows:

$$m_{jk} = 1, \quad k = 0 \quad (5.7)$$

ensures that there is at least one component at each stage.

$$m_{jk} - m_{jk-1} \le 0, \quad k = 1, \ldots, \tilde{m}_j, \ j = 1, \ldots, N \quad (5.8)$$

Here, $\tilde{m}_j$ is the maximum number of allowable redundant units at each stage and N is the total number of stages. We ensure that the k^{th} redundant unit is included only if $(k-1)^{th}$ redundant unit is included. In order to guarantee that each stage has at least one component, the variable m_{j0}, $1 \le j \le 8$, is initialized to 1. The remaining m_{jk}, ($1 \le j \le 8$, $1 \le k \le 4$) denote the additional number of redundant units at each stage. The total number of redundant units is the desired unknown. For a traditional SONET/SDH self-healing ring with 1 : 1 protection $m_{j0} = 1$, $m_{j1} = 1$, $1 \le j \le 8$, where $m_{j0} = 1$ designates the working component and $m_{j1} = 1$, designates the protection component. Both components are uniformly distributed throughout the ring. The solution obtained with integer programming using data from Table 1 is shown in Table 2. This table shows the integer solution $m_{jk} = (0/1)$, at each stage. That is , $m_{jk} = 1$ if a component is allocated or $m_{jk} = 0$ otherwise. The actual number of components, including the redundant units, at stage j is the sum of allocated units and is given by

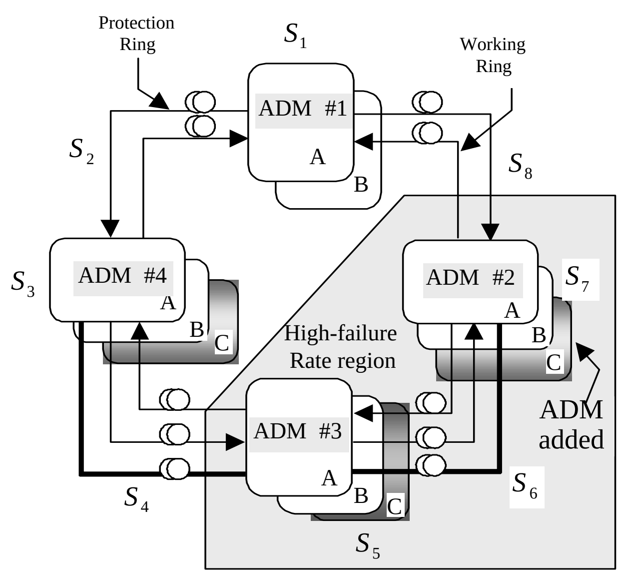

Figure 3: USHR in Figure 2 with optimum number of redundant ADMs and links

$$m_j = \sum_{k=0}^{4} m_{jk}, \quad 1 \le j \le 8. \quad (5.9)$$

Applying (5.9) to the integer solution in table 2 we have

$$\begin{array}{ccc} m_1 = 2 & & m_5 = 3 \\ m_2 = 2 & & m_6 = 3 \\ m_3 = 2 & m_7 = 2+1 \text{ (additional)} & \\ m_4 = 3 & & m_8 = 2. \end{array} \tag{5.10}$$

Upon comparing $\bar{m}_j$, $1 \leq j \leq 8$, given by (5.1) with m_j in (5.10), we note that an additional redundant unit is required in stages 4, 5, 6, and 7 $(m_{43} = 1, m_{53} = 1, m_{63} = 1, m_{73} = 1)$ to achieve the survivability level specified in table 1. Using the optimum number of components at each stage given by (5.10), we obtain the optimum survivable WDM ring network shown in Figure 3. The computed values for m_{63} and m_{73} are three and two, respectively. Since each fiber in stage 6 must be terminated by an ADM in stage 7, m_{73} must be adjusted such that $m_{63} = m_{73} = 3$ as shown in Table 2. The additional redundant unit added in stage 7 is also shown in Figure 3 .

m_{jk}	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
$j=1$	1	1	0	0	0
$j=2$	1	1	0	0	0
$j=3$	1	1	0	0	0
$j=4$	1	1	1	0	0
$j=5$	1	1	1	0	0
$j=6$	1	1	1	0	0
$j=7$	1	1	$0 \rightarrow \mathbf{1}^*$	0	0
$j=8$	1	1	0	0	0

Table 3: Adjusted Integer Solution required for Full Connectivity

6. Conclusion

In some networks, design engineers are willing to pay more to achieve a desirable survivability. Often, the willingness to pay more for a higher level of survivability is constrained by the physical characteristics of the networks and other factors such as cost. Achieving this objective requires a framework for estimating and predicting the number of redundant units. In networks such as SONET, the number of redundant units is kept uniform in the ring for simplicity. However, varying the redundancy along the network non-uniformly, in order to meet the network-wide requirements posed by a varying failure propensity, is a viable option in WDM networks. This paper provides a framework for such optimization of survivability in WDM networks.

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