Differential Equation Model for Accommodation Magnetization

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***Abstract*—We use the differential equation method of computing the accommodation magnetization in a modified Preisach model. We present the properties of this model for a Gaussian medium, and show that the resulting model has neither the congruency property nor the deletion property.**

***Index Terms*—Accommodation, hysteresis, magnetic media, Preisach modeling.**

I. Introduction

ACCOMMODATION is a rate-independent modification of minor loops cycled between the same two fields. It differs from thermal magnetic aftereffect, which is rate dependent. Both effects cause minor loops to drift toward a limit, and both effects are maximum near the remanent coercivity. In fact, if one cycles the minor loops at a constant rate, the two effects would be almost indistinguishable. The accommodating loop would drift depending upon how many times one went around the minor loop, while the other would drift in time, even if there were no change in applied field.

The concept of accommodation has been known for some time [1]. However, the first model for calculating it was presented more recently [2]. Several papers have been written on measuring it such as [3], [4], and recently, rotational accommodation has been reported [5], that is minor loops that do not close due to a rotating magnetic field. Since the Preisach model cannot compute accommodation due to the deletion property, it has to be modified. The principle of this model is the same as that in [2], but is easier to compute. It is based on a particular instability of the Preisach function as the magnetization changes. Other instabilities, such as that in the moving model and in the variable variance model, do not lead to accommodation.

The accommodation model is based upon the interaction between hysterons, so that when one hysteron switches, it affects the switching characteristics of its nearest neighbor. It is noted that hysterons that are physically close to each other can be quite far apart in the Preisach plane. Furthermore, when a neighbor switches, the hysteron characteristic can change appreciably. Due to this motion, the division of the Preisach plane into saturated regions, separated by a boundary consisting of vertical and horizontal segments whose positions depend upon the sequence of past applied field extrema, is violated. Thus, one can end up with particles in a region of the Preisach plane with opposite magnetization from its neighbors and with applied fields too weak to remagnetize them.

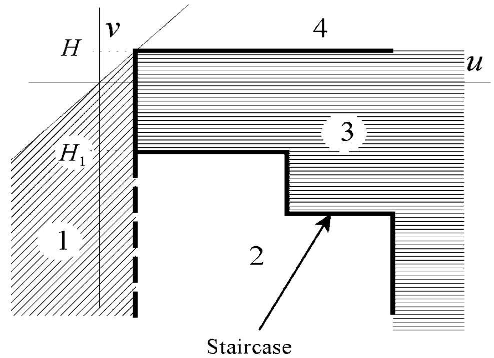

Fig. 1. Division of the Preisach plane for an increasing field.

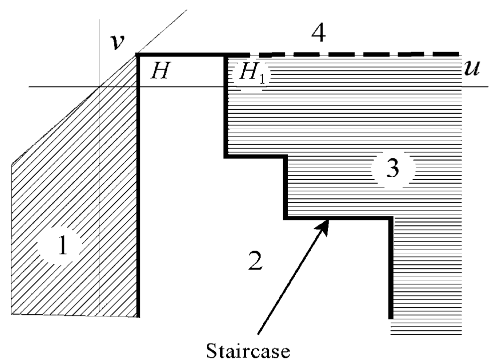

Fig. 2. Division of the Preisach plane for a decreasing field.

In this paper, we will use the differential equation method of computing the magnetization from the Preisach model as the basis for the accommodation model, since both the classical model and the accommodation model divide the Preisach plane in the same way. The difference between the models is that in the classical Preisach model, all regions are either saturated in the positive or the negative direction. In the accommodation model, the Preisach plane is divided into four regions as shown in Figs. 1 and 2 for increasing applied fields and decreasing applied fields, respectively.

The hysterons in region 1 all have up-switching fields less than the applied field, and hence are saturated in the positive direction in both models. Similarly, the hysterons in region 4 all have down-switching fields greater than the applied field, and hence are saturated in the negative direction in both models. The

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boundary between regions 1 and 2 and the boundary between regions 3 and 4 are determined by the current value of the applied field. The staircase between regions 2 and 3 contains the history of the magnetizing process.

The two models differ in regions 2 and 3 where all the hysterons should be unaffected by the applied field. In the classical model they remain saturated, but in the accommodation model they are subject to accommodation. The difference between these two regions is that in the classical Preisach model they would have been saturated in different directions. These magnetizations are the initial values for the accommodation model and are slowly diluted by the magnetizing process.

For simplicity, we will use the classical Preisach model as the basis. Thus, we do not have any reversible component of magnetization, and there is no motion of the Preisach function in the plane. These limitations can be easily removed. Furthermore, we shall illustrate the model by assuming that the Preisach function is Gaussian. The model can easily be modified for any shape Preisach function. However, with all these limitations, the accommodation model has neither the congruency property nor the deletion property that the classical Preisach model has.

II. Calculation of Accommodation

For a Gaussian medium, it has been shown in [6] that the susceptibility of the decreasing applied field χ_0 of the nonaccommodation model is where

$$\chi_0 = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(H-\bar{h}_k)^2}{2\sigma^2}\right] \left[\operatorname{erf}\left(\frac{H_1+\lambda H-\kappa\bar{h}_k}{\tau\sqrt{2}}\right) - \operatorname{erf}\left(\frac{(1+\lambda)H-\kappa\bar{h}_k}{\tau\sqrt{2}}\right)\right] \quad (1)$$

where

$$\lambda = \frac{\sigma_k^2-\sigma_i^2}{\sigma^2}, \quad \sigma = \sqrt{\sigma_i^2+\sigma_k^2}, \quad \tau = \frac{2\sigma_i\sigma_k}{\sigma}, \quad \kappa = 1-\lambda \quad (2)$$

$\bar{h}_k$ is the average critical field of the hysterons, and σ_i and σ_k are the standard deviation of the interaction fields and the critical fields, respectively. The term H_1 is the value of the previous extrema. For a sequence of Hs, the extrema are pushed onto a stack. If a maximum follows a previous smaller maximum, then the previous maximum is popped from the stack. There is a similar formula for increasing applied field.

At a particular step in a magnetizing process with the applied field increasing, the Preisach plane is divided into four regions, as shown in Fig. 1. Let p_k be the integral of the normalized Preisach function over region k. Then

$$\sum_{k=1}^{4} p_k = 1. \quad (3)$$

The classical Preisach model computes the magnetization using

$$m = p_1 + p_2 - p_3 - p_4. \quad (4)$$

Thus

$$p_1 + p_2 = \frac{1+m}{2} \quad \text{and} \quad p_3 + p_4 = \frac{1-m}{2}. \quad (5)$$

For a Gaussian medium, it is shown in [6] that with an applied field H, we have

$$p_1 = \frac{1+\operatorname{erf}\left(\frac{H-\bar{h}_k}{\sigma\sqrt{2}}\right)}{2} \quad \text{and}$$
$$p_4 = \frac{1-\operatorname{erf}\left(\frac{H+\bar{h}_k}{\sigma\sqrt{2}}\right)}{2}. \quad (6)$$

So it follows that

$$p_2 = \frac{1}{2}\left[m - \operatorname{erf}\left(\frac{h-\bar{h}_k}{\sigma\sqrt{2}}\right)\right] \quad (7)$$
$$p_3 = \frac{1}{2}\left[\operatorname{erf}\left(\frac{h+\bar{h}_k}{\sigma\sqrt{2}}\right) - m\right]. \quad (8)$$

These values are valid for both the classical Preisach model and for the accommodation model.

Let Q_k be the magnetic state of region k, and p_k be the integral of the normalized Preisach function in region k. For the classical Preisach model, $Q_1 = Q_2 = 1$ and $Q_3 = Q_4 = -1$. Thus, m_k, the contribution to the magnetization in region k, is $p_k Q_k$. For the accommodation model, the change in the Qs is caused by the motion of hysterons in the Preisach plane. In particular, we are concerned with the hysterons that move from one region into others. The average magnetization of all the hysterons is m. Thus, if there is a change in the applied field, some dilution takes place and we model the rate of change of the Qs by

$$\frac{dQ_2}{dH} = (m - Q_2)\beta\left|\frac{dm}{dH}\right| \quad (9)$$

and

$$\frac{dQ_3}{dH} = (m - Q_3)\beta\left|\frac{dm}{dH}\right|. \quad (10)$$

The factor β measures the effective motion of hysterons whenever the magnetization changes and is a measure of the stability of the Preisach function. If $\beta = 0$, the function is stable and the Qs do not change when the magnetization changes and there is no accommodation. There is no change in Q if the magnetization does not change; hence, the rate of change is proportional to the magnitude of the rate of change of m. The effect of (9) and (11) is that in general Q_2 drifts from $+1$ to m and Q_3 drifts from -1 to m.

There is an additional correction to the Qs because we choose to assign a single value of Q to a region. To distinguish that from the previous Q, we shall use its average and denote it by $\langle Q \rangle$. There is a change whenever a region grows by annexing part of an adjacent region. For example, when ΔH is negative and region 2 grows at the expense of region 1. The region that it annexes has a $Q = 1$, but the particles in that region do not experience a field large enough to switch them. Thus, the mean Q

increases from Q_2 to the weighted average of Q_2 and 1. Therefore, if the change in p_1 is negative, then

$$\langle Q_2\rangle = \frac{Q_2 p_2 - \Delta p_1}{p_2 - \Delta p_1} \quad \text{if } \Delta p_1 < 0. \tag{11}$$

It is seen that in the limit as p_2 goes to 0, $\langle Q_2\rangle$ goes to 1. However, if Δp_1 is also small, (11) can become indeterminate, so in an implementation of a program, whenever p_2 becomes small, it is best to explicitly set $\langle Q_2\rangle$ equal to 1.

Similarly, for an increasing field, region 3 grows at the expense of region 4, and we have

$$\langle Q_3\rangle = \frac{Q_3 p_3 + \Delta p_4}{p_3 - \Delta p_4} \quad \text{if } \Delta p_4 < 0. \tag{12}$$

In the limit of small p_3, we set $\langle Q_3\rangle = -1$. It is noted that region 2 only grows at the expense of region 1 whenever H decreases, and region 3 only grows at the expense of region 4 whenever H increases. These are the only two corrections that have to be made.

In the accommodation model, $\langle Q_2\rangle$ and $\langle Q_3\rangle$ are diluted by hysterons of the opposite magnetization. At this point, we can compute the magnetization with the accommodation model in two ways. Instead of using (4), we now compute the net normalized magnetization by substituting these $\langle Q\rangle$s into

$$m = \sum_{k=1}^{4} \langle Q_k\rangle p_k \tag{13}$$

where in the accommodation model, the ps are the same as in the nonaccommodating model, but the Qs are different. It must be noted that as the process continues both the ps and the Qs change, making it difficult to use (14).

Alternately, we can compute the magnetization for the accommodating model from the susceptibility. Using the above $\langle Q\rangle$s, we can modify the susceptibility for $\Delta H > 0$ as

$$\chi = \frac{1-\langle Q_3\rangle}{2}\chi_0 + \frac{1-\langle Q_2\rangle}{2\sigma\sqrt{2\pi}} \exp\left[-\frac{(H-\bar{h}_k)^2}{2\sigma^2}\right] \times \operatorname{erf}\left(\frac{H_1 + \lambda H + \kappa \bar{h}_k}{\tau\sqrt{2}}\right) + p_2 \frac{d\langle Q_2\rangle}{dH} + p_3 \frac{d\langle Q_3\rangle}{dH} \tag{14}$$

with a similar expression for $\Delta H < 0$. For the accommodating model, there are four terms to the susceptibility above. The first two in (14) are along the line $u = H$ and are due to the change in boundary between the regions. The first, from $v = H_1$ to $v = H$, is similar to the only contribution to the susceptibility of the nonaccommodating model, except that instead of the Q changing from -1 to 1, now it changes from Q_3 to 1. The other terms are zero in the nonaccommodating model. Along the remainder of the line $u = H$, that is from $v = -\infty$ to $v = H_1$, now Q changes from Q_2 to 1, giving rise to the second term in (14). The last two terms are due to the change in Q in the various regions. The third term in (14) is the contribution in region 2 of the change in Q_2, and the fourth term in (14) is the contribution in region 3 of the change in Q_3. There is no contribution to the susceptibility from regions 1 and 4, since the Qs do not change there. Note that if Q_2 and Q_3 are independent of applied field and are equal to $+1$ and -1, respectively, then this reduces to the nonaccommodating model, and $\chi = \chi_0$.

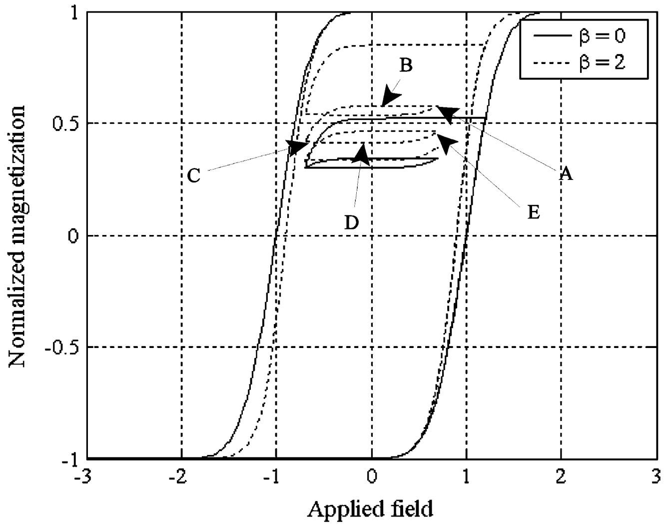

Fig. 3. Hysteresis loops from the accommodation model with $\beta = 2$ (dotted line) and for no accommodation $\beta = 0$ (solid line).

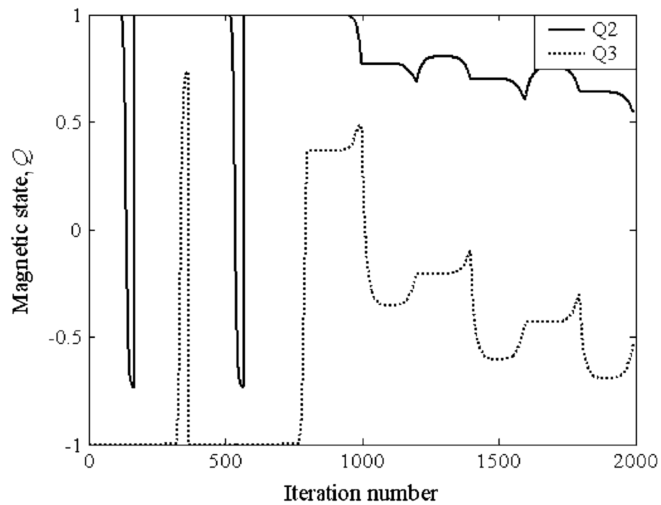

Fig. 4. Variation of Q_2 and Q_3 with time for the magnetizing process above.

III. Model Results

We have developed a MATLAB program for this algorithm and included it in the Appendix. This program applies a sequence of initialization fields followed by cycling between a pair of extrema a certain number of times. For simplicity, we did not include a reversible component in the model, nor did we use the moving model. The interval between a pair of extreme fields was divided into a fixed number of intermediate fields.

The magnetization is computed by first computing the ps using (6)–(8). We then computed the Qs using (11) and (12), and finally computed the magnetization using the modified Qs and the new magnetization is computed from the susceptibility using (14). We used the Runge–Kutta algorithm to solve the differential equation for the regions. At each step, we used this program to obtain the following results.

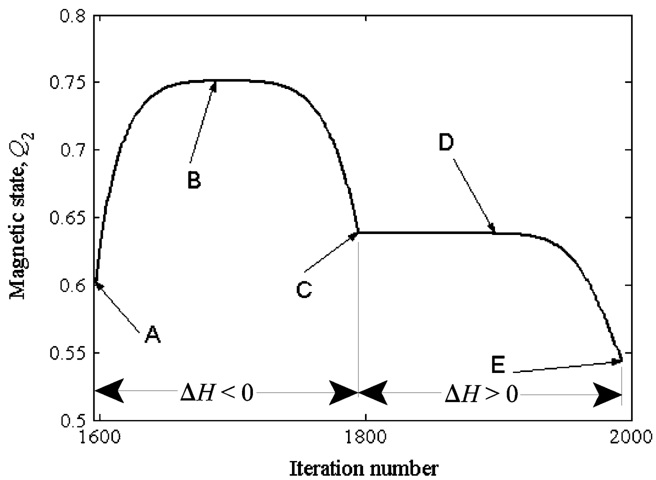

Fig. 5. Variation of Q_2 over a single cycle.

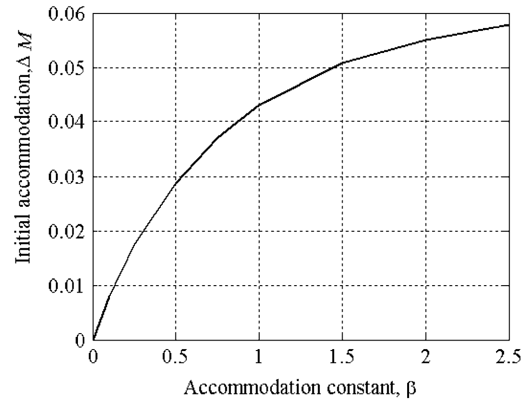

Fig. 6. Variation of ΔM with β, for $\bar{h}_k = 1, \sigma_k = 0.2$, and $\sigma_i = 0.2$.

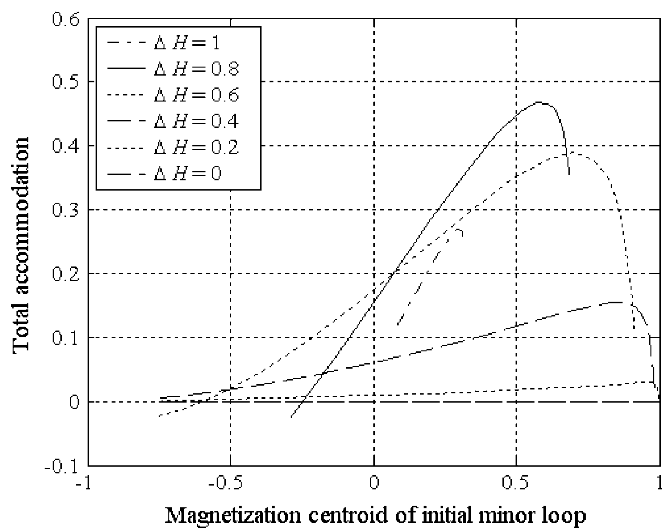

Fig. 7. Variation in the total accommodation as a function of the centroid of the initial loop for different half-widths of minor loops and initial magnetizations for $\beta = 2.0$.

A sequence of fields was applied to the model and the results are illustrated in Fig. 3. The sequence of fields, normalized to the coercivity, first drew the major loop, then went from negative saturation to 1.2, and then went back and forth between -0.7 and $+0.7$. It is seen that the effect of increasing β from 0 to 2 has the effect of decreasing the width of the major loop. This is due to the lower magnetization magnitude for a given field, because of the decrease of the magnitudes of Q_2 and Q_3 from 1. This can be seen in Fig. 4 from the plot of Q_2 and Q_3 with the step number of the calculation. The negative "spike" in Q_2 occurs during the descending portion of the major loop and positive "spike" in Q_3 occurs during the ascending portion of the major loop. After computing the major loop, we drew a sequence of minor loops, and it is seen that there is a gradual downward drift in the Qs which is caused by accommodation.

In order to explain the variation in the Qs, we have taken a single cycle of Q_2 and expanded it as shown in Fig. 5. We see that there are four distinct regions: from A to B, from B to C, from C to D, and from D to E. In the first region, from A to B, Q_2 increases because the region 2 is growing at the expense of 1, which is saturated positively thus increasing the average value of Q_2, that is, the effect of (11) is felt. In the second region, from B to C, there is no change at first, because the magnetization is not changing, and then as the magnetization starts changing, Q_2 decreases until the reversal point C. In the third region, from C to D, there is no change in the magnetization and the region does not change in size, so Q_2 does not change. Finally, from D to E, the fourth region, the magnetization changes and so Q_2 again decreases. Due to accommodation the value of Q_2 at the end of the cycle, point E, is different from the value at the beginning of the cycle, point A. Eventually, when the system achieves an equilibrium, the minor loop will be in a limit cycle with the value of Q_2 at the end of the cycle equal to value at the beginning of the cycle.

We will define the amount of accommodation as the difference in the magnetization between the beginning and the end of the first accommodation loop. The amount of accommodation depends directly on the value of β, but in addition, it depends upon the width of the accommodating loop and the initial magnetization at the start of the accommodating cycles. In particular, although changing β changes the shape of the major loop, there is no accommodation of the major loop. Also, as the width of the minor loop goes to zero, there is no accommodation, since the magnetization does not change.

The amount of accommodation depends upon the sequence of applied fields. We define the initial accommodation ΔM as the maximum difference between the end of the second-order reversal curve and the beginning of the first-order reversal curve at the same applied field. We maximized the difference in magnetization by adjusting the accommodating field limits. A plot of the β dependence on ΔM is shown in Fig. 6. Once the other parameters of the Preisach model have been obtained, this type of plot should be useful to estimate the value of β for a real material from measurements. We note that this plot would be different for different values of $\sigma_{\rm i}$ and $\sigma_{\rm k}$, which in these calculations were both set to 0.2 of H_C, which was normalized.

In order to characterize the total accommodation, we have plotted in Fig. 7 the total accommodation, that is the change in the magnetization of the centroid of a symmetrical minor loop for different values of applied field half-widths, ΔH. To obtain

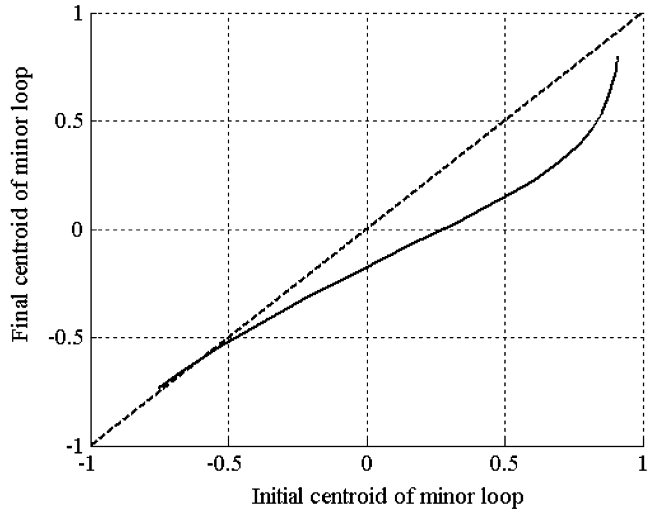

Fig. 8. Centroid of a limit cycle as a function of the centroid of the initial minor loop with (solid line) and without accommodation (dotted line).

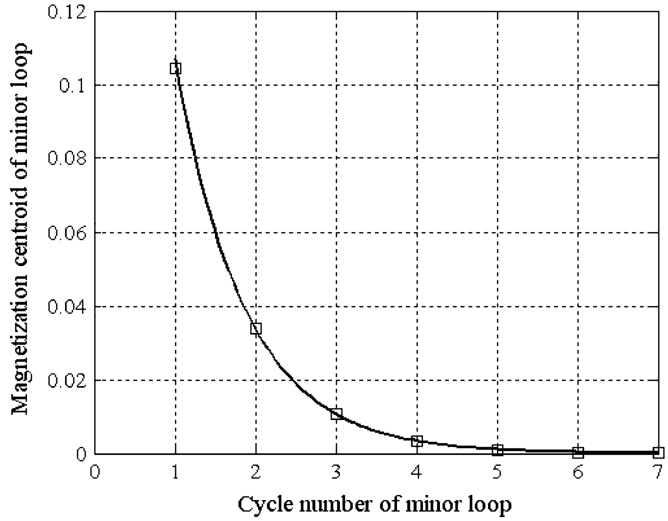

Fig. 9. Motion of centroid of minor loop with cycle number. The squares are the computed numbers and the solid line is the exponential fit.

this data, we started from different fields, H_1, on the ascending major loop, went to $-\Delta H$, and then cycle between ΔH and $-\Delta H$. The centroid of the limit cycle is different depending upon H_1, because the initial values of p_2 and p_3 are different due to the different initial magnetizations. This is an important difference between this model and [2]. In that paper, regions 2 and 3 were combined in a single region and the final value for Q of that region was independent of the starting point.

We note that for $\Delta H = 0$, there is no change in the centroid. The range of the initial centroid is limited by the major loop so that as ΔH increases, the possibility of fitting a wider minor loop inside the major loop becomes more limited; however, the total accommodation increases until the extrema of the minor loop approaches fields near the coercivity. The maximum total accommodation from an ascending major loop occurs when the initial loop is near but not touching the descending major loop. The maximum accommodation occurs around $\Delta H = 0.8$, the solid line in Fig. 7, which is approximately the same as the coercivity for this set of parameters. Even though we set the coercivity of the classical model $(\beta = 0)$ to unity, when we increased β to 2, the coercivity decreased to 0.8.

In Fig. 8, we plot the final centroid of the minor loop as a function of the initial centroid for $\Delta H = 0.6H_c$. We see that starting from the ascending major loop, the accommodation is downward until the initial centroid falls below $-0.35M_S$. This is about as far as we can go with the initial centroid since that is the value that one obtains when one starts the minor loop at ΔH on the major loop.

In order to show how the accommodation approaches the limit cycle, we have plotted the centroid of the minor loops as a function of the cycle number in Fig. 9. To obtain this curve, we used the values of $\beta = 2$ and $\Delta H = 1$. It is seen that the motion of the centroid decreases exponentially with cycle number.

We note that if we plot the height of the minor loops as a function of the cycle number, for $\beta = 2$ and $\Delta H = 1$, as shown in Fig. 10, the initial minor loop is taller than the subsequent loops, indicating that this model does not have the congruency property. In fact, the height of the limit cycle is about half of the height of the initial loop. Furthermore, repeated application of the same field does not delete its effects, since it is in a different state. Thus, this model does not have the deletion property either.

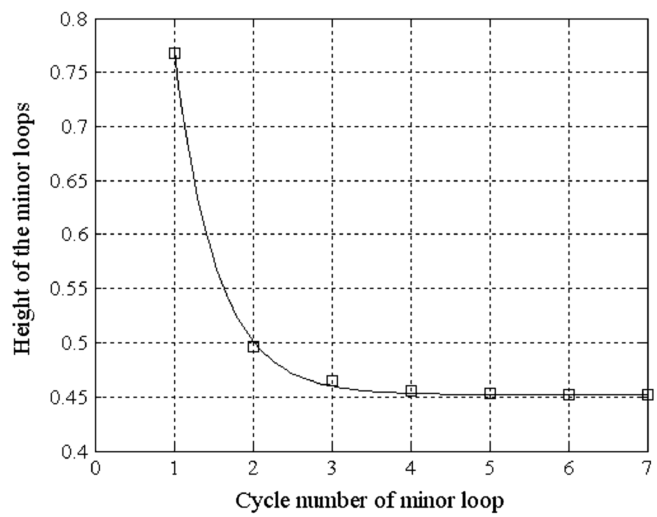

Fig. 10. Exponential fit to minor loop height variation with cycle number.

IV. CONCLUSION

We have presented a differential equation model for accommodation and the results of that model for a Gaussian medium. The model is limited by neither the congruency property nor the deletion property. The amount of accommodation depends upon the width of the minor loops and their starting point. We present these results to indicate what experiments are necessary to validate this model. These results are very promising and we intend in the future to generalize the model to include state-dependent reversible magnetization, moving Preisach function and a variable variance model by using an operative field and to include thermal magnetic aftereffect.

APPENDIX

MATLAB Program Used To Obtain Fig. 3

```
% accommodation.m
% by Edward Della Torre, Levent Yanik, A. Emre Yarimbiyik and Michael J. Donahue
% Note that the program has no reversible component.
% The program normalizes magnetization to the saturation magnetization and the applied
field to the coercivity.
% The case without accommodation (beta = 0) is plotted for convenience.
clear
% Parameters which will be kept constant throughout the program
sigma_i = .2; sigma_k = .2;  % Standard deviations in the critical field
hkbar = 1;          % Mean critical field
sigma = sqrt((sigma_i)^2 + (sigma_k)^2);
lambda = (sigma_k^2 - sigma_i^2)/sigma^2;
opl = 1 + lambda;
khb = (1 - lambda) * hkbar;
ts2 = (2 * sigma_i * sigma_k)/sigma * sqrt(2);
twos2 = 2 * sigma^2;
pref = (1/(sigma * sqrt(2 * pi)));
% Initializations
sp = 2; stack = 1e2 * [-1 1];
chi = [0 0 0 0];
rk = [0 0 0 0];
m = [1 1]; Q2 = [1 1]; Q3 = [-1 -1];
p(1) = 1; p2(1) = 0; p3(1) = 0; p4(1) = 0;
% Default sequence
% ---------------------------------------------------------------------
H_SEQ = [3 -3 3 -1.2]; % Initial sequence
beta = 2;             % Accommodation constant
no_ac_lp = 5;          % Number of accommodation cycles
h01 = -0.7;            % Extrema of accommodation cycles
h02 = 0.7;             % Extrema of accommodation cycles
resolution = 100;
% ---------------------------------------------------------------------
% Display default input sequence/user input
  disp('DEFAULT PARAMETERS AND SEQUENCE OF FIELDS:');
  disp('1- Initial sequence: [3 -3 3 -1.2]');
  disp('2- Accommodation constant: 2');
  disp('3- Number of accommodation cycles: 5');
  disp('4- Extremas of accommodation cycles: h01 = -0.7 and h02 = 0.7');
  disp('5- Resolution: 100');
user_input = 10;
while user_input ~= 0
user_input = input('Choose corresponding number to change parameter (enter 0 if you
are done):')
  if user_input == 1
    H_SEQ = input('Enter initial sequence between brackets:');
  elseif user_input == 2
    beta = input('Enter accommodation constant (>0):');
  elseif user_input == 3
    no_ac_lp = input('Enter number of accommodation cycles (>0):')
  elseif user_input == 4
    h01 = input('Enter minima of accommodation cycles (h01):')
    h02 = input('Enter maxima of accommodation cycles (h02):')
  elseif user_input == 5
    resolution = input('Enter resolution (>100 for more appropriate results):')
  elseif user_input ~= 0
    disp('PLEASE ENTER A NUMBER FROM 0 to 5 NEXT TIME.');
  end
end
% ---------------------------------------------------------------------
for kk = 1 : no_ac_lp
    H_SEQ = [H_SEQ h01 h02];
end
SEQ_LEN = length(H_SEQ);
Hint = 10 * [9 8 7 6];
for k = 1:SEQ_LEN - 1
    temp = linspace(H_SEQ(k), H_SEQ(k + 1), resolution);
    Hint = [Hint temp(2 : length(temp))];
end
% Calculate hysteresis trajectory
for i = 2:length(Hint) - 1
    kk(i) = i;
  deltaH(i + 1) = Hint(i + 1) - Hint(i);
  pr = deltaH(i + 1) * deltaH(i);
if pr < 0
    sp = sp + 1;
    stack(sp) = Hint(i);
  end
  H1 = stack(sp);
  difH = Hint(i + 1) - stack(sp - 1);
  if deltaH(i) > 0    % increasing applied field
    if pr >= 0 & difH > 0
      sp = sp - 2; stack = stack(1:sp); H1 = stack(sp);
    end
    e1 = erf((opl * Hint(i + 1) + khb)/ts2);
    e2 = erf((H1 + lambda * Hint(i + 1) + khb)/ts2);
    chi(i + 1) = pref * exp(-(Hint(i + 1) - hkbar)^2/twos2) * (e1 - e2);
    chi_halfstep = pref * exp(-(Hint(i) + deltaH(i)/2 - hkbar)^2/twos2) *
(e1 - e2);
  else           % decreasing applied field
    if pr >= 0 & difH < 0
      sp = sp - 2; stack = stack(1:sp); H1 = stack(sp);
    end
    e1 = erf((H1 + lambda * Hint(i + 1) - khb)/ts2);
    e2 = erf((opl * Hint(i + 1) - khb)/ts2);
    chi(i + 1) = pref * exp(-(Hint(i + 1) + hkbar)^2/twos2) * (e1 - e2);
    chi_halfstep = pref * exp(-(Hint(i) + deltaH(i)/2 + hkbar)^2/twos2) *
(e1 - e2);
  end
% Applying Runge–Kutta methods for solving differential equation
rk(1) = chi(i);
rk(2) = chi_halfstep;
rk(3) = chi_halfstep;
rk(4) = chi(i + 1);
avr_deriv = (rk(1) + 2 * rk(2) + 2 * rk(3) + rk(4))/6;
m(i + 1) = m(i) + deltaH(i) * avr_deriv;
if pr < 0, m(i + 1) = m(i); end
lenm = length(m);
% Including Accommodation
bdMabsdH(i) = beta * abs((m(i + 1) - m(i)));
p1(i) = (1 + erf((Hint(i) - hkbar)/(sigma * sqrt(2))))/2;
p2(i) = ((1 + m(i))/2) - p1(i);
p4(i) = (1 - erf((Hint(i) + hkbar)/(sigma * sqrt(2))))/2;
p3(i) = ((1 - m(i))/2) - p4(i);
```

```
dp4 = p4(i) - p4(i - 1); dp1 = p1(i) - p1(i - 1);
ma(i) = p1(i) + p2(i) * Q2(i) + p3(i) * Q3(i) - p4(i);
if p2(i) > .0005, Q2(i + 1) = Q2(i) + (ma(i) - Q2(i)) * bdMabsdH(i); else,
Q2(i + 1) = 1; end
if p3(i) > .0005, Q3(i + 1) = Q3(i) + (ma(i) - Q3(i)) * bdMabsdH(i); else,
Q3(i + 1) = -1; end
if dp1 < 0, Q2(i + 1) = (p2(i) * Q2(i + 1) - dp1)/(p2(i) - dp1); end
if dp4 < 0, Q3(i + 1) = (p3(i) * Q3(i + 1) + dp4)/(p3(i) - dp4); end
% Plot routine
figure(1)
set (1, 'Color', 'white')
lenm = length(m); lenma = length(ma);
plot(Hint(1:lenm), m, 'b', Hint(1:lenma), ma, 'r', Hint(lenm), m(lenm), 'b*', Hint(lenma),
ma(lenma), 'r*')
betax = ['\beta = 'num2str(beta)]; axis([-3 3 -1 1]), grid, legend('\beta =
0', betax)
xlabel('Applied field'), ylabel('Normalized magnetization')
pause(0)
end
```

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