MULTIRESOLUTION REPRESENTATION OF URBAN TERRAIN BY L_1 SPLINES, L_2 SPLINES AND PIECEWISE PLANAR SURFACES

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Abstract

Cubic L_1 and L_2 interpolating splines based on C^1 smooth, piecewise cubic Sibson elements on a tensor-product grid are investigated. Computational tests were carried out for an 800 m by 800 m area of Baltimore, Maryland represented by an 801×801 set of 1-meter-spacing (posting) data set. Interpolating splines at coarser resolutions were computed along with ℓ_1, ℓ_2, and ℓ_∞ errors relative to the 800 m by 800 m data set. Piecewise planar interpolations at the coarser resolutions were also computed along with the above errors for comparative purposes.

1. Introduction

Currently, irregular geometric surfaces and, in particular, terrain are often represented by piecewise planar surfaces on triangulated networks (often called "TINs" or "triangulated irregular networks" when the triangles are irregular in shape). However, generating an accurate, error-free surface within a triangulated network framework requires extremely fine triangulations in regions of rapid change and therefore storage and manipulation of huge amounts of data. The conceptual superiority of using smooth surfaces for representation of terrain and of irregular geometric surfaces in general has long been recognized. Recently, a new class of cubic "L_1" splines that perform well in preserving the shape of data sets has been developed (Lavery, 2000a, 2000b, 2001). We will compare L_1 splines with a class of conventional "L_2" splines and with piecewise planar surfaces.

2. Cubic L_1 Splines, Cubic L_2 Splines and Piecewise Planar Surfaces

The cubic splines $z(x, y)$ used in this paper consist of C^1 smooth, piecewise cubic Sibson elements (Han and Schumaker, 1997; Lavery, 2001) on regularly spaced rectangular grids with nodes $(x_i, y_j) = (c_x i, c_y j), i = 0, 1, ..., I, j = 0, 1, ..., J$, where c_x and c_y are known constants. These cubic splines, which exist on the domain $D = (x_0, x_I) \times (y_0, y_J)$, are characterized by their values $z_{ij} = z(x_i, y_j)$ and the values of their derivatives $z^x_{ij} = \frac{\partial z}{\partial x}(x_i, y_j)$ and $z^y_{ij} = \frac{\partial z}{\partial y}(x_i, y_j)$ at the nodes (x_i, y_j). At each node (x_i, y_j), the elevation z_{ij} is given. To calculate a cubic interpolating spline, one must compute the values of the derivatives z^x_{ij} and z^y_{ij}. The z^x_{ij} and z^y_{ij} of a cubic L_1 spline are calculated by minimizing the following weighted sum of the absolute values of the second derivatives of the spline and a regularization term

$$\iint_D \left[\left|\frac{\partial^2 z}{\partial x^2}\right| + 2\left|\frac{\partial^2 z}{\partial x \partial y}\right| + \left|\frac{\partial^2 z}{\partial y^2}\right|\right] dx\, dy + \varepsilon \sum_{i=0}^{I} \sum_{j=0}^{J} \left[|z^x_{ij}| + |z^y_{ij}|\right] \tag{1}$$

over all Sibson-element surfaces z that interpolate the data z_{ij}. The z^x_{ij} and z^y_{ij} of a conventional cubic L_2 interpolating spline are calculated by minimizing the following weighted sum of the squares of the second derivatives of the spline and a regularization term

$$\iint_D \left[\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + 4\left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 z}{\partial y^2}\right)^2\right] dx\, dy + \varepsilon^2 \sum_{i=0}^{I} \sum_{j=0}^{J} \left[(z^x_{ij})^2 + (z^y_{ij})^2\right] \tag{2}$$

over all Sibson-element surfaces z that interpolate the data z_{ij}. The regularization parameter ϵ in (2) is the same as the ϵ in (1). The integral in (2) was discretized in the same manner as the integral in (1). A piecewise planar surface z is calculated by dividing each rectangle $[x_i, x_{i+1}] \times [y_j, y_{j+1}]$ into two triangles by drawing the diagonal from the corner (x_i, y_j) to the corner (x_{i+1}, y_{j+1}) and letting z inside each triangle be the linear interpolant of the data at the three corners of the triangle.

3. Multiresolution Representation of Urban Data

Computational tests were carried out on a set of 801×801 data that consists of an 800 m by 800 m portion of a 1000×1000 set of 1-meter-posting digital elevation data for downtown Baltimore, Maryland surrounding Oriole Park at Camden Yards. The data set was obtained from the Joint Precision Strike Demonstration Project Office (JPSD PO) Rapid Terrain Visualization (RTV) ACTD. In Fig. 1, we present the surface for the 800 m by 800 m, 1-meter-posting subset of the Baltimore data set mentioned above.

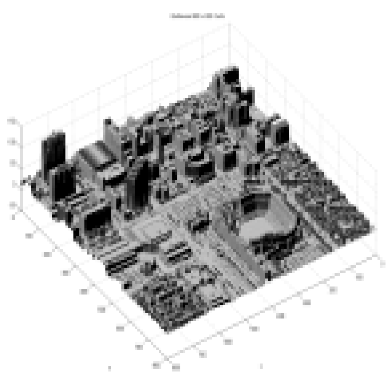

Fig. 1. Surface based on 1-meter-posting for 800 m by 800m area of Baltimore, Maryland.

The first row of the tables below contain the spacings, designated by "s" in meters. In the left hand column of the tables the following notation is used: $\Delta\ell_1 = \|z_{[L_1,s]} - \text{data}\|_{\ell_1}$, $\Delta\ell_2 = \|z_{[L_1,s]} - \text{data}\|_{\ell_2}$, $\Delta\ell_\infty = \|z_{[L_1,s]} - \text{data}\|_{\ell_\infty}$, where data is the original 801×801 data used to plot Fig. 1.

Table 1. Norms of differences between cubic L_1 splines on coarse grids and original data.

s (m)	5	10	20	40
$\Delta\ell_1$	1.314	2.207	3.709	5.987
$\Delta\ell_2$	3.640	5.058	7.582	11.34
$\Delta\ell_\infty$	94.50	108.4	103.1	104.2

Table 2. Norms of differences between cubic L_2 splines on coarse grids and original data.

s (m)	5	10	20	40
$\Delta\ell_1$	1.488	2.450	4.014	6.305
$\Delta\ell_2$	3.726	5.144	7.702	11.69
$\Delta\ell_\infty$	101.2	113.0	98.50	104.1

Table 3. Norms of differences between piecewise planar surfaces on coarse grids and original data.

s (m)	5	10	20	40
$\Delta\ell_1$	1.389	2.346	3.902	6.099
$\Delta\ell_2$	3.690	5.130	7.545	11.15
$\Delta\ell_\infty$	89.82	94.85	95.57	101.1

Overall, this evidence indicates that L_1 splines preserve shape better for this terrain data set than do L_2 splines. With respect to the piecewise planar surface interpolation the criteria for preservation of shape depends strongly on the measure of difference between the interpolation and the original data. Piecewise planar performs better than the L_2 spline for this data set given the three measures of performance used. The comparison with the L_1 spline depends on the error measure.

4. Conclusion

The results in this paper indicate that L_1 splines are excellent candidates for representation of urban terrain. In this article, we have investigated the approximation properties of L_1 interpolating splines on increasingly coarse grids, ignoring intermediate data. In the future, computational experiments with L_1 smoothing splines, which use all of the data, including the data between the coarse-grid nodes, will be carried out. It is expected that the performance of L_1 splines will be further enhanced by doing this.

References

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