Moment-Preserving Modeling with Image Applications

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Abstract *Linear least-square modeling is a powerful modeling approach. Optimal piece-wise constant modeling is an important special case. In this paper we will develop a piece-wise constant modeling method that preserves the regional means and regional first absolute central moments, thus preserving important visual properties in images. The method will be used for two practical image applications: (1) image preprocessing techniques for better edge detection, and (2) dynamic range adjustment of images to meet display constraints. Our method yields better performance than existing standard techniques for those applications.*

Keywords: modeling, moment preservation, edge detection, dynamic range adjustment.

1 Modeling with Regional Mean-Moment Preservation

Linear least-square modeling is a powerful tool [4] and can have important applications in imaging. One special case is optimal piece-wise constant modeling: an image is divided into several subregions, and then the pixel values in each subregion are replaced by an optimal constant value. Optimality is with respect to the mean-square error, or signal-to-noise ratio (SNR), between the original image and its piece-wise constant model.

One key aspect to piece-wise constant modeling is how to partition an image into subregions so that the modeling preserves certain statistical properties that are relevant to visual perception. Two statistical measures worth preserving are the regional (i.e., local) means and regional first absolute central moments. This kind of modeling and two important applications, namely, edge detection and dynamic range adjustment, are the focus of this paper.

This section will establish that optimal piece-wise constant modeling is nothing but piece-wise averaging. Afterwards, consideration is given to region partitioning under which piece-wise constant modeling preserves the regional means and first absolute central moments. The next two sections will address the two aforementioned applications.

Theorem 1 *Let F be a function (an image) over a region R, and assume that R is partitioned into p non-overlapping subregions R_j for $j = 1, 2, ..., p$, where R_j has N_j points. The optimal least-square approximation (or model) M of F where M is constant over each subregion satisfies:*

$$\textit{for all } X_k \in R_j,\ M(X_k) = \frac{1}{N_j} \sum_{X_i \in R_j} F(X_i). \tag{1}$$

That is, $M(X_k)$ is the average of F over subregion R_j.

Proof: The proof is straightforward and is thus omitted. ∎

Next we address mean- and moment-preserving partitioning. It will be asserted that the mean is preserved, that is, $mean(M) = mean(F)$, regardless of the partitioning of R. Moreover, if the subregions are chosen in a certain way, the first absolute central moment (FACM) is also preserved, where $FACM(F) = \frac{1}{N}\sum_{i=1}^{N} |F_i - mean(F)|$. Since the 1st absolute central moment is a measure of average variation around the mean, it has statistical and visual significance, making its preservation a desirable property.

We will present an algorithm to partition a region R into subregions in such a way that piece-wise modeling preserves the first absolute central moment. Indeed, the partitioning algorithm will guarantee that the subregional means and first absolute moments will be preserved, thus maintaining certain localization features. We call such partitioning *moment-preserving partitioning* (MPP). The partitioning algorithm works recursively to partition R into a collection P of any arbitrary number $p \geq 2$ of non-overlapping subregions.

Algorithm MPP(**Input:** R, p; **output:** P)
begin

1. Let $P = \{R\}$.
2. Choose any subregion S in P, and partition it into two subregions S_1 and S_2 where $S_1 = \{X_i \in S \mid F_i \leq mean(F/S)\}$, and $S_2 = \{X_i \in S \mid F_i > mean(F/S)\}$. Note that F/S denotes the function F limited to subregion S, and that $mean(F/S) = \frac{1}{|S|}\sum_{X_i \in S} F_i$, which is the mean of F in subregion S.
3. Remove S from P; put S_1 and S_2 in P.
4. If P has less than p subregions, go to Step 2. Otherwise, return.

end

Note that this recursive partitioning can be represented by a tree: the root is the whole region R, and the leaves are the subregions of the final partition. Any internal (non-leaf) node S, which we call a *macrosubregion*, is the union of two or more subregions in the final partition; those subregions are the descendent leaves of S, and form a partition of S. The next theorem justifies the name "moment-preserving".

Theorem 2 *Let F be a function over a region R, and let P be a partition of R produced by the MPP Algorithm. Let M be the piece-wise constant model of F corresponding to the partition P. Then, the following statements hold*
i) $mean(M) = mean(F)$ and $FACM(M) = FACM(F)$.
ii) For each macrosubregion S, $mean(M/S) = mean(F/S)$ and $FACM(M/S) = FACM(F/S)$.

Proof: The proof is by some elaborate induction on p. It is omitted for brevity. ■

The preservation of the regional means and first absolute central moments will have applications in preprocessing of images for better edge detection, and in dynamic range adjustment for image display. Those applications will be addressed next.

2 Preprocessing for Better Edge Detection

Edge detection is one of the standard and most basic operation in image processing and analysis, and has been studied extensively [1, 2, 6, 8, 9]. Most of the commonly used edge detection algorithms compute the partial derivatives of an image I, and then threshold $\sqrt{\frac{\partial I}{\partial x}^2 + \frac{\partial I}{\partial y}^2}$ to yield the edges.

In the regions where changes are not abrupt enough, the existing techniques described may fail to detect edges. One way to remedy the situation is to enhance the images to sharpen the edges before edge detection is performed. Typical sharpening methods include high-pass filtering, which fail to adjust to local variations in sharpness and detail within images. Another sharpening technique is histogram equalization [3], which works best when the contrast within an image is very low; however, when the contrast is fairly normal, this technique does not improve the images, neither does it lead to better edge detection.

An alternative preprocessing (sharpening) method is given here. It is simply the moment-preserving piece-wise constant modeling. Since edge pixels are likely to belong to subregions distinct from the subregions of the nearby pixels, the average of an edge subregion is likely to differ sharply from the average of the subregion of the nearby pixels. The difference between the averages is often sharper than between the original pixels.

We evaluated experimentally this new preprocessing technique. Figure 1 shows the image Lena (Fig. 1(a)) and its edges as detected with Sobel edge detection (SED) [9] (Fig. 1(b)), with histogram equalization followed by SED (Fig. 1(c)), and with our new modeling technique followed by SED (Fig. 1 (d)). The figure clearly shows that our technique yields more edge detection than the other two techniques. Similar experiments using other operators such as Prewitt [8], Roberts, and Marr-Hildreth [6] show that our technique is superior to all of them.

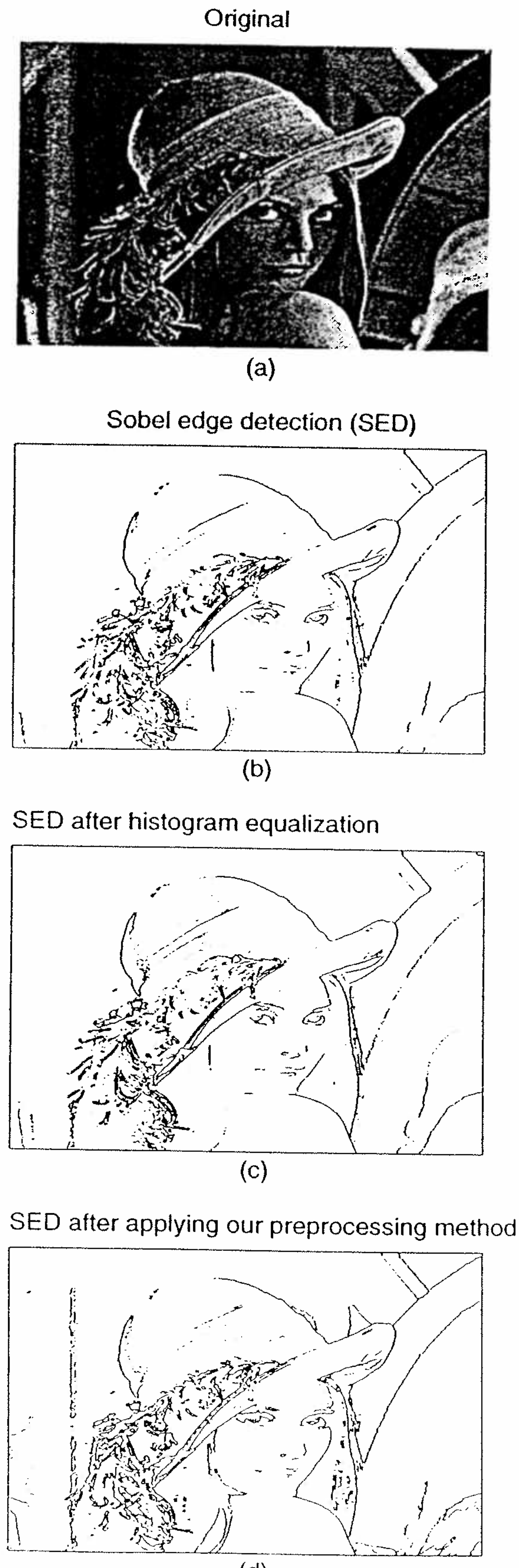

Figure 1: Our edge detect method vs. others

3 Dynamic Range Adjustment of Images

Dynamic range adjustment is the reduction of density resolution, that is, number of bits per pixel. It is performed when the density resolution is too high for a given display system or too costly in storage requirements. For example, the density resolution of an image may be 12 or 16 bits, while the display monitor resolution is only 8 bits.

One standard way to reduce the density resolution n down to some level d is to keep the d most significant bits of each pixel value. This thresholding technique is equivalent to piecewise constant — but nonoptimal — modeling of the image. To see this, partition the image into several subregions where in each subregion the pixel values do not differ in the d most significant bits. By reducing each pixel to its d most significant bits, all pixel values in each subregion become equal, thus proving that the resulting image is piece-wise constant. However, the piece-wise constants are not oprimal in the least-square sense because the optimal constant per subregion is the average in the subregion. Clearly, the d most significant bits are less than the average. Another drawback of this piece-wise constant modeling is that it is not necessarily moment-preserving, and is thus less sensitive to human vision. Another method for dynamic range adjustment is Max-Lloyd quantization [5, 7], which is optimal in the least-square sense. However, it does not preserve the absolute central moments.

An alternative and superior method for dynamic range adjustment is to use the Moment-Preserving-Partitioning Algorithm of section 1, and then apply piece-wise constant (optimal) modeling. This method has the added advantage of preserving the regional means and the regional absolute central moments, leading to better subjective visual quality of the resulting images. Once the subregions of the image have been identified and averaged, the subregions are labeled 1,2,...,p, and the averages are put in an array $Y[1:p]$, $Y[i]$ being the average of subregion i. If the new density resolution is meant to be d bits per pixel, take $p = 2^d$. Every pixel is then replaced by the label of the subregion to which the pixel belongs. Clearly, every pixel takes d bits to represent. At display time, the display devices map every pixel value (index) i to actual pixel value $Y[i]$. This mapping is typically implemented in hardware in most current display systems. This hardware capability is available because in many graphics algorithms, pixels are represented by various indexing schemes. Therefore, this new dynamic range adjustment technique can be readily put to use.

We have implemented the new dynamic range adjustment technique, evaluated it experimentally, and compared it to the standard thresholding-based density reduction technique. We reduced the density resolution of Lena (shown in Fig. 1(a)) from 8 bits per pixel down to 5, 4, 3, 2, and 1 bit per pixel, using both our new method and the thresholding method, and measured the signal-to-noise ratio (SNR) between the original image of Lena and each of the density-reduced images. The SNR's of our new method were consistently higher by 8 decibel points than the SNR's of the thresholding method, which is quite significant. Note that in evaluating the SNR of the thresholded images, the missing bits of every pixel were replaced by 0's, to give a more fair estimate of the SNR, and make the comparison with the new technique valid.

4 Conclusions

In this paper we developed a regionally moment-preserving piecewise-constant modeling technique for images, and applied it to two standard image applications: edge detection and dynamic range adjustment. Our experimental evaluations show that the new technique gives significantly better results than the standard methods for those applications.

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