# Soft Decision Metrics for Turbo-coded FH M-FSK Ad Hoc Packet Radio Networks

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***Abstract*—This paper addresses turbo-coded non-coherent FH *M*-FSK ad hoc networks with a Poisson distribution of interferers where multiple access interference can be modeled as symmetric α-stable (SαS) noise and α is inversely proportional to the path loss exponent. The Bayesian Gaussian metric does not perform well in non-Gaussian (α≠2) noise environments and therefore an optimum metric for Cauchy (α=1) noise and a generalized likelihood ratio (GLR) Gaussian metric requiring less side information (amplitude, dispersion) are presented. The robustness of the metrics is evaluated in different SαS noise environments and for mismatched values of the interference dispersion and channel amplitude in an interference-dominated network with no fading or independent Rayleigh fading. Both the Cauchy and GLR Gaussian metric exhibit significant performance gain over the Bayesian Gaussian metric, while the GLR Gaussian metric does so without the knowledge of the dispersion or amplitude. The Cauchy metric is more sensitive to the knowledge of the amplitude than the dispersion, but generally maintains better performance than the GLR Gaussian metric for a wide range of mismatched values of these parameters. Additionally, in an environment consisting of non-negligible Gaussian thermal noise along with multiple access interference, increasing the thermal noise level degrades the performance of the GLR Gaussian and Cauchy metric while for the observed levels both maintain better performance than the Bayesian Gaussian metric.**

***Keywords-soft decision metrics, symmetric α-stable noise, non-coherent detection, ad hoc networks***

## I. INTRODUCTION

It was shown in [1] that multiple access interference in a spread spectrum (SS) network with a Poisson distribution of interferers using the same modulation and power has a symmetric α-stable (SαS) distribution where α is inversely proportional to the path loss exponent. This work was extended in [2] to include the effects of fading and shadowing for a frequency hopping (FH) network and in [3] interference was analyzed in a direct sequence (DS) network with variable coherent modulation. Non-coherent receivers for uncoded systems in α-stable noise were given in [2, 4]. Our previous work [5] derived soft decision metrics for a general case of coded orthogonal signaling in SαS noise. This paper focuses on the performance of these metrics in a turbo-coded FH ad hoc network, including fading channels, and investigates the robustness of the metrics to imperfect side information.

## II. SYSTEM MODEL

A turbo-coded FH SS ad hoc network with a Poisson distribution of interferers using the same modulation and power is considered. User hops are synchronized (non-synchronized hopping would just scale the dispersion of the multiple access interference [1]) and one symbol is transmitted per hop. Information bits are binary encoded, transmitted using *M*-FSK, and detected non-coherently using soft decision decoding. The output of the *i*th correlator, $0 \le i \le M-1$, is modeled as

$$\mathbf{Z}_i = \frac{a_i}{R^{m/2}}\mathbf{S}_i + \mathbf{Y}_i + \mathbf{N}_i \tag{1}$$

where all vectors are two-dimensional, representing the in-phase and quadrature components, $a_i$ is the amplitude of the received signal (including fading, if present), $R$ is the transmitter-receiver distance, $m$ is the path loss exponent, $\mathbf{N}_i$ is Gaussian thermal noise with variance $\sigma_N^2$, $\mathbf{Y}_i$ is the multiple access interference at the output of the demodulator, modeled as additive SαS noise where $\alpha=4/m$ for $m>2$ (i.e., free space path loss excluded) [1,2] and $\mathbf{S}_i$ is the desired signal given by

$$\mathbf{S}_i = \begin{cases} [\cos\theta_i \ \sin\theta_i]; & i = i' \\ 0 & ; i \ne i' \end{cases} \tag{2}$$

where $\theta_i$ is the relative phase of the signal, and the $i'$th frequency is modulated.

Multiple access interference, modeled as SαS noise, can be characterized by two parameters: the characteristic exponent α, that is a function of the path loss exponent, and dispersion γ that is a function of the path loss exponent and the fading/shadowing statistics [2]. For non-fading environments the dispersion is given by [2]

$$\gamma = \frac{\lambda\pi}{qM}\frac{\Gamma(1-\alpha/2)}{2^{\alpha}\Gamma(1+\alpha/2)} \tag{3}$$

while for fading environments [2]

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$$\gamma = \frac{\lambda\pi}{qM}\frac{\Gamma(1-\alpha/2)}{2^{\alpha}\Gamma(1+\alpha/2)}E[A^{\alpha}] \quad (4)$$

where $\lambda$ is the density of the interferers in the network per unit area, $q$ is the number of hopping bins, $\Gamma(x)=\int_0^{\infty} x^{z-1}\mathrm{e}^{-x}\mathrm{d}x$ is the gamma function, E[·] is the expectation operator and $A$ is the random fading amplitude. For example, given a path loss exponent of four, multiple access interference is modeled as S1S noise and the dispersion for the non-fading case is given by $\gamma=\lambda\pi/qM$ while for unit power Rayleigh fading $\gamma=\lambda\pi\sqrt{\pi}/2qM$ .

## III. DECISION METRICS

Each received symbol represents $\log_2 M$ coded bits. A soft-decision metric in the form of a log-likelihood ratio is computed at the receiver for each coded bit and is fed to a soft-decision decoder. The log-likelihood ratio of the $j$th coded bit, $c_j$, is defined as

$$L_j(\mathbf{z},\mathbf{a}) = \log\frac{\Pr[c_j=1\,|\,\mathbf{z},\mathbf{a}]}{\Pr[c_j=0\,|\,\mathbf{z},\mathbf{a}]} \quad (5)$$

where $\mathbf{z}=[\mathbf{z}_0\ \mathbf{z}_1\ \ldots\ \mathbf{z}_{M-1}]$ represents outputs of the $M$ in-phase and quadrature correlators, and $\mathbf{a}=[\mathrm{a}_0\ \mathrm{a}_1\ \ldots\ \mathrm{a}_{M-1}]$.

Log-likelihood ratios were derived assuming an interference-dominated network where Gaussian thermal noise is negligible and multiple access interference is modeled as SαS noise [5]. Since closed form expressions for the probability density function of SαS noise exist only for α=1 (Cauchy distribution) and α=2 (Gaussian distribution), the metrics were obtained assuming these noise environments. Note that α=2 is not applicable in modeling the multiple access interference [1], however it is of interest to assess the performance of metrics designed for Gaussian noise in a non-Gaussian interference environment. The following gives the assumed noise densities and corresponding decision metrics while details of the derivations can be found in [5].

### A. Gaussian metric

Assuming S2S (Gaussian) noise, the noise density is given as

$$f_{\mathrm{Y}}(\mathbf{y}) = \frac{1}{4\pi\gamma}\exp\left(-\frac{\|\mathbf{y}\|^2}{4\gamma}\right). \quad (6)$$

The Gaussian metric for non-coherent detection, optimum in S2S noise, is given by [5]

$$L_j(\mathbf{z},\mathbf{a}) = \log\frac{\sum_{i:c_j=1} e^{-a_i^2/4\gamma} I_0\left(\frac{a_i w_i}{2\gamma}\right)}{\sum_{i:c_j=0} e^{-a_i^2/4\gamma} I_0\left(\frac{a_i w_i}{2\gamma}\right)} \quad (7)$$

where $I_0(x)=\frac{1}{2\pi}\int_0^{2\pi} e^{x\cos\theta}d\theta$ is the zeroth-order modified Bessel function of the first kind, $w_i=\|\mathbf{z}_i\|$ and the summations are over $M/2$ signals for which $c_j$=1 and $c_j$=0, respectively.

### B. Cauchy metric

Assuming S1S (Cauchy) noise, the noise density is given as

$$f_{\mathrm{Y}}(\mathbf{y}) = \frac{\gamma/2\pi}{\left(\gamma^2+\|\mathbf{y}\|^2\right)^{3/2}} . \quad (8)$$

The Cauchy metric for non-coherent detection, optimum for S1S noise, can be shown to be [5]

$$L_j(\mathbf{z},\mathbf{a}) = \log\frac{\sum_{i:c_j=1} E\left(\sqrt{\frac{2\delta_i}{\beta_i+\delta_i}}\right)\frac{\left(\gamma^2+w_i^2\right)^{3/2}}{(\beta_i-\delta_i)\sqrt{\beta_i+\delta_i}}}{\sum_{i:c_j=0} E\left(\sqrt{\frac{2\delta_i}{\beta_i+\delta_i}}\right)\frac{\left(\gamma^2+w_i^2\right)^{3/2}}{(\beta_i-\delta_i)\sqrt{\beta_i+\delta_i}}} \quad (9)$$

where $E(k)=\int_0^{\pi/2}\sqrt{1-k^2\sin^2\varphi}\,d\varphi$ is the complete elliptic integral of the second kind, $\beta_i=\gamma^2+w_i^2+a_i^2$ and $\delta_i=2a_iw_i$.

### C. Generalized Likelihood Ratio Metric

The Bayesian Gaussian and the Cauchy metric depend on the amplitude, **a**, and interference dispersion γ. While the Bayesian approach to eliminating the dependence does not appear to be tractable, even if distributions of **a** and γ are known, another approach is to use the generalized likelihood ratio (GLR) paradigm that maximizes the likelihood function with respect to the unknown parameters. The GLR Gaussian metric, obtained from the assumption of S2S noise by applying the GLR paradigm, that does not require knowledge of amplitude and dispersion can be shown to be [5]

$$L_j(\mathbf{z}) = \log\frac{\sum_{i:c_j=1}\left(\sum_{k=0,k\neq i}^{M-1} w_k^2\right)^{-M}}{\sum_{i:c_j=0}\left(\sum_{k=0,k\neq i}^{M-1} w_k^2\right)^{-M}} . \quad (10)$$

For $M$=2 (10) simplifies to $L_j(\mathbf{z}) = 2\log\left(w_1^2/w_2^2\right)$.

## IV. SIMULATION RESULTS

The performance of the given metrics is first evaluated in an interference-dominated network (negligible thermal noise) and different noise environments corresponding to typical values of the path loss exponent. The robustness of the Cauchy metric to mismatched values of the interference dispersion and the channel amplitude is also presented. Additionally, performance is given for an environment consisting of multiple access interference and thermal noise. Both independent Rayleigh fading and non-fading environments were investigated.

Performance results are obtained through Monte Carlo simulations of a rate 1/2 binary parallel concatenated convolutional (turbo) coded BFSK FH system with constituent encoder generator polynomial $(15,13)_8$. The number of decoding iterations for a pair of soft-input/soft-output log-MAP decoders is eight. The frame size is 1024 information bits. We present the results in terms of frame error rate (FER) versus the normalized interference parameter, $\widetilde{N} = R^2\pi\lambda/qM$ , interpreted as the "expected number of interferers closer to the receiver than the transmitter per frequency slot" [1]. Therefore, $\widetilde{N}$ is a normalized measure of the interference; higher values imply larger distances covered by the transmission, higher interferer density and/or fewer hopping bins.

### *A. Performance in different noise environments*

Figs. 1-3 compare performance of the decision metrics in an interference-dominated network with independent Rayleigh fading and multiple access interference modeled as SαS noise for path loss exponents of $m$=3,4 and 5, respectively. These are typical path loss exponent values for mobile terrestrial communications. We observe that the Cauchy metric yields better performance than the GLR Gaussian metric while both metrics exhibit significant performance gain over the Bayesian Gaussian metric for the given noise environments. The GLR Gaussian metric achieves this performance, unlike the other two metrics, without knowledge of the amplitude or dispersion. The advantage of the Cauchy metric over the other metrics increases with higher impulsiveness of the interference corresponding to smaller α values. The same conclusions hold in a non-fading environment, for which results are omitted here for brevity.

Results indicate that in an S1S noise environment with independent Rayleigh fading and FER=0.02, for example, the use of the Cauchy metric enables, approximately, over 70% longer transmission distances compared to the GLR Gaussian metric and an order of magnitude longer than the Bayesian Gaussian metric. Increasing the transmission distance, and thereby decreasing the number of hops in a multihop ad hoc network, translates into increased network capacity when the power is kept fixed.

### *B. Robustness to mismatched side information*

The results above assume perfect knowledge of the signal amplitude and interference dispersion for the Cauchy and Bayesian Gaussian metrics. The robustness of the Cauchy metric to fixed mismatched values of the dispersion and amplitude are presented in Figs. 4 and 5 for an interference-dominated network with S1S noise and independent Rayleigh fading. In Fig. 4, mismatched values of the dispersion are given as multiples of the actual dispersion. In Fig. 5, the effect of substituting the average channel amplitude for the instantaneous amplitudes in the metrics is shown.

We observe in Fig. 4 that for a wide range of mismatched values of the dispersion the Cauchy metric still outperforms the other metrics. Additionally, the Cauchy metric exhibits higher sensitivity to an overestimate than an underestimate of the noise dispersion. Previous results [6] on the sensitivity of the turbo decoder to mismatched values of the signal-to-noise ratio (SNR) in an additive white Gaussian noise (AWGN) channel indicate less sensitivity of the decoder to overestimating than underestimating the SNR. Given that the dispersion is inversely proportional to SNR in an AWGN channel our results are in agreement with the findings in [6].

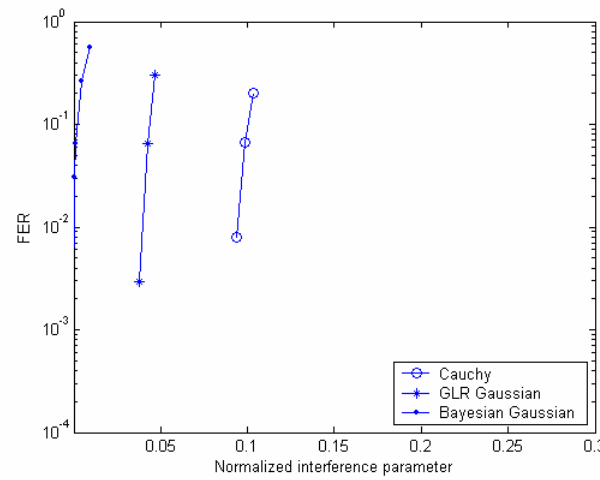


Figure 1. Performance in S(α=4/3)S noise with fading

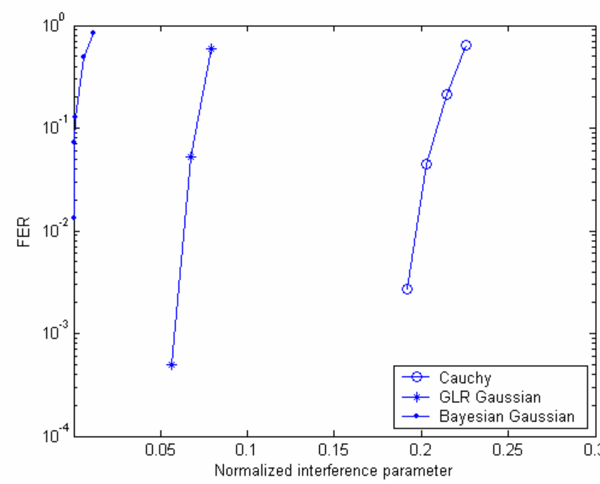


Figure 2. Performance in S(α=1)S noise with fading

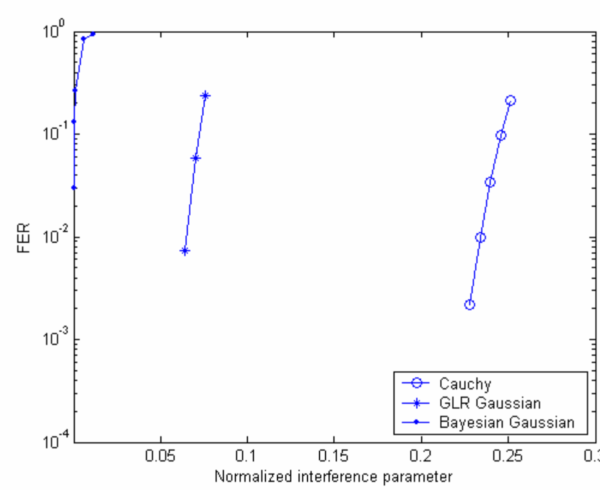


Figure 3. Performance in S(α=4/5)S noise with fading

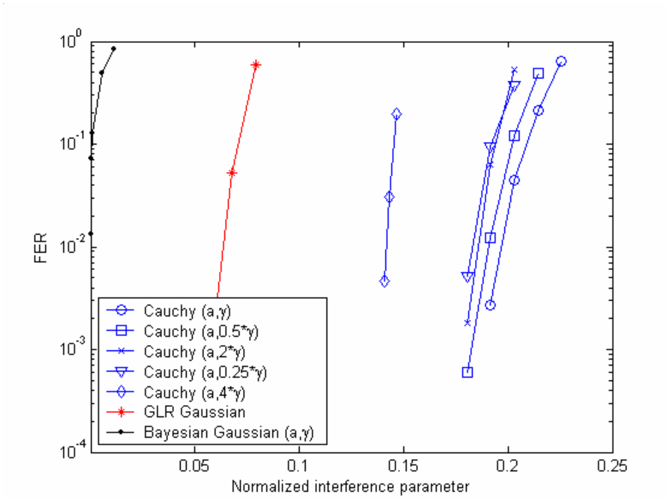


Figure 4. Performance of the Cauchy metric for fixed mismatched values of dispersion in S(α=1)S noise with fading

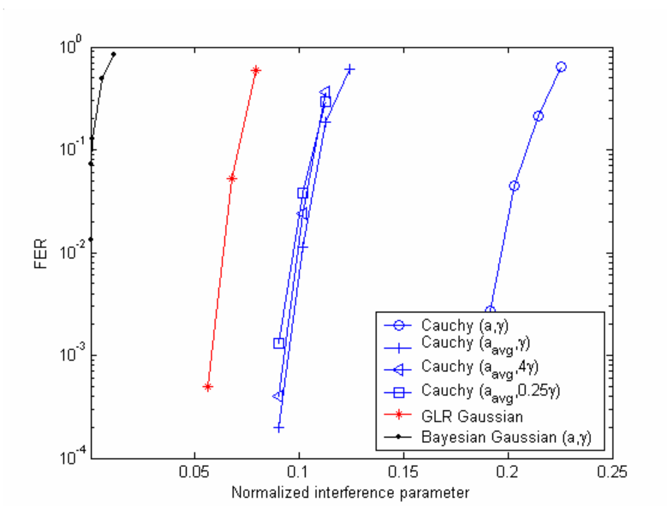


Figure 5. Performance of the Cauchy metric for fixed mismatched values of dispersion and amplitude in S(α=1)S noise with fading

Although the Cauchy metric exhibits performance degradation when the average value of the channel amplitude is applied instead of the instantaneous value, as seen in Fig. 5, it still outperforms the other metrics. Furthermore, the combination of mismatched dispersion values and use of the average channel amplitude does not add significant performance loss. This observation indicates a higher sensitivity of this metric to the channel amplitude than to the interference dispersion.

### *C. Performance in interference and thermal noise*

Performance in an environment consisting of Gaussian thermal noise in addition to multiple access interference modeled as Cauchy noise is given in Fig. 6 for the non-fading case. The thermal noise level is varied according to $\delta = \sigma_N^2 / 2\gamma$ in order to observe its influence on the performance of the metrics. The dispersion value used for the input to the decoder is $\gamma$ (dispersion of the multiple access interference). Results indicate that increasing the thermal noise level relative to the multiple access interference degrades the performance of the Cauchy and GLR Gaussian metric, but both maintain an advantage over the Bayesian Gaussian metric. However, the performance advantage of the Cauchy metric over the GLR Gaussian metric decreases as the thermal noise level increases, which is expected as the overall noise and interference becomes more Gaussian.

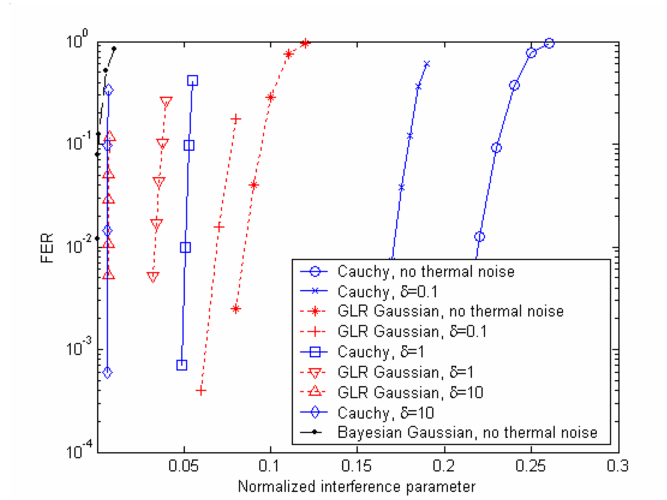


Figure 6. Performance in an environment with S(α=1)S noise and thermal noise

## V. SUMMARY

Performance of soft decision decoding metrics is evaluated for a turbo-coded FH ad hoc network with a Poisson distribution of interferers in both fading and non-fading environments. The GLR Gaussian and Cauchy metric exhibit significant performance gain over the Bayesian Gaussian metric for the observed noise environments. As opposed to the other two metrics, the GLR Gaussian metric achieves this performance without knowledge of the dispersion or amplitude. It was shown that the Cauchy metric maintains a performance advantage over the GLR Gaussian metric for a wide range of mismatched values of the dispersion and amplitude.

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