Micromagnetic eddy currents in conducting cylinders

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The inclusion of eddy currents into micromagnetic programs is important for the proper analysis of dynamic effects in conducting magnetic media. This paper introduces a numerical implementation for eddy current calculations, in a limited geometry, for a thick domain wall. In the special case of a zero-thickness wall, our results are directly comparable with an analytical model previously presented. Our calculations provide some computational results for testing more complex programs. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1851872]

I. INTRODUCTION

Recently interest has been expressed in introducing eddy currents into micromagnetic problems.[1–3] We have developed a one-dimensional model micromagnetic program to solve for the dynamic magnetization in conducting cylinders as a test bed for determining errors in these programs. This model involves solving the coupled problems of eddy current and magnetization calculations. This model permits one to determine any effect of wall bending on its characteristics, since the wall's radius of curvature decreases as it approaches the center of the wire. We found that the wall energy per unit area increased slightly with the decreasing radius.

The model geometry consists of an infinitely long circular cylinder of radius R consisting of a uniaxial material whose easy axis, z, coincides with the cylinder's axis. Initially the cylinder is uniformly magnetized in the positive z direction. Applying a constant field in the negative z direction eventually reverses the magnetization. The reversal consists of nucleating a Bloch wall at the surface that propagates towards the center, where it is eventually annihilated at the cylinder's axis. The moving wall induces eddy currents that impede the wall's progress without creating demagnetizing fields.

An external magnetic field acts as a boundary condition on the magnetic field inside the material. The difference between the internal magnetic field from the surface magnetic field is due to the shielding effects of eddy currents. Thus, the magnetic field tries to penetrate the material and in doing so changes the magnetization, which in turn, generates the eddy currents that keep it from penetrating. At each time step, one has to simultaneously relax both the magnetization and the magnetic field.

We have recently presented a model for testing limiting cases of a micromagnetic model, for a zero-thickness wall, that included eddy currents.[4] Here, we present the effect of a finite-thickness wall, the effect of wall curvature, and the problems with jointly computing wall shape and dynamics. For simplicity, we again leave out gyroscopic effects and assume that the domain structure propagating toward the center is a Bloch wall. The calculation properly reduces to the recently presented results if the wall has zero thickness.

Figure 1 shows how the magnetization angle, with respect to the z axis, varies as a function of the normalized radius for several time steps in the calculation as the wall progresses. For the arbitrary parameters chosen, in the first 6 steps, the wall forms. During the next 40 steps, the wall propagates towards the center where it is annihilated.

II. FORMULATION OF THE PROBLEM

The exchange energy between a pair of spins is

$$W_{ex} = -J_{ex}\mathbf{s}_1\mathbf{s}_2. \tag{1}$$

If we assume that the magnetization varies linearly between a pair of calculation nodes j and k, then the exchange energy for the atoms in that row is given by

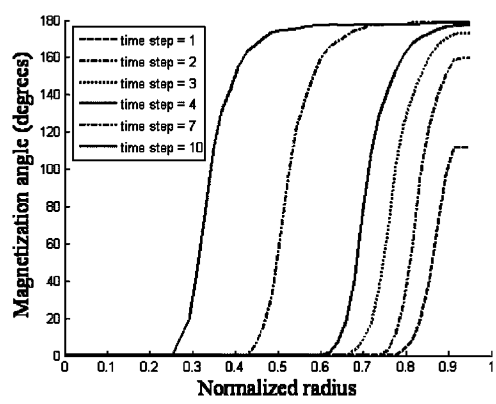

FIG. 1. A plot of the magnetization angle as a function of radius for various time steps.

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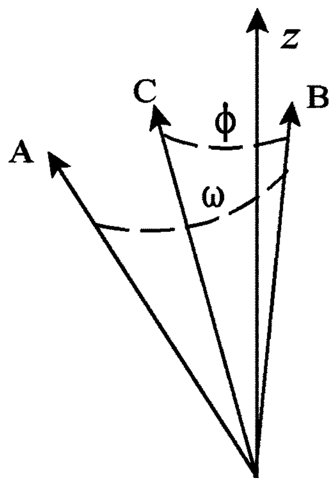

FIG. 2. Relationship of spherical angles.

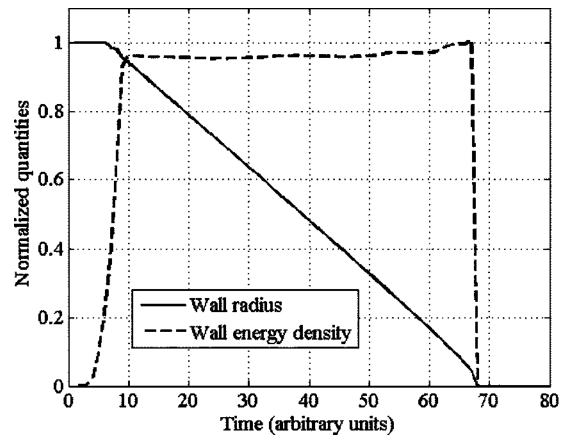

FIG. 3. A plot of a normalized wall radius (solid line) and a normalized wall energy density (dashed line) with time for a constant applied field.

$$w_{\mathrm{ex},jk} = -J_{\mathrm{ex}}\frac{h}{\delta}\cos\left[\frac{\delta}{h}(\alpha_j - \alpha_k)\right], \tag{2}$$

where α_k is the angle that the magnetization $\mathbf{M}_k$ makes with respect to the z axis, since we are assuming that $\phi=0$, h is the distance between nodes, and δ is the distance between magnetic unit cells. To get the total exchange energy we have to sum this over all the computation points.

As shown in Fig. 2, let A be the spin on the computation row; B is the spin on the adjacent row, which is obtained by rotating an interpolated spin, C, from the computation row by an angle ϕ equal to $\tan^{-1}(h/r)$, about the z axis. Let spin A make an angle A with respect to the z axis, and spin B make an angle B with respect to the z axis. The angle ω between A and B is computed using the spherical angle formula.

If there are n intervening atoms in between, then

$$w_{\mathrm{ex}} = Jn\cos\left[\frac{1}{n}\cos^{-1}(\cos A\cos B + \sin A\sin B\cos\phi)\right]. \tag{3}$$

To this, we have to add the total anisotropy energy, which for uniaxial anisotropy is

$$W_{\mathrm{anis}} = 2\pi\int_0^R \rho K\sin^2\omega d\rho. \tag{4}$$

The Zeeman energy is given by

$$W_{\mathrm{Zeeman}} = -2\pi\int_0^R \mu_0 H_z(\rho)M_s\cos[\omega(\rho)]\rho\, d\rho. \tag{5}$$

If the applied field and, consequently, the magnetization change with time, an electric field will be induced. By Faraday's law, the curl of this electric field is given by

$$\mathrm{curl}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} = -\mu_0\left(\frac{\partial\mathbf{H}}{\partial t} + \frac{\partial\mathbf{M}}{\partial t}\right). \tag{6}$$

The electric field, neglecting the effect of the z component inside the wall,[4] is then given by

$$\mathbf{E}(r) = -\frac{\mu_0\mathbf{1}_y}{r}\int_0^r\left[\frac{\partial H_z(\rho,t)}{\partial t} + \frac{\partial M_z(\rho,t)}{\partial t}\right]\rho\, d\rho, \tag{7}$$

where $\mathbf{1}_y$ is a unit vector in the y direction. This electric field will induce eddy currents. If the time scale is appropriate, then these currents can be computed using Ohm's law, $\mathbf{J} = \sigma\mathbf{E}$. By Ampere's law, the field in the interior of the material will differ from the surface field by the eddy currents. Thus, the field at any point $H_z(r)$ is given by

$$\frac{dH_z(r)}{dr} = K_w(r), \tag{8}$$

where K_w is the surface current density per unit length at the wall.

The iterative method that we used at each time step involves calculating the magnetization due to the local field, computing the eddy currents due to the change in magnetization, then adding the field induced by the eddy currents to the applied field to obtain the local field and repeating until there is convergence. Then we advance to the next time step. The magnetization can be computed by either applying the Landau–Lifshitz–Gilbert (LLG) equation to study high-speed phenomena or by minimizing the energy of the system. In this version of the program, we used energy minimization for simplicity and since the magnetization relaxation is much faster than the relaxation of the eddy currents.

One of the problems where several symmetrical solutions are possible, such as this one, is how to break the symmetry. We chose to do this by searching for a solution for α in the range $[0,\pi]$. For materials with uniaxial and similar anisotropies there is a unique minimum in this range; also this breaks the symmetry.

The spatial discretization Δr is determined by the resolution that we wish to obtain of the domain wall. For example, if the width of the domain wall is l_w and we wish to have n_w points in the wall, then

$$\Delta r = l_w/n_w. \tag{9}$$

The temporal discretization is determined by the ability

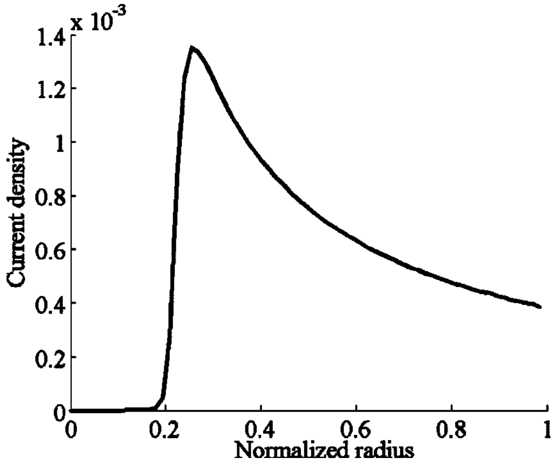

FIG. 4. Eddy current density as a function of position partway through the magnetization reversal.

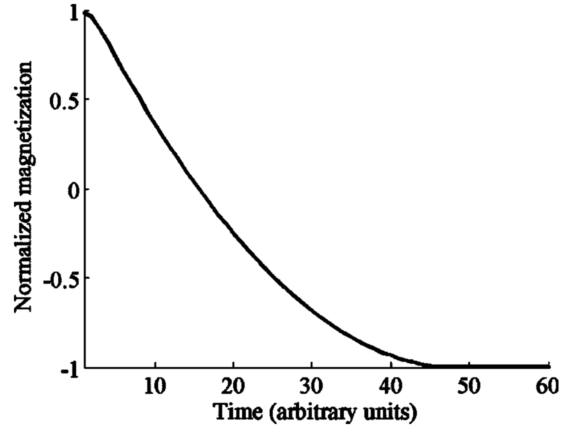

FIG. 5. Normalized magnetization vs time for a constant applied field.

of the induced eddy currents to slow the wall down. In particular, the change in the magnetic field from the surface to the midpoint of the wall, R, is

$$\Delta H = \int_r^R K(\rho)d\rho \approx -K_w(R-r), \tag{10}$$

If the material has no coercivity, ΔH is equal to the applied field H_{app}. The current density at the wall is given by

$$K_w = \sigma \Delta \Phi / \Delta t, \tag{11}$$

where $\Delta\Phi$ is the change in the magnetic flux, for a unit length along the cylinder between the midpoint of the wall and the center, in one time step. This flux change is roughly equal to the product of μ_0, twice the saturation magnetization (since the magnetization changes from $-M_S$ to M_S), and the distance that the wall moves in one time step. We would like this distance to be comparable to Δr. Thus,

$$\Delta t \approx \frac{2\mu_0 \sigma M_s \Delta r}{K_w} = \frac{2\mu_0 \sigma M_s (R-r)\Delta r}{H_{\text{app}}}. \tag{12}$$

It is seen that if one chooses a smaller time step, the induced currents will be larger. We shall choose a time step so that ΔH is a fraction α of H_{app}, where α is of the order of unity. Thus, we choose the time step

$$\Delta t = \frac{2\mu_0 \sigma M_s (R-r)\Delta r}{\alpha H_{\text{app}}}. \tag{13}$$

All things being equal, this means that we could take larger time steps as we approach $r=0$.

III. MODEL RESULTS

In these calculations, we chose material parameters so that the wall thickness was 0.1 times the wire radius. We arbitrarily define the wall radius as the position at which the magnetization angle is $\pi/2$. We see from Fig. 3, a plot of the wall position with respect to time for a constant applied field, that the radius varies linearly once the wall is formed until just before it starts to annihilate. We expected the wall to accelerate as it approached the center since the amount of flux change for a given change in the radius decreases. However, the eddy currents also increase since they have a larger area to flow in. This is illustrated in Fig. 4, which is a plot of the eddy current density for a wall at 0.2 times the normalized radius.

The magnetization change is due to the shrinking of the core. Since the volume of the core varies as the square of the wall's radius, and since the wall position is linear in time, as shown in Fig. 5, we expect the magnetization to vary parabolically in time. If we compare this result with that of a zero-thickness wall, discussed in Ref. 4, we see that there is agreement.

We defined the wall width w_w as

$$w_w = \frac{2}{d\omega/dr}, \tag{14}$$

and the wall energy per unit area for a unit length along the wire as the sum of anisotropy and exchange energies divided by the length of the wall at the wall radius. We then computed the wall thickness and found it to be essentially constant once the wall was fully formed and until it annihilated. The wall energy density variation on the other hand increased slightly with decreasing the radius.

ACKNOWLEDGMENTS

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