Model Tests to Investigate the Effects of Geometrical Imperfections on the NIST Calculable Capacitor

Anne-Marie Jeffery, *Member, IEEE*, Lai H. Lee, and John Q. Shields

***Abstract*— The calculable capacitor at National Institute of Standards and Technology (NIST) links the U.S. capacitance unit to the SI unit and has a relative standard uncertainty of 2×10^{-8}. Geometrical imperfections are one of the largest sources of this uncertainty. Tests with a model calculable capacitor have been done to better evaluate and reduce this uncertainty. These included evaluations of the effect of eccentricity of the blocking electrodes used to define the capacitor's length and of uniform taper on all the calculable capacitor bars. In addition, the effect of tilt of the blocking electrode and taper in three or fewer bars was investigated. A new cone-shape for the tip of the blocking electrode was also studied.**

***Index Terms*— Calculable capacitor, capacitance measurements.**

I. Introduction

THE CALCULABLE capacitor is a cross-capacitor whose capacitance can be found from a single length measurement [1]. The National Institute of Standards and Technology (NIST) calculable capacitor consists of four vertical cylindrical bars arranged at the corners of a square. The length of the capacitor, which determines its capacitance, is defined by cylindrical blocking electrodes partially inserted between the four bars at both extremities. The length is measured with a Fabry–Perot interferometer where the optical flats are mounted at the ends of the top and bottom blocking electrodes. The actual measurement is differential and is done by moving the upper blocking electrode between two different positions that correspond to 0.2 and 0.7 pF. This produces a capacitance difference of 0.5 pF. The NIST calculable capacitor is described in [2].

One of the largest sources of uncertainty in the calculable capacitor experiment is due to geometrical imperfections in the bars, and in the alignment of the bars with each other and with the blocking electrodes. The previously reported value [3] of the relative standard uncertainty due to geometrical effects in the NIST calculable capacitor was 7×10^{-9}. However, recent linearity tests, which consisted of measuring 0.1 pF increments along the length of the calculable capacitor, have resulted in the assignment of a larger uncertainty for geometrical imperfections [4]. These tests do not include the nonlinearity due to close approach of the blocking electrodes to each other, which is observed at the 0.2 pF position. This effect was measured and removed from the data. The new relative standard uncertainty is the standard deviation of the measurement of the 0.1 pF intervals, and is 15×10^{-9}. We believe that two of the main contributors to this uncertainty are the eccentricity of the central blocking electrodes and local imperfections along the main bars.

Clothier [5], who was the originator of the present design used at NIST, performed a series of tests with a model capacitor to evaluate these effects. Clothier investigated the effect of eccentricity of the blocking electrode, the effect of local imperfections on the bars, and the effect of a uniform taper of the bars. His tests showed that a small spike at the ends of the blocking electrodes reduces the effect of local imperfections in the bars and compensates for the effect of a uniform taper in the bars. There is still a change, however, in capacitance when the spike is near a local imperfection. If a measurement is made when the movable blocking electrode is in such a position, there will be an error.

Similar tests were performed by Igarashi [6] and Cutkosky when the NIST calculable capacitor was constructed [2]. We have repeated these tests as part of our reevaluation of the calculable capacitor chain [4], which is a series of measurements, that links the units of capacitance and resistance to the SI system. Through these tests we have checked that the spike on the NIST calculable capacitor provides adequate compensation for imperfections in the geometry of the capacitor, and have obtained a better estimate of the effect of local imperfections. In the present work we also looked at the effect of tilt of the blocking electrodes and the effect of a uniform taper of three or fewer bars.

The tilt of the blocking electrodes is of interest, since in the NIST calculable capacitor the tilt of the blocking electrodes changes between the two measurement positions. The tilt of the upper blocking electrode is not adjustable, and changes as it is moved between the 0.2 and 0.7 pF position. The tilt of the lower blocking electrode is adjusted so that the interferometer's optical flats mounted at the ends of the blocking electrode are parallel. We are able to evaluate the tilt of the blocking electrodes, as well as their alignment with respect to the bars, by means of probe measurements. These consist of capacitance measurements between an insulated band on the movable upper blocking electrode and each of the calculable capacitor bars. This band is insulated from the rest of the blocking electrode for these measurements but is connected to the electrode during normal measurement. These probe measurements give an estimate of the spacing between the blocking electrode and the bars. The taper of each bar

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relative to the blocking electrode can be found by making probe measurements at intervals along the bars. Since there are two of these insulated bands on the blocking electrode, its tilt can also be determined. These measurements have indicated that all four main bars do not have a uniform taper; one has more of an outward taper than the others.

We have also tried changing the shape of the tip of the blocking electrodes using a cone instead of the usual spike. It was anticipated that a cone-ended blocking electrode would be less sensitive to local imperfections.

II. Experiment and Results

The tests were performed with a model calculable capacitor that has the same dimensions as the NIST calculable capacitor. The bars are horizontally mounted in air and measurements can be made with a resolution of 1 aF (10^{-18} F). In order to remove the effect of capacitance change as the blocking electrode is moved, a short double-ended blocking electrode is used. In this way the blocking electrode can be moved along the length of the bars without changing the capacitance. The region in which this electrode could be moved along the bars without capacitance change was determined and measurements made only in that region. Other effects can then be observed.

A. Eccentricity Tests

The double-ended blocking electrode was centered between the four bars. The electrode was then moved in 0.2 mm increments toward the gap between two adjacent bars, and the change in cross-capacitance ΔC measured. The cross-capacitance C is the average of the two cross-capacitances between the two pairs of opposing bars. The electrode was also moved toward a bar from the central position to see if the effect was the same as movement toward a gap. Following Clothier's analysis [5], the change ΔC in the capacitance due to the electrode being off-center can be expressed as $\Delta C = k\varepsilon^2$, where k is a constant and ε is the displacement from the central position. Our value for k is 1.2×10^{-4} aF/μm^2 for our present blocking electrode. This agrees with Clothier's value for k, if a scale factor is used to account for the difference in the size of the bars and their spacing. Displacement toward a bar gave the same results as displacement toward the gap between bars.

B. Tilt Tests

A tilt was created by placing a polytetrafluoroethylene (PTFE) ring near one end of the double-ended electrode to keep it centered while a point near the other end was displaced by 0.2 mm increments. The change in cross-capacitance was measured. We found that the measured change in capacitance ΔC as a function of the tilt of the electrode can be predicted by both assuming the tilt effect is due purely to a change in the eccentricity of the ends of the blocking electrode and by using the relationship for eccentricity, $\Delta C = k\varepsilon^2$, from the previous section. Since the actual length of the blocking electrode is not its effective electrical length, when calculating the displacement, ε, of the effective electrical end of the electrode from the central position, an estimate of the electrical length must be used. This gives us a means to estimate the effect of tilt in the NIST calculable capacitor.

We have attempted to explain the variation in the linearity measurements of the 0.1 pF interval increments using the change in tilt and eccentricity obtained from the probe measurements, but were unsuccessful. It is possible that between the 0.2 and the 0.7 pF positions, the lower blocking electrode undergoes a translation as well as a tilt. This means that our estimations of uncertainty due to the tilt using eccentricity will be at best a rough estimate. In the next version of the NIST calculable capacitor, the tilt of the upper blocking electrode will have the capacity to be varied, while the lower electrode will remain fixed.

C. Effect of Local Imperfections and Uniform Taper in the Bars

For this series of tests, three double-ended blocking electrodes, one flat-ended electrode, one spike-ended, and one cone-ended, all of the same length, were used. These blocking electrodes all have the same diameter as the ones used in the NIST calculable capacitor. The spike on the spike-ended electrode has the same dimensions as the one used in the NIST calculable capacitor (10.01 mm diameter, 22.45 mm length). The angle for the apex of the cone for the cone-ended electrode was 39°, and was chosen as a result of experiments with different angles.

To simulate the effect of a local imperfection on the bars, a narrow strip of metal foil, 3 mm wide and 127 or 50 μm thick, is wrapped around each of the four bars. A uniform taper in the bars is simulated by wrapping a wider metal foil strip (102 mm wide) around 1, 2, 3, or 4 bars near their ends. The spike should compensate for the presence of the foil, meaning that the average cross-capacitance between the bars after the edge of the blocking electrode has passed the edge of the foil, should return to the same value as before the electrode approached the edge of the foil. The wider strip also simulates the effect of a local imperfection as the electrode approaches the edge of the foil; much of our analysis of local imperfections was done by observing the effect as the blocking electrode approaches the edge of the wider strip of foil. In each case, the double-ended blocking electrode is moved along the bars until its front edge is 2 cm past the front edge of the foil. The average cross-capacitance is measured as the electrode is moved along the bars.

The results from these tests are shown in Fig. 1 and are similar to Clothier's results [5]. The magnitudes of the changes in capacitance are equivalent to those observed by Clothier when the difference in thickness of foil used is accounted for. The diameters of the bars and the blocking electrode of the NIST calculable capacitor are approximately twice the corresponding diameters in the calculable capacitor described by Clothier, but the effect from imperfections on the bars are expected to be the same. The change in capacitance is plotted against d, the distance from the edge of the foil to the flat edge of the blocking electrode to be consistent with Clothier's results.

One difference between Clothier's results and the results reported here is for the spike-ended electrode in case (c) in Fig. 1. Both spikes compensate for a uniform taper, but as

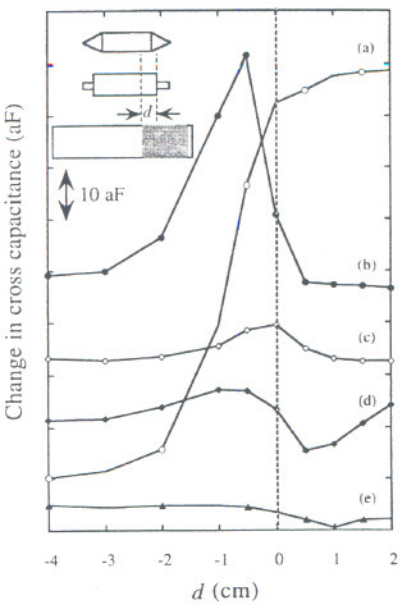

Fig. 1. Capacitance changes as the blocking electrode is moved along the bars. The different cases are (a) the flat-ended electrode with 102 mm wide, 50 μm thick foil, (b) with 3 mm wide, 127 μm thick foil, (c) the spike-ended electrode with 102 mm wide, 50 μm thick foil; (d) with 3 mm wide, 127 μm thick foil; and (e) the cone-ended electrode with 3 mm wide, 127 μm thick foil. In each case, the foil was applied to all four bars. For the spike-ended electrode, d is the distance from the edge of the foil to the flat edge of the blocking electrode, which is 2.24 cm from the electrode end. In the case of the cone-ended electrode, d is the distance from the edge of the foil to a point on the electrode 2.24 cm from the electrode end to be consistent with the spike electrode.

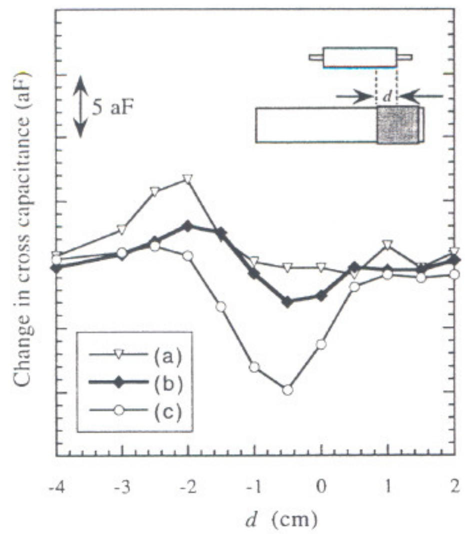

Fig. 2. Capacitance change as blocking electrodes with different diameter spikes are moved along the bars past the edge of 50 μm thick, 102 mm wide foil wrapped on the bars. The spike diameters are: (a) 10.0 mm (diameter of spike presently used), (b) 7.6 mm, and (c) 5.3 mm.

our spike approaches the edge of the foil there is an increase in capacitance instead of a decrease. This opposite effect is also seen for the narrow strip of foil in case (d) of Fig. 1. We observed that this difference is due to the relative diameter of the spike, ours being larger. We experimented with several spike diameters to determine if there was one between these two cases that produced no change in capacitance. The "in-between" case showed an increase and then a decrease in capacitance, with the overall change being similar in magnitude to the two other cases. The result is that the change in capacitance from the initial value is halved, reducing the effect of a local imperfection. Changes in the cross-capacitance for the spikes with the three different diameters that produce the capacitance increase, capacitance decrease, and the in-between case are shown in Fig. 2. Each spike is at its ideal length (i.e., the length at which the compensation is optimum for a uniform taper).

The tests with foil on three or fewer bars had to be done with very thin foil (25 μm) because second-order effects were significant at greater thickness and these effects mask the compensating effect of the spike. These tests simulate the case where all of the bars do not have the same uniform taper. On average, the spike appears to compensate even when the taper is only on three or fewer bars. Any changes seen were smaller than the resolution of the experiment. This is an important result because it ensures that in our case we do not have an additional error being introduced by the bar with the larger outward taper.

D. Cone-Ended Electrode

All the previously described measurements were also performed with the cone-ended electrode. The eccentricity measurements gave a slightly smaller k of 1.1×10^{-4}, which means that the cone is slightly less sensitive to eccentricity. The effect of tilt with the cone was approximately the same as observed with the spike. The cone-ended electrode showed smaller changes in capacitance for both the narrow [Fig. 1, case (e)] and the wider strip of foil as compared to the spike-ended electrode. However, the cross capacitance does not return to its initial value implying that the angle of the cone's apex was not optimum. Because of the difficulty in varying this angle, obtaining the correct angle was not pursued. In any case, a cone would not be practical since the blocking electrode tip has to be wide enough for the laser beam used for the length measurement, to pass through its center. A modified cone shape (a cone with a short spike on top) was used instead. This cone had the same angle as the one used in the previous experiment, but its tip was removed and replaced with a short spike.

Tests with this modified cone were done with 50 μm thick, 102 mm wide foil on all four bars. This allowed us to look at the effect of local imperfections as the electrode approaches the edge of the foil and ensure that it still compensates for a uniform taper. Several diameters for the small spike on the end of the modified cone were tried. For each diameter it was necessary to make measurements until the correct length that compensates for a uniform taper was found. Initially we started with the modified cone with a short spike of 7.6 mm diameter, which was the diameter of the spike that produced the "in-between" case mentioned before. However, the best results were obtained for a modified cone with a short spike of 6.4 mm diameter. This modified cone yields better results since the smaller diameter spike produces a smaller change in capacitance as the front edge of the spike passes

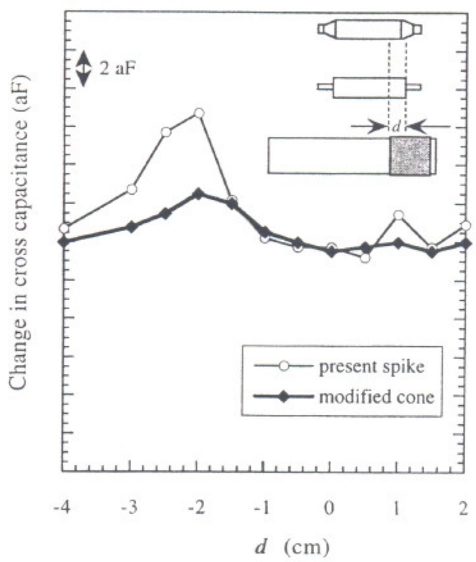

Fig. 3. Comparison of the effect between the spike-end and the modified cone-end as the blocking electrode moves past the edge of 50 μm thick, 102 μm wide foil wrapped on the bars.

the edge of the foil. The cone shape eliminates the change in capacitance observed when the flat edge of the blocking electrode approaches the edge of the foil. This modified cone showed a change of 2.5 aF near an imperfection of 0.05 mm on all four bars. A modified cone with a smaller diameter spike of 5.1 mm did not show any further reduction. This change near an imperfection is smaller than the change of 6 aF observed for a similar imperfection with the spike presently used on the NIST calculable capacitor. The results for both shapes are plotted in Fig. 3. For the actual calculable capacitor measurement this translates into a relative change of 3×10^{-9} of the 0.5 pF measurement for a local imperfection of 0.13 μm. This estimate for a local imperfection was determined from probe measurements made at 8 mm intervals at several places along the bars. While this is an improvement of a factor of three from the relative change of 9×10^{-9} that would be obtained with our present spike, the amount of work required does not warrant changing the tip shape at this time. Any changes will be made when a second version of our calculable capacitor is built.

III. Conclusions

Our eccentricity tests agreed well with Clothier's results, and the tilt tests have shown that the tilt of the blocking electrode can be evaluated as an eccentricity effect. The spike on the NIST calculable capacitor appears to be adequate and compensates even for the taper of three or fewer bars. In the range of spikes that compensate for a uniform taper, there exists an "in-between" case that reduces even further the effect of local imperfections. Changing to a tip with a modified-cone shape will also significantly reduce the effects from local imperfections. However, any changes to the tip shape will only be implemented in the next version of the NIST calculable capacitor.

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