STOCHASTIC ANALYSIS OF AC-GENERATED PARTIAL-DISCHARGE PULSES FROM A MONTE-CARLO SIMULATION

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INTRODUCTION

It has been shown from recent experimental investigations [1–3] that the statistical properties of pulsating partial-discharge (PD) phenomena are significantly influenced by effects of pulse-to-pulse and/or phase-to-phase memory propagation. The sources of the memory are dissipating residuals from the discharge pulses such as ion space charge, surface charge, and molecular species in excited states.

Previous "deterministic" models for ac-generated PD [4], although useful in providing insight into the physical basis for discharge patterns, do not account for the observed statistical behavior. Recent attempts [5] to develop Monte-Carlo simulations of PD that produce the required statistical variability have not been tested to determine that they account for known memory effects. In the present work, a Monte-Carlo simulation of ac-generated PD pulses is described which can properly account for effects of phase-to-phase memory propagation. The results of the simulation have been tested by determination of various unconditional and conditional pulse amplitude, phase-of-occurrence, and integrated charge distributions. The stochastic behavior of the simulated pulses is shown to be similar to that found experimentally for ac-generated PD in point-to-dielectric discharge gaps [2, 6].

PHYSICAL MODEL

The theoretical model used to make the simulation reported here is applicable to discharge gaps in which at least one of the surfaces is composed of a dielectric material, e.g., a metal point-dielectric plane gap, or a dielectric-dielectric gap such as might correspond to a void in a solid insulator. The basic assumptions of the model are:

1. The PD pulses are initiated by electrons released from surfaces by quantum mechanical tunneling, e.g., a Fowler-Nordheim field emission [7].

2. The amplitudes of the pulses have a quasi-normal (Gaussian) distribution about a mean sufficient to cause a drop in the local electric-field strength to quench the discharge and prevent immediate reinitiation of the next pulse, and small enough to prevent a complete field reversal at the discharge site.

3. The charge deposited on the dielectric surface by a PD event and the corresponding local field reduction are directly proportional to the PD amplitude.

4. The dielectric surface charge is quasi-permanent, i.e, it decays at a rate much slower than the frequency of the applied voltage.

5. The applied voltage is sinusoidal, i.e.,

$$V(t) = V_o \sin \omega t \tag{1}$$

where V_o is the amplitude and ω the frequency; and the local surface field at the discharge site has a magnitude proportional to $| V(t) |$, i.e, $| E(t) | \propto | V(t) |$.

According to the first assumption above, the rate of electron release from a surface at any time t is given by

$$r_e^{\pm}(t) = C_1^{\pm} | E(t) |^2 \exp(-C_2^{\pm} / | E(t) |), \tag{2}$$

where the positive and negative superscripts refer to the two possible directions of the field and $C_1^{\pm}, C_2^{\pm}$ are constants which are in general different for $+$ and $-$ if there are gap asymmetries or differing dielectric and metal electrodes. These constants can be expressed in terms of effective work functions Φ_+ and Φ_- [7,8], i.e.,

$$C_1^{\pm} = (1.54 \times 10^{-6} / \Phi_{\pm}) \times 10^{(4.52 \Phi_{\pm}^{-0.5})}, \tag{3}$$

$$C_2^{\pm} = 6.83 \times 10^9 \Phi_{\pm}^{1.5}, \tag{4}$$

where $\Phi_{\pm}$ is in units of eV if $E(t)$ is in units of V/cm.

The probability that an electron will be released at an arbitrary time between t and $t + \Delta t$ is given by

$$P_i^{\pm}(t)\Delta t = 1 - \exp(-r_e^{\pm}(t) \, \Delta t). \tag{5}$$

In the computer simulation, the period of the applied voltage, $T = (2\pi\omega)^{-1}$, is segmented into equal increments, Δt, where $T \gg \Delta t$, and $r_e^{\pm}(t)$ is evaluated for the field value at time $t = n\Delta t + \Delta t/2$, where n is an integer increment number such that $T \geq n\Delta t \geq 0$. An electron is assumed to be ejected at time t if $R \leq P_i^{\pm}(t)\Delta t$, where R is a uniformly distributed random number generated by the computer in the interval (0, 1).

Using assumption (2), the probability that a PD pulse formed at a time t will have an amplitude in a range $q^{\pm}$ to $q^{\pm} + dq^{\pm}$ is determined from a Gaussian random number generator so that it is given by

$$P_a^{\pm}(q^{\pm}, t) \, dq^{\pm} = [2\pi\sigma^{\pm}(t)]^{-\frac{1}{2}} \exp\left[\frac{1}{2}\left(\frac{q^{\pm} - \mu^{\pm}(t)}{\sigma^{\pm}(t)}\right)^2\right] dq^{\pm}, \tag{6}$$

where, for cases considered here, $\mu^{\pm}$ and $\sigma^{\pm}$ are given by

$$\mu^{+}(t) = (E(t) - E_I^{+}) + (0.2 + 0.1R)E_I^{+} \tag{7}$$

$$\sigma^{+} = (0.35 + 0.1R)E_I^{+} \tag{8}$$

$$\mu^{-}(t) = (E(t) + E_I^{-}) - (0.1 + 0.1R)E_I^{-} \tag{9}$$

$$\sigma^{-} = -(0.6 + 0.1R)E_I^{-}. \tag{10}$$

Here $E_I^{\pm}$ are adjustable constants that determine the mean local field reduction due to a discharge pulse and satisfy the condition $|E_c| \geq |E_I^{\pm}| > 0$, where E_c is a critical field required for growth of a discharge pulse that satisfies the condition $P_i^{\pm}(E_c)\Delta t \ll 1$.

The third assumption above implies that the drop in local surface field due to the i th PD event can be expressed as

$$\Delta E_i^{\pm} = \kappa q_i^{\pm}, \tag{11}$$

where κ is a constant.

From assumption (4) and (5), the local surface field strength at any given time is given by

$$E(t) = E_o \sin \omega t - \sum_{j=1}^{N} \Delta E_{ij}^{\pm}\,(\phi_{ij}^{\pm}), \tag{12}$$

where $\Delta E_{ij}^{\pm}$ is the field drop due to the i th PD pulse in the j th voltage cycle and N is number of cycles that have occurred up to time t. The phase-of-occurrence, $\phi_{ij}^{\pm}$, of the i th PD pulse is defined to lie in the interval $(0, 2\pi)$. Thus the time, t_{ij}, at which the i th pulse occurred in the j th cycle is given by

$$t_{ij} = \left(\frac{\phi_{ij}}{2\pi} + j - 1\right) T, \tag{13}$$

where $t_{ij} < t$.

STOCHASTIC ANALYSIS OF SIMULATION

The theoretical model described in the previous section has been used as the basis for developing a computer program that generates a continuous sequence of random phase-correlated pulses that mimic the observed behavior of ac-generated PD phenomena. Figure 1 shows a plot of the local field as determined by Eq. (12) for three randomly selected cycles together with the applied field. These data were obtained under the same conditions that apply to the results shown in the next section. The corresponding pulse amplitudes were determined from Eq. (11) for $\kappa = 1$.

As successive pulses are generated by the simulation, their phases and amplitudes are accumulated by the computer into "bins" used to determine a set of various conditional and unconditional probability distributions in "real time". The computer software routine that controls this sorting of pulses is described elsewhere [9].

The distributions determined for the particular case considered in this report include: $p_0(\phi_i^{\pm})$, $p_1(\phi_i^{\pm} \mid Q^{\mp})$, and $p_2(q_i^{\pm} \mid \phi_i^{\pm}, Q^{\mp})$ as defined in previous work [1, 3, 9]. Here $Q^{\pm}$ is the sum of all positive or negative pulse amplitudes for a particular half-cycle, i.e.,

$$Q^{\pm} = \sum_i q_i^{\pm}. \tag{14}$$

The distributions are defined such that, for example, $p_0(\phi_i^-)d\phi_i^-$ is the probability that the i th negative pulse in a cycle has a phase-of-occurrence between ϕ_i^- and $\phi_i^- + d\phi_i^-$; $p_1(\phi_i^- \mid Q^+)d\phi_i^-$ is the probability that it will have a phase-of-occurrence in this range if Q^+ for the previous half-cycle has a "fixed" value, and $p_2(q_i^- \mid Q^+, \phi_i^-)dq_i^-$ is the probability that the i th negative pulse has an amplitude between $q_i^- + dq_i^-$ if both Q^+ and its phase, ϕ_i^-, are fixed. The fixed variables are specified to lie within narrow ranges. The conditional distributions provide a direct indication of memory effects. If, for example, it is found that $p_0(\phi_i^-) \neq p_1(\phi_i^- \mid Q^+)$ for at least some value of Q^+, then it can be stated that the most probable phase-of-occurrence for the i th negative pulse depends on the total charge, Q^+, associated with all PD events on the previous half-cycle. (Note that the amplitude of a PD is conventionally expressed in units of charge [1, 3]).

RESULTS AND DISCUSSION

To obtain the results shown here, the simulation was performed using the values $E_0 = 2 \times 10^8$ V/cm, $\Phi_+ = 0.95\,\mathrm{eV}$, $\Phi_- = 0.57\,\mathrm{eV}$, $E_I^+ = 8.74 \times 10^7$ V/cm, $E_I^- = -3.87 \times 10^7$ V/cm. Data were accumulated for 10^6 cycles using 10^3 time increments, Δt, per cycle. The simulated PD under these conditions produce an asymmetric pattern in which one or two large pulses appear on the positive half-cycle and three to seven smaller pulses typically appear on the negative half-cycle (see Fig. 1). This behavior is similar to that seen in some point-dielectric discharge gaps [2, 6].

Results for unconditional and conditional phase-of-occurrence distributions $p_0(\phi_i^-), p_1(\phi_i^- \mid Q^+)$, $i = 1, 2, 3, 4$ and $p_0(\phi_i^+), p_1(\phi_i^+ \mid Q^-), i = 1, 2$ are shown respectively in Figs. 2 and 3. The corresponding integrated charge distributions $p_0(Q^+)$ and $p_0(Q^-)$ are shown in Fig. 4. The windows that specify the upper and lower fixed value ranges for Q^+ and Q^- used to obtain the conditional distributions shown in Figs. 2 and 3 are also indicated in Fig. 4. It is evident from the conditional distributions that the larger the absolute value of $Q^{\mp}$ the lower is the phase $\phi_i^{\pm}$ at which pulses occur on the subsequent half-cycle. This behavior is consistent with experimental results [2, 3, 6] and clearly demonstrates the existence of significant memory propagation.

It was determined that the bimodal structure in $p_0(Q^+)$ is due primarily to

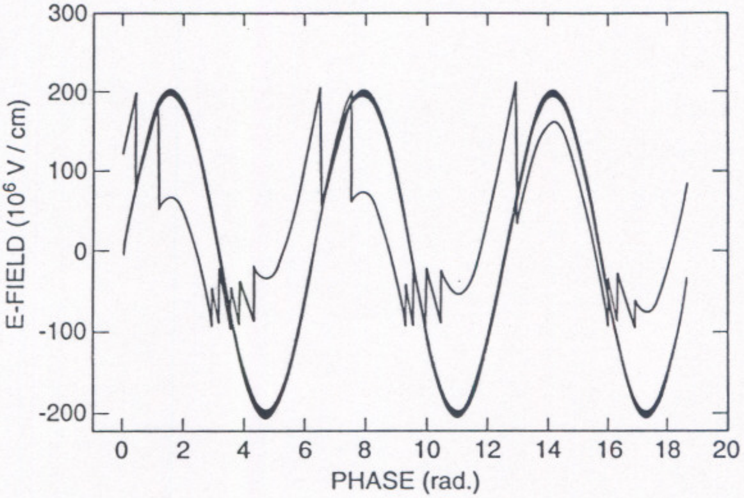

Figure 1. Three randomly selected cycles from a Monte-Carlo PD simulation. Shown is the applied field (thick line) and the local field given by Eq. (12) (thin line).

the difference in total charge on the positive half-cycle resulting from one and two discharge events. This characteristic has also been seen in experimental results. Because the distributions shown in Figs. 2–4 are not independent when

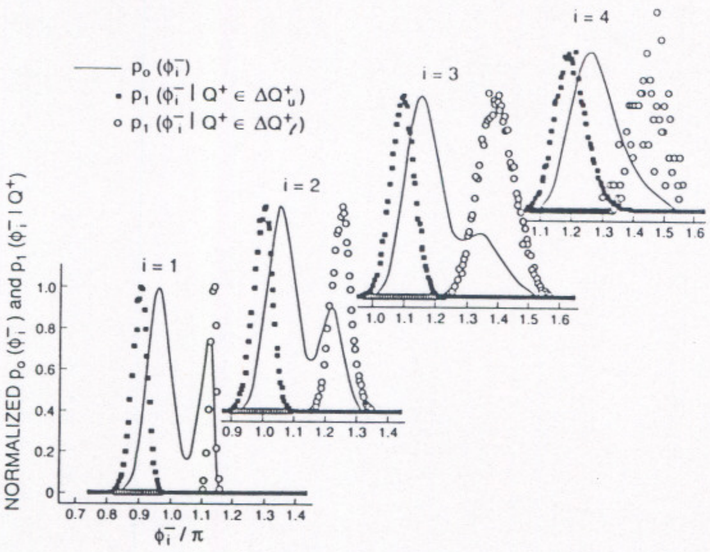

Figure 2. Unconditional (lines) and conditional (points) phase-of-occurrence distributions for the first, second, third, and fourth negative PD pulses. All distributions have been normalized to the maximum value.

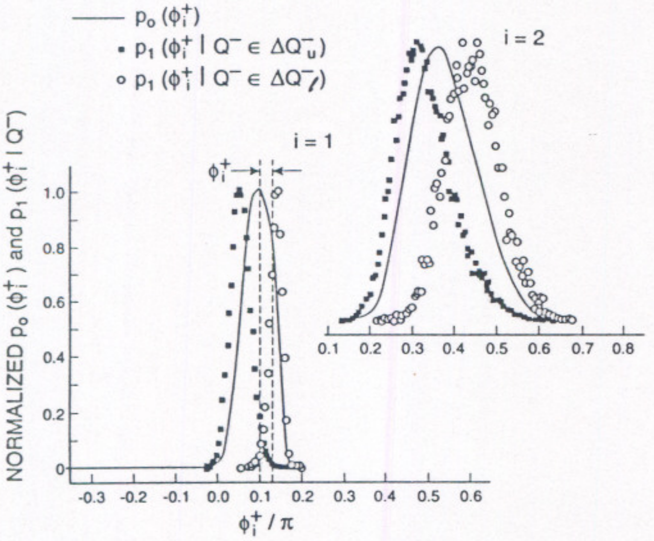

Figure 3. Unconditional (lines) and conditional (points) phase-of-occurrence distributions for the first and second positive PD pulses. Indicated by the vertical lines is the phase window used to specify ϕ_1^+ for the second-order conditional pulse-amplitude distribution shown in Fig. 5. All distributions have been normalized to the maximum value.

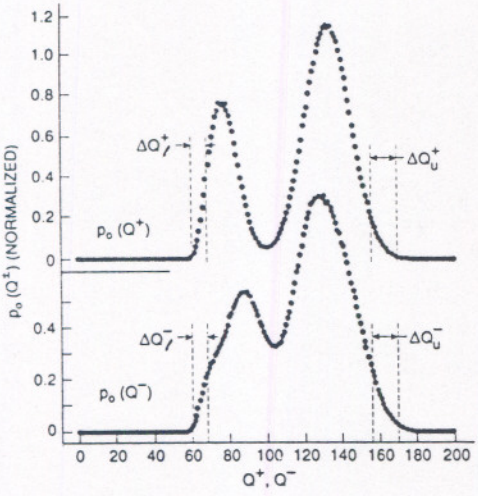

Figure 4. Integrated charge distributions for positive and negative pulses. The vertical lines indicate the windows used to specify the Q^+ and Q^- values for determination of the conditional distributions in Figs. 2, 3, and 5.

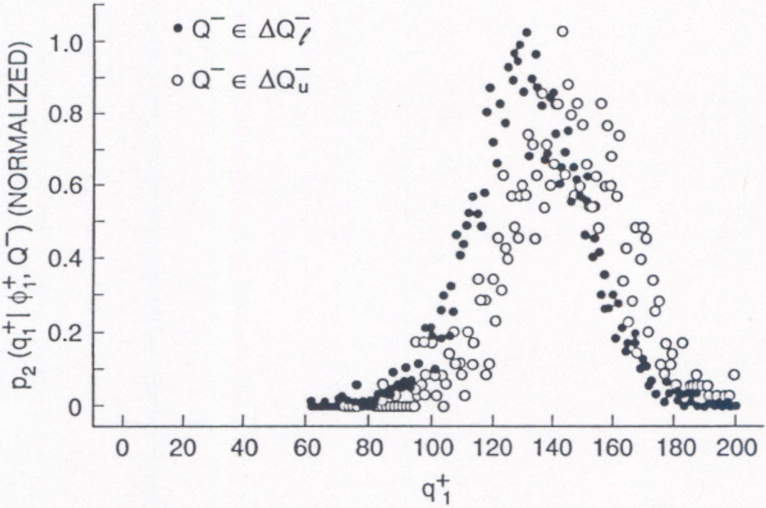

Figure 5. Second-order conditional amplitude distribution for the first positive PD pulse.

memory is important, the bimodal structure originating in $p_0(Q^+)$ can also be reflected in the other unconditional distributions such as $p_0(\phi_1^-)$. The distributions $p_0(\phi_i^-), p_1(\phi_i^- \mid Q^+)$, and $p_0(Q^+)$ are related, according to the law of probabilities, by the integral expression

$$p_0(\phi_i^-) = \int_0^\infty p_0(Q^+) p_1(\phi_i^- \mid Q^+) dQ^+. \tag{15}$$

Bimodal structure in the PD phase distributions has also been seen in experimental results [6].

The second-order conditional distribution $p_2(q_1^+ \mid \phi_1^+, Q^-)$ is shown in Fig. 5. The appropriate windows for ϕ_1^+ and Q^- are those indicated respectively in Figs. 3 and 4. Again, consistent with experimental results [3], it is seen that the larger the absolute value for Q^- in the previous half-cycle, the larger will be the value of the amplitude for the first positive pulse that occurs at a particular phase in the next half-cycle.

It has thus been demonstrated from the results presented here that it is possible, using reasonable assumptions, to simulate PD patterns that have the same stochastic properties as those observed experimentally. Tests of the stochastic behavior are essential in validating theoretical models of pulsating PD phenomena. Simulated PD pulses may also prove useful in testing the performance of systems used to measure stochastic behavior, e.g., conditional pulse-amplitude and phase distributions [9].

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REFERENCES

1. R. J. Van Brunt and S. V. Kulkarni, "Stochastic Properties of Trichel-Pulse Corona: A non-Markovian Random Point Process," Phys. Rev. A, Vol. 42, pp. 4908–4932 (1990).

2. R. J. Van Brunt and E. W. Cernyar, "Influence of Memory Propagation on Phase-Resloved Stochastic Behavior of ac-Generated Partial Discharges," Appl. Phys. Lett. Vol. 58, pp. 2628–2630 (1991).

3. R. J. Van Brunt, "Stochastic Properties of Partial-Discharge Phenomena," IEEE Trans. Elect. Insul. Vol. 26, pp. 902–948 (1991).

4. A. I. Bennett, "Endurance of Electrical Insulation Under Sinusoidal Voltage Stress," Annual Report of the CEIDP, IEEE, NY pp. 80–88 (1987).

5. F. Fruth and L. Niemeyer, "The Importance of Statistical Characteristics of Partial Discharge Data," IEEE Trans. Elect. Insul., Vol. 27, pp. 60–69 (1992).

6. R. J. Van Brunt, E. W. Cernyar, P. von Glahn, and T. Las, "Variations in the Stochastic Behavior of Partial-Discharge Pulses with Point-to-Dielectric Gap Spacing," Conf. Rec. 1992 IEEE Int. Symp. on Elect. Insul., IEEE, NY, pp. 349–353 (1992).

7. R. H. Fowler and L. Nordheim, "Electron Emission in Intense Electric Fields," Proc. Roy. Soc. London, Ser. A, Vol. 119, pp. 173–181 (1928).

8. R. V. Latham, H. V. Vacuum Insulation-Physical Basis, Academic Press, NY, pp. 51–79 (1981).

9. R. J. Van Brunt and E. W. Cernyar, "System for Measuring Conditional Amplitude, Phase, or Time Distributions of Pulsating Phenomena," NIST J. Res. (submitted, 1992).