

# Linewidth measurement by high-pass filtering: a new look

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Earlier workers have noticed that high-pass filtering produces a sharp dark line in precisely the location of the geometrical image of an edge. They proposed using this fact as an aid in measuring linewidth in microscopy but found that the other edge of the line caused significant error. In this paper, I examine that error as a function of normalized linewidth and normalized spatial-filter width and find that it may be limited to  $\pm 5\%$  or so, provided that the spatial filter subtends between 0.25 and  $0.3\times$  the numerical aperture of the objective and that the linewidth exceeds about twice the resolution limit.

When a high-pass filter is located in the frequency plane of a coherent-optical processor, sharp edges in the object stand out brightly in the image; this is commonly called edge enhancement. Some years ago, Birch<sup>1</sup> and later Swing<sup>2</sup> noticed that the bright edge contained a sharp zero of intensity located precisely at the geometrical image of the edge. They proposed using this sharp zero as an aid in measuring linewidth but found that a small error appeared due to the presence of the other edge of the line. Swing proposed an elegant method to minimize the error by tailoring the width of the frequency-plane filter to the approximate width of the line being measured. Nevertheless, as far as I know, high-pass filtering has been used comparatively little in microscopy.<sup>3</sup>

In this paper, I will take a fresh look at the problem and show that, if the filter is chosen properly, the width of any line wider than about twice the resolution limit of the objective may be measured to an accuracy of 5% or thereabouts. This is substantially higher accuracy than that of conventional visual microscopy or photomicroscopy, where the error could be as large as one resolution limit or so.

The calculation is precisely the same as that of Birch: We begin by calculating the diffraction image of an edge when there is a central stop in the frequency plane. We carry out the calculation in one dimension, assume diffraction-limited optics, and, for mathematical convenience only, assume a conventional two-lens optical processor with unit magnification. Unlike Birch, however, we make the results completely general by normalizing the variables so that (1) the dimension  $x$  in the image plane is expressed in units of the resolution

limit  $\lambda/2$  N.A. of the objective, and (2) the angular width (half-angle)  $w$  subtended by the frequency-plane stop is expressed as a fraction of the lens's N.A. The result is a normalized version of Birch or Swing's results; the amplitude  $E(x)$  of the image of the edge is

$$\pi E(x) = Si(\pi x) - Si(\pi x/w), \quad (1)$$

where  $Si$  is the sine integral,<sup>5</sup>

$$Si(z) = \int_0^z (\sin t/t) dt. \quad (2)$$

Figure 1 shows the filtered images of a single edge when  $w = 0.2$  and  $0.4$ . As Birch noticed, it shows a zero at precisely the location of the geometrical image of the edge; I call this the primary zero. In addition, there are secondary zeros one or more resolution limits away. Their locations depend on  $w$ . The first secondary zero moves from  $x = 3.3$  when  $w = 0.2$  to  $1.5$  when  $w = 0.4$ , and the first secondary maximum grows in importance.

A slit may be regarded as two parallel edges, so we may calculate the filtered image of a slit by adding two displaced edge amplitudes. When the edges approach one another, their diffraction images begin to overlap; the amplitude at the geometrical-image points will not be zero unless one of the secondary zeros of one edge chances to fall precisely at the geometrical image of the other edge. (Forcing that to happen for each linewidth is the basis of Swing's optimization procedure.) The position of the primary zero is, therefore, displaced by the proximity of the other edge of the line; this causes an error if the linewidth is assumed to be equal to the separation of the primary zeros.

How large is this error and how can it be minimized? To answer this question, I calculated the normalized intensity,

$$\pi^2 I(x) = [E(x) + E(x + S)]^2, \quad (3)$$

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Received 19 February 1983.

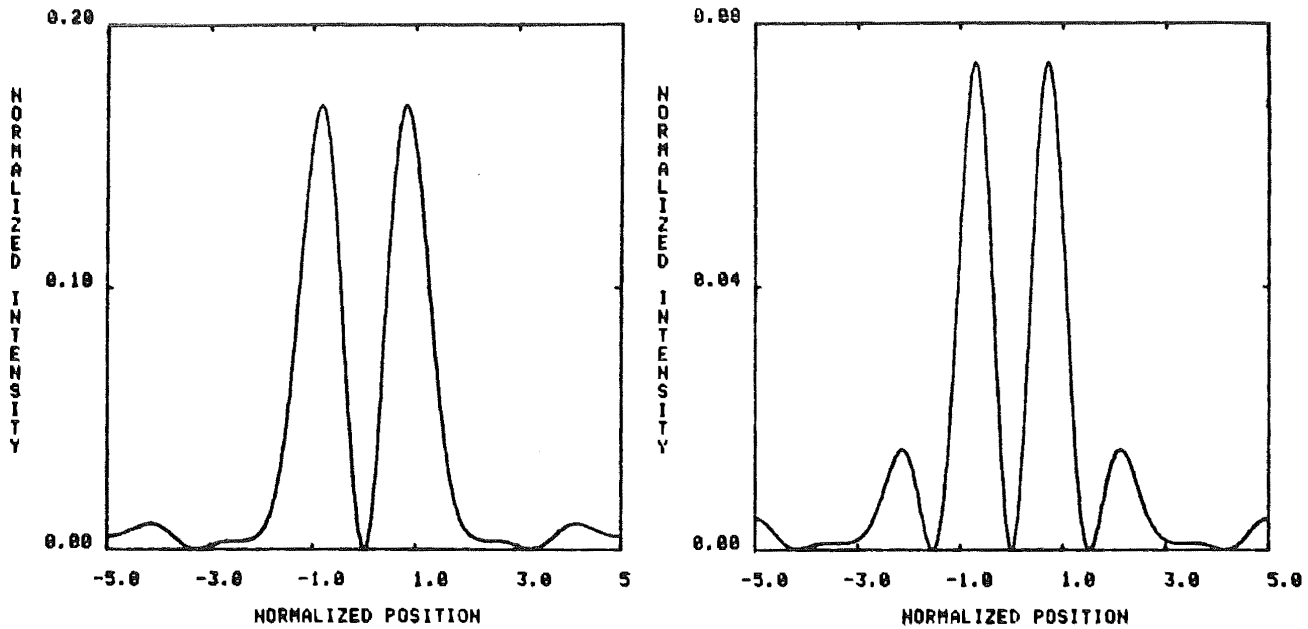


Fig. 1. Filtered images of a single edge normalized to incident intensity: (left)  $w = 0.2$ ; (right)  $w = 0.4$ . Positions of secondary zeros have shifted, and secondary maxima have increased in intensity.

for various values of  $w$  and slit width  $S$ . The intensity of the incident beam is 1.

Figure 2 shows filtered images of a slit whose width is twice the resolution limit of the objective. The intensity is normalized to the incident intensity. The normalized filter widths are 0.2, 0.4, and 0.6. (The case of no filtering is included for comparison.) In the first case, the measured separation  $S'$  between the primary zeros is 2.16, not the expected value of 2, whereas in the second case it is 1.68 and in the third 1.36. The intensities of the central maxima decrease rapidly as  $w$  increases; in addition, the intensities of the higher-order maxima increase relative to that of the central maxima. As a result, calculations are not carried beyond  $w = 0.6$ .

The measured separation  $S'$  differs from the width  $S$  of the geometrical image and varies with  $w$ ; the percent error  $e$  may be defined as

$$e = (S' - S)/S; \quad (4)$$

we wish to minimize this quantity for as large a range of values of  $S$  as possible. Figure 3 shows several plots of  $e$  as a function of normalized filter width  $w$  for some representative values of  $S$  (expressed in units of resolution limit).

Examination of a larger number of such plots led me to conclude that the curves are bounded approximately by the envelopes shown in Fig. 4 as long as the image width  $S$  remains larger than twice the resolution limit. (When  $S$  is appreciably smaller than that value,  $e$  becomes quite large.) The envelopes approach each other most closely in the region where  $w = 0.25-0.3$  or where the filter blocks  $\sim 0.3$  of the lens's numerical aperture. For that value, the error of measuring any linewidth is between  $-8$  and  $+2.5\%$ . Therefore, if we arbitrarily add

a correction of  $+3\%$  to any measurement, the result will be accurate to  $\pm 5\%$ .

The dashed curves show the effect of restricting  $S$  to three resolution limits or greater; the acceptable range of  $w$  increases somewhat, but the precision increases only a small amount. (A slightly narrower waist appears near  $w = 0.5$ , but this may be an unacceptably large value of  $w$ .)

Because we have used normalized parameters, this result is completely general; it applies to any coherent system, no matter what the magnification or the divergence of the beam that illuminates the object. Likewise, except for the region around  $w = 0$ , the figures apply equally well to opaque or transparent lines or to phase objects as long as they are not physically too thick. Therefore, the principle may be applied to conventional microscopic systems, although the optimal angular subtense of the filter may differ slightly in a system that has cylindrical symmetry.

Besides the use of a laser source that oscillates in a 00 transverse mode, only one modification of a conventional microscope is required: The condenser must now have relatively high optical quality so that the diffraction pattern in the transform plane is sharp; a microscope objective would be a suitable condenser. (Problems that result from the use of coherent light are common to all coherent-imaging systems, and I will not dwell on them here.)

Swing has noted that the frequency plane of most microscope objectives is located inaccessibly within the lens itself. This is true if the object is illuminated with a collimated beam. However, with laser illumination there is no need for a special illumination system (like Kohler or critical illumination). I have found that the object may be located in a sharply diverging beam, so

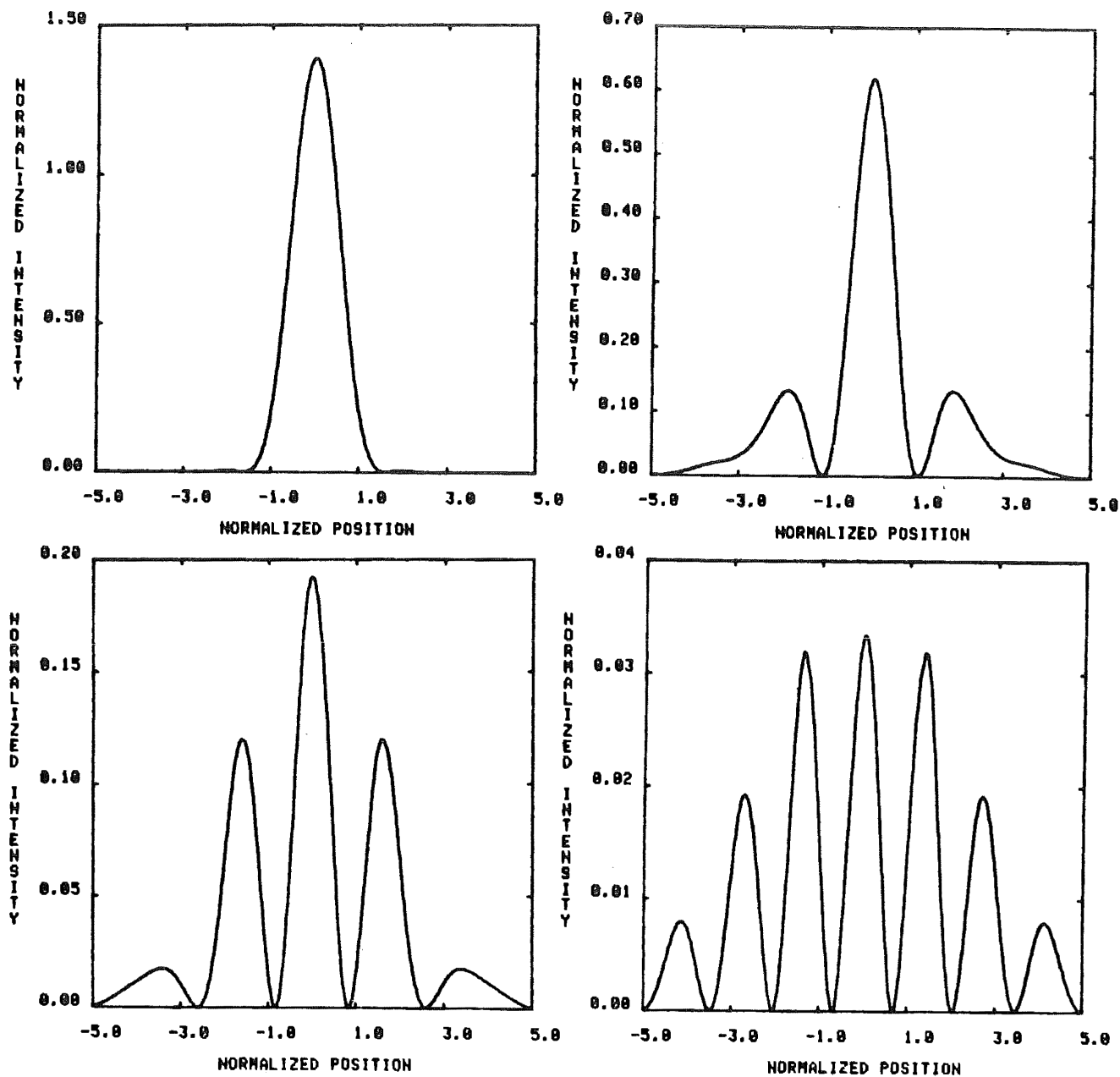


Fig. 2. Images of slit whose width is two resolution limits. Upper left—no filtering. Upper right— $w = 0.2$ ; apparent linewidth is 2.16, not 2. Lower left— $w = 0.4$ ; apparent linewidth is 1.68. Lower right— $w = 0.6$ ; apparent linewidth is 1.36, and third and fourth maxima are now significant.

that the frequency plane (which is just the plane of the image of the source<sup>4</sup>) is shifted well outside the objective. Alternately, the image may be relayed in the manner of Ref. 3 and the filter located in the frequency plane of the relay lens. In either case, the transform will exhibit field curvature and spherical aberration because the lens is not specially corrected for Fourier-transform optics. Since the filter consists of a simple opaque dot on a plane surface, these will have an effect similar to tapering the transmittance of the filter edges and ought

not be a serious problem since the waist of Fig. 2 is relatively gradual.

Thus I anticipate that a practical system can be made with conventional microscope parts with a microscope objective replacing the condensing lens. The resolution limit of such a system will, as a practical matter, be limited to about twice the value of  $0.61\lambda/\text{N.A.}$  that applies in an ordinary microscope. Quantitative measurement of linewidth should be accurate to  $\pm 5\%$  or so, even for the narrowest lines measured.

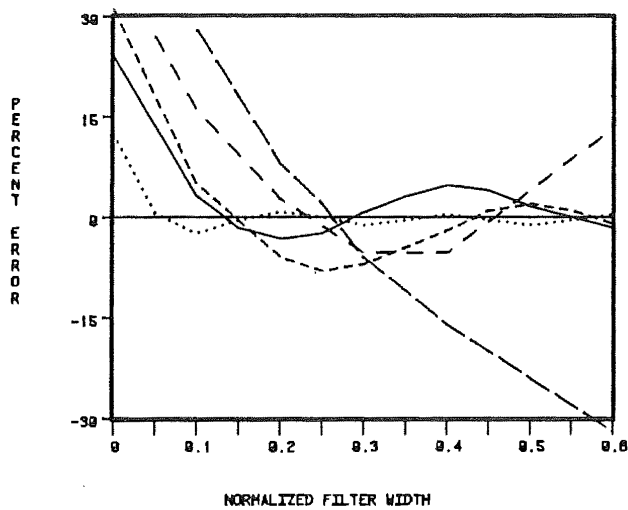


Fig. 3. Percent error as a function of normalized spatial-filter width for five different linewidths: — — —, linewidth equals 2 resolution limits; - - - - -, 3; ·····, 4; — · — ·, 5; and ·····, 10.

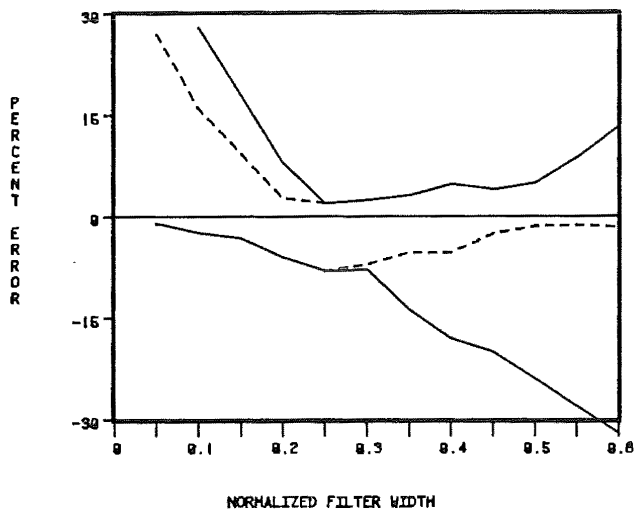


Fig. 4. Approximate envelopes of a complete set of curves like those in Fig. 3: — — —, all linewidths of  $\geq 2$  resolution limits; - - - - -,  $\geq 3$  resolution limits.

## References

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