

JOSEPHSON STANDARDS FOR AC VOLTAGE METROLOGY

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Abstract

We have designed a new generation Josephson array that replaces the traditional superconductor-insulator-superconductor (SIS) junctions with superconductor-normal metal-superconductor (SNS) junctions. These new arrays generate inherently stable voltages and respond to broadband inputs that can be programmed to generate metrologically accurate ac waveforms. To do this we use an input to an SNS junction arrays that consists of a long (up to 8 Mbits), repetitive, digital pulse train that is clocked at frequencies up to 12 Gbit/s. In preliminary experiments using arrays of 1000 junctions, this technique has been used to synthesize both stable dc levels and sinewaves of a few millivolts in amplitude at frequencies up to 1 MHz. The continuing effort is focused on increasing the clock frequency and the number of junctions in the array to achieve metrologically practical voltages of a few volts.

Introduction

In this paper we describe conventional dc Josephson voltage standards in terms of a summation of quantized pulses that are synchronized to an external reference frequency. We then show how we have synthesized time dependent voltages using a bipolar digital pulse sequence. The time dependent voltage is precisely calculable based on the Josephson representation of the volt. Preliminary experiments confirm this idea for voltages of a few millivolts and output bandwidth up to 1 MHz.

In the language of digital electronics, a Josephson junction may be thought of as a one-shot, that is, a device that can be triggered to generate a standard pulse. The unique feature of these Josephson pulses is that, while their height and width depend on the embedding circuit, their time integral is exactly equal to $1/K_J = 2.067\ 833\ \text{mV}\cdot\text{ps}$. $K_J = 483\ 597.9\ \text{GHz/V}$ is a defined constant that is approximately $2e/h$, where h is Planck's constant and e is the elementary charge. The

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ability of Josephson junctions to generate quantized pulses forms the basis for the representation of the SI volt. In this representation, large series arrays of N Josephson junctions are triggered by a fixed microwave frequency f . The amplitude of the microwave frequency is such that any given junction may generate up to about 5 pulses during each microwave drive cycle. In this case it is easy to show that the average voltage across the array is given by

$$V = mf/K_J, \quad (1)$$

where m is an integer representing the number of pulses per junction summed over all N junctions. Most existing Josephson voltage standards use hysteretic SIS junctions in which spontaneous transitions in the value of m may occur. The time scale of these transitions varies from a few seconds to many hours depending on the quality of the Josephson array and the noise in its bias current. For dc measurements the spontaneous transition problem is easily resolved with software. Josephson voltage standards using SIS junctions have been implemented in about 50 standards laboratories. They have improved the uniformity of dc standards around the world by about three orders of magnitude over previous standards based on electrochemical cells.¹

Metrologists are now seeking ways to exploit the Josephson effect in applications that require stable and programmable dc voltages and for the synthesis of ac voltage. The most advanced circuit for achieving this goal uses 32 768 SNS junctions in a binary sequence of 9 arrays.² The internal resistance of the SNS junctions results in a nonhysteretic I-V curve that is inherently stable. As a result the problem of unwanted transitions in the quantum integer m is eliminated. The arrays can be independently selected to generate a quantized scale of output voltages over the range from -1.2 V to +1.2 V. This new circuit is already being used in applications where perfectly stable and programmable voltages are required.

A staircase approximation to any waveform can be generated by rapidly switching selected arrays. Unfortunately, the switching transients that occur in moving from one voltage to the next add substantial uncertainty to the generated waveform. This uncertainty makes the switched array technique inferior to other ac voltage metrology methods above frequencies of about 100 Hz^{3,4}.

Solving the Switching Transient Problem

Both the single-array dc standard and the binary programmable array standards control the output voltage by changing the quantum number m in Eq. (1). The output voltage can also be controlled by changing the excitation frequency f . Unfortunately, in the case of a sine wave excitation, the range of junction bias current over which Eq. (1) holds (usually called the step amplitude) collapses rapidly to zero as the frequency decreases. This means that it is practical to control the voltage via the frequency only over about a 2:1 frequency range. Recently Benz et al. have shown that if the sine wave excitation is replaced with a pulse excitation, the step amplitude is independent of the pulse repetition frequency for all frequencies below a characteristic frequency $f_c = I_c R_n / K_J$, where I_c is the junction's critical current and R_n is the junction's normal state resistance.⁵⁻⁶ The optimum pulse width is $\tau \approx 1/(2\pi f_c)$.

As shown in Fig. 1, a programmable voltage source based on this idea consists of a single large array of N junctions distributed along a wide bandwidth transmission line. When a pulse

propagates down the line, it induces a quantized voltage pulse with an area of n/K_J (n is an integer) across each junction it passes. Thus a pulse train at frequency f propagating down the line generates an average voltage nNf/K_J across the ends of the array. A complex output waveform can be generated by gating the pulse train with a long digital word generator. A knowledge of the digital code, the clock frequency, and the number of junctions in the array is sufficient to precisely calculate the output waveform. Since there is only one array, the number of Josephson pulses occurring in any time increment is calculable and the uncertainty associated with switching between arrays is eliminated.

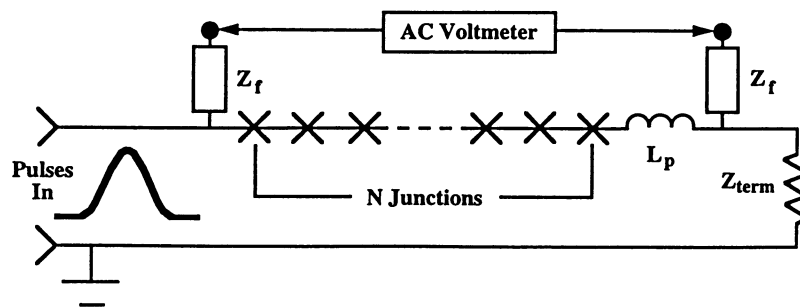


Fig. 1 A programmable voltage source based on driving an array of Josephson junctions with a pattern of pulses.

Josephson Delta Sigma D/A Converter

The pulse quantizing behavior of a Josephson junction can be readily understood by the numerical simulation of a resistively shunted junction (RSJ) model as shown in Fig. 2. In this model, the junctions shown as an X in Fig. 1 are modeled mathematically by a resistor R_n in parallel with a pure Josephson element. Physically, R_n is the resistance of the normal metal junction barrier and it has a value of about 0.004Ω . The total current through the junction is given by

$$I = V/R_n + I_c \sin \{ (2\pi K_J) \int V dt \} , \quad (2)$$

where $I_c \approx 5$ mA is a constant of the junction and V is the voltage across the junction. The bottom traces in Fig. 2 show a sequence of inputs $I(t)$ in which the pulses on the left span a range of peak amplitude from 0 to 2.4 mA, those in the center span the range from 2.9 to 5.9 mA, and those on the right the range from 6.8 to 9.6 mA. The middle curves of Fig. 2 show the corresponding junction voltage responses. For input pulses up to 2.4 mA, the junction voltage is a transient with a time integral of exactly 0. Above a threshold of 2.9 mA, the junction generates a voltage pulse. The shape of this voltage pulse varies with the input current pulse amplitude, but its time integral is exactly $1/K_J$. Above a second threshold of 6.8 mA, the time integral of the voltage response doubles to exactly $2/K_J$. The top set of curves plot the time integral (area) of the voltage pulses and illustrate the exact quantization into areas of 0, $1/K_J$, or $2/K_J$. In a series array of N junctions the voltages across all junctions add. If the input current pulse amplitude falls in the range that generates just one output pulse for each junction, then the total output pulse time integral is exactly N/K_J .

Figure 3 is a block diagram of the process that is used to synthesize a sinewave or any other

periodic waveform from quantized Josephson pulses. The part within the dashed box, usually called the modulator, is a computer program that digitizes one period of the input sinewave into N_s two state (-1 or +1) digital samples. The mathematical details of the modulator algorithm are beyond the scope of this paper.⁷ Suffice it to say that the modulator output is a long digital sequence where the density and sign of the 1's are proportional to the input $S(t)$. For a repetitive waveform, this code is calculated just once and stored in a circulating memory. To re-create the waveform as an output voltage in real time, the memory is clocked at a frequency $N_s f_1$, where f_1 is the desired output frequency. Each output digit generates either a negative, or positive current pulse that is launched into the Josephson array stripline. The Josephson array quantizes the time integral of the pulses. The pulse-induced voltage across the array consists of the desired waveform plus additional harmonics of f_1 that are known as quantization noise. Since most of the quantization noise occurs at frequencies far above f_1 , a low pass filter can eliminate it, leaving only the desired waveform $S(t)$. The modulator algorithm is chosen to minimize the quantization noise spectrum near the signal frequency thus giving a very high signal-to-noise ratio in the filter pass band. Systems that generate analog signals by summing many pulses, like that shown in Fig. 3, are called delta-sigma digital-to-analog converters.

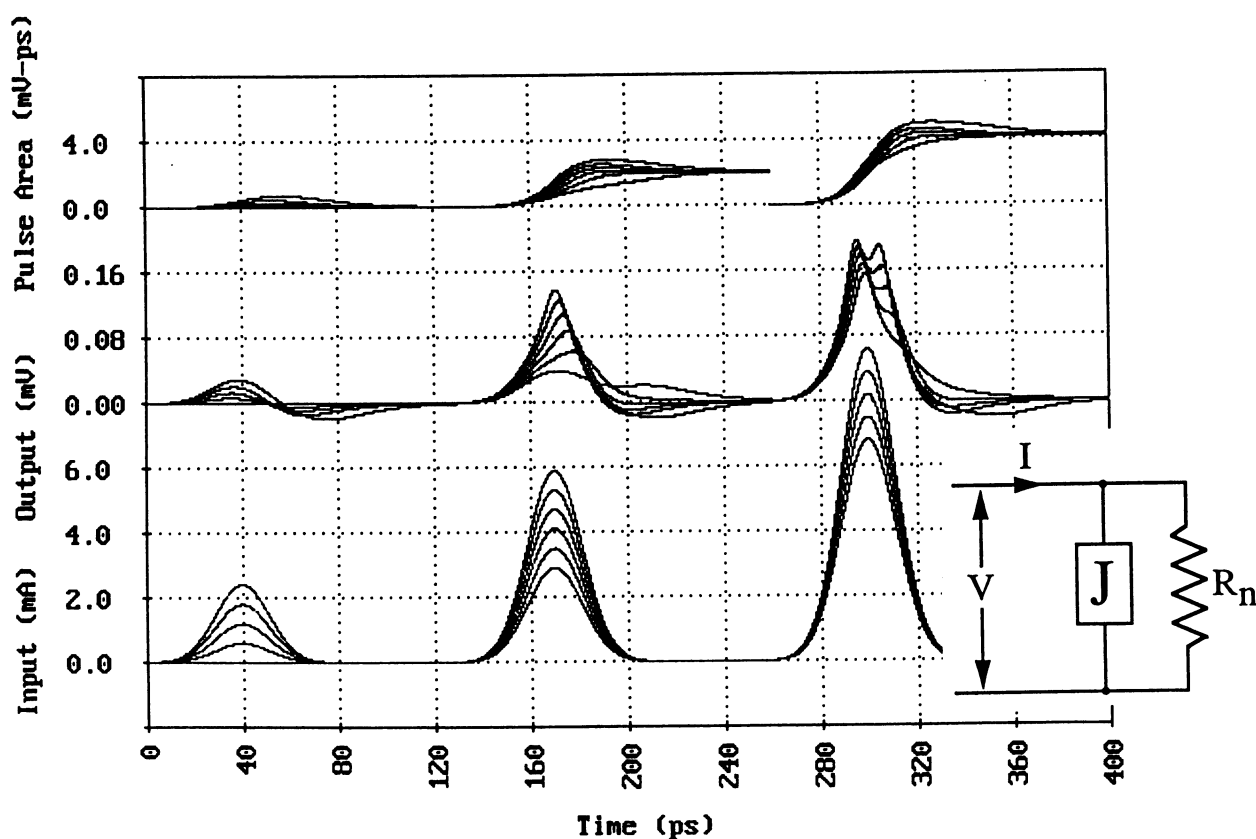


Fig. 2 The input current (bottom), junction voltage (middle), and time integral of the junction voltage (top) for a pulse driven Josephson junction. The curves illustrate the quantization of the time integral of the junction voltage. The inset shows the equivalent circuit for the resistively shunted junction model used in the simulation.

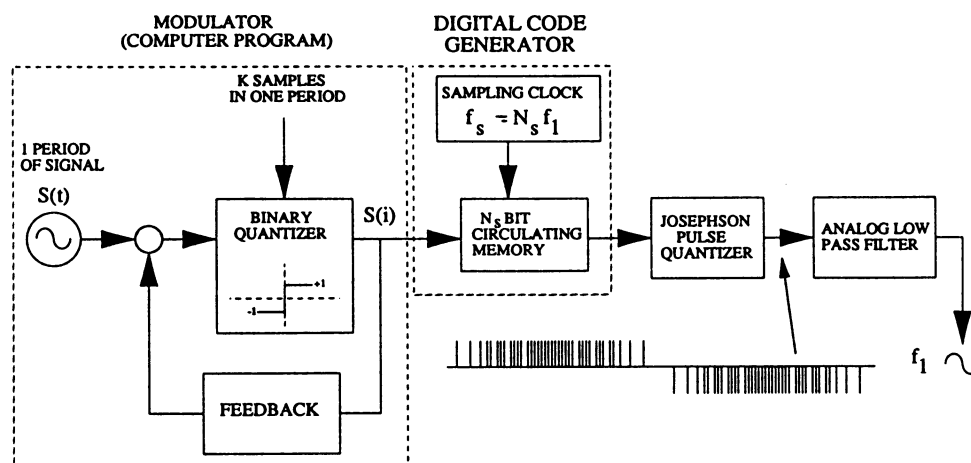


Fig. 3 A block diagram of a delta-sigma digital-to-analog converter based on pulsed Josephson junctions.

Fourier Analysis of the Josephson D/A Converter Output

Let us consider the Fourier spectrum of an example digital code. To simplify the calculation, we choose for the code a sequence of 2560 binary digits representing 4 periods of a 1 MHz offset sine wave. Since the sine wave is generated by repeating the digital sequence indefinitely, the Fourier transform of the digital code is a line spectrum with a dominant line at $f_i = 1$ MHz. This line spectrum for a perfect digital code is shown in Fig. 4, where for clarity we have connected the points representing the power in each harmonic. The spike at 1 MHz is the desired sine wave signal and all of the other lines are quantization noise. Note that the quantization noise within one decade of the signal frequency is down by about 100 dB. If we could generate such a perfect digital code then there would be no need for the Josephson array because a perfect digital code is by definition quantized. In fact, the output of typical digital code generators has both correlated and uncorrelated amplitude and phase noise. When we recompute the Fourier spectrum including realistic values for the amplitude and phase noise, the result is the curve labeled "real digital code". This spectrum is far too noisy to be useful for metrology purposes. When we add the quantizing effect of the Josephson array we arrive at the spectrum labeled "Josephson quantized code", a result almost identical to the spectrum of the ideal digital code. Thus, even though the Josephson array does nothing to improve time jitter, and it does not quantize the pulse amplitudes, the fact that the time integral of the pulses is quantized is sufficient to produce a nearly ideal spectrum. Finally we add the filter function to arrive at the output filtered spectrum shown in Fig. 4. In a metrology application we might now ask what contribution the quantization noise makes to the rms value of the signal at f_i . This is readily computable by comparing the rms value of the line at f_i with the rms value of the total filtered output spectrum. In this example, the quantization noise increases the rms value by only 1 part in 10^9 . In practice, other effects discussed below will dominate the uncertainty of the output voltage.

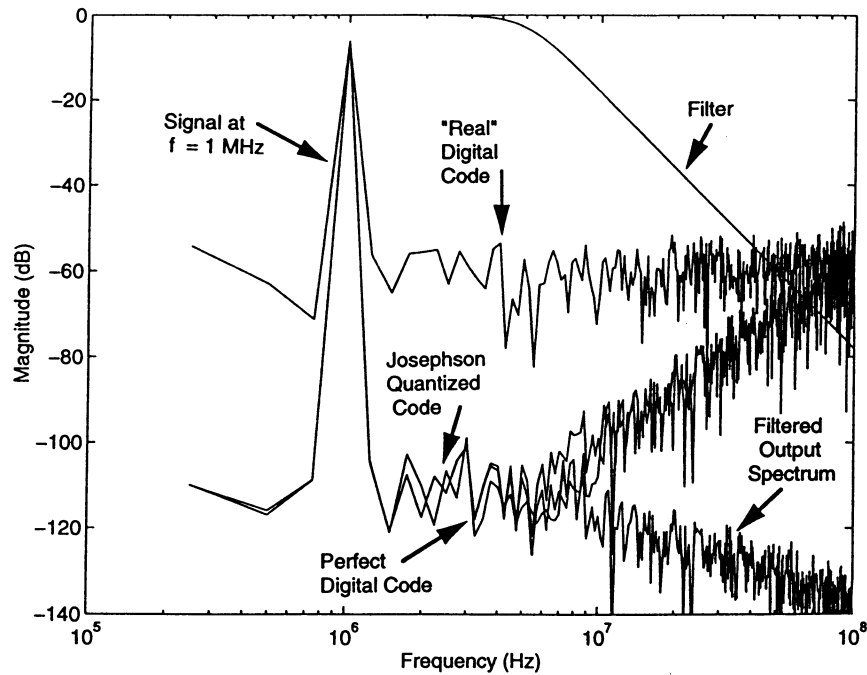


Fig. 4 The simulated Fourier spectra of the signals in the Josephson delta-sigma D/A converter.

Experimental Results

We have designed and fabricated the circuit of Fig. 5 to prove the feasibility of these ideas. It consists of a single 8 mm long array of 1000 junctions along the center conductor of a 50 Ω coplanar stripline. The stripline is terminated in a 50 Ω resistor and connections to the ends of the array are made through low pass filters. Equation 1 shows that the maximum voltage range of this test circuit is only a few millivolts. This is sufficient for our feasibility study but much larger circuits with up to 100 000 junctions will be required for a practical metrology device.

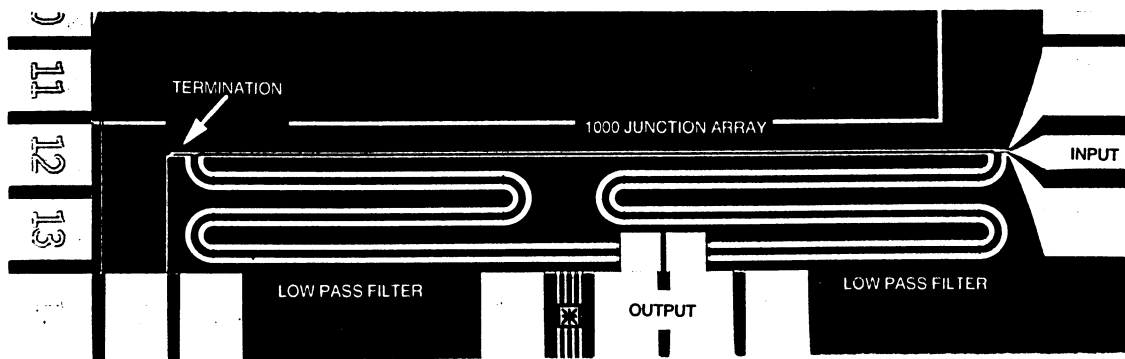


Fig. 5 A photograph of the 1000 junction array test circuit.

Figure 6 is our first attempt to verify experimentally the theoretical results of Fig. 4. In this case we used a 2.56 Mbit long code to synthesize a 1 kHz sine wave. The spectrum on the left

shows the fundamental (peak 1) and the first few harmonics of the output of the digital code generator. These large harmonics are caused by the fact that the voltage levels of the digital code are correlated with the density of 1's in the code. The spectrum of the corresponding array output is shown on the right. In this case the Josephson array circuit has reduced the unwanted harmonics by about 22 dB.

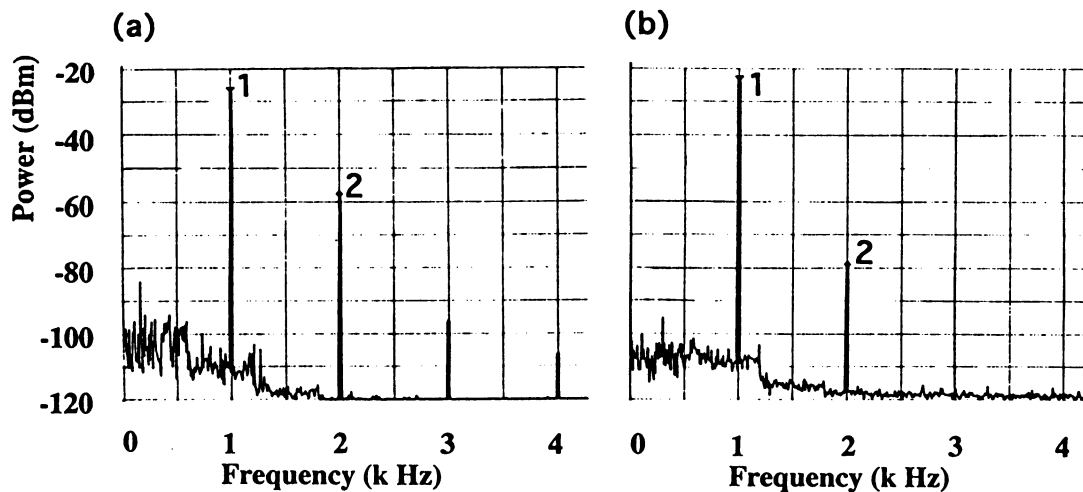


Fig. 6 Measured Fourier spectra showing how the quantizing effect of the array improves signal purity. (a) Digital code generator spectrum, (b) Amplified spectrum of Josephson array voltage.

This verifies the principle that the quantizing effect of the array will greatly reduce the effect of amplitude noise in the pulse input. However, Figure 4 suggests that the improvement should be greater than 50 dB rather than just 22 dB. The reason for this discrepancy is believed to be insufficient common mode rejection in our measurement circuit as discussed below.

Influence of the Microwave Circuit Design on the Amplitude Uncertainty

The design of the passive components Z_f and Z_{term} in Fig. 1 is critical to achieve the lowest possible uncertainty in the amplitude of the generated voltage V_{out} . Z_{term} is required to eliminate pulse reflections at the end of the line. Any such reflections result in nonuniform input pulse coupling to the junctions along the line. If the input pulse amplitude for any junction falls outside the quantization margins illustrated in Fig. 2, then the output amplitude is no longer calculable. However, Z_{term} generates an error in the measured voltage through the voltmeter's finite common mode rejection ratio (CMRR). In our first experiment, the common mode voltage at the signal frequency is more than 100 times greater than the signal V_{out} generated by the junctions. Thus, to achieve a 1 part in 10^6 uncertainty in V_{out} , the voltmeter would need a CMRR of greater than 160 dB – a most unreasonable requirement. Future circuits will be designed to minimize common mode voltages.

A second challenge is to design the output filters Z_f at either end of the array. These filters need to have a high impedance at the clock frequency so as not to disturb the input pulses and they must be a low impedance at the signal frequency so as not to attenuate the signal output. Achieving the required broadband response, within the constraints of planar integrated circuit design, is a considerable challenge.

In addition to the junctions, the coplanar transmission line also has a distributed inductance represented schematically by a lumped inductance L_p in Fig. 2. The spectrum of the pulse train contains a strong component at the signal frequency, and this component induces an unwanted voltage across L_p . While this voltage is in quadrature with the signal voltage, it still contributes a significant error at frequencies above 10 kHz. Similarly, stray capacitive or inductive coupling between the input pulse train and the output leads contributes another source of uncertainty in the output voltage. These effects, however, can be measured and corrected because they add a component to V_{out} that varies linearly with the input pulse amplitude. (Over the normal range of operation, the junction contribution to V_{out} is independent of input pulse amplitude.) The measurement is complicated by the CMRR problem described above, but the result at 1 kHz signal frequency is on the order of $0.2 \mu\text{V}$ across L_p per volt at the input to the coplanar line.

Another significant uncertainty component arises from the effects of attenuation and reflection in the coaxial cable to the ac voltmeter. AC voltage metrology traditionally minimizes this uncertainty by using short lengths of cable and by defining a plane where the ac voltage is known. For our prototype pulse-driven array standard the defined ac voltage is on a chip inside a liquid helium Dewar and at least 1 m of measurement cable is required. The magnitude of the errors that result depend on the type of cable used, its length, and the signal frequency. As an example, the error in the 1 MHz signal voltage measured by an ac voltmeter with a high input impedance is approximately 500×10^{-6} for a 1 m length of RG-58 coaxial cable. Since the error is proportional to the square of the length, it could be reduced by two orders of magnitude by using a helium cryostat designed to allow the use of 0.1 m cable length.

Conclusion

Implementations of Josephson voltage standards prior to this work use a fixed frequency to trigger quantized junction pulses that are averaged into a dc voltage. In this paper we demonstrate how a complex pattern of pulses driving an array of 1000 junctions produces a smooth sinewave output with high resolution and accuracy. Efforts are underway to increase the number of junctions and the clock frequency to achieve a voltage range of ± 1 V. Success in this effort will lead to a new programmable Josephson voltage standard that can produce quantum mechanically accurate dc and ac voltages with a bandwidth of 0 to about 1 MHz.

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