Monte Carlo estimation of noise-parameter uncertainties

J. Randa and W. Wiatr

Abstract: Uncertainties in noise-parameter measurements, obtained using a Monte Carlo simulation of the measurements, are presented. Sets of data were generated to simulate measurements on a low-noise amplifier, with given uncertainties in the underlying measurements, including the standard noise temperature (hot or cold), ambient temperature, reflection coefficients of the terminations, scattering parameters of the amplifier, power measurements, and variations in the connections. Each set of simulated measurement results was analysed to determine the "measured" noise parameters, and the standard deviation of the set of measured noise-parameter values was computed to determine the uncertainty in each noise parameter. Results are presented for the noise-parameter uncertainties for different values of the underlying measurement uncertainties.

1 Introduction

At present, most noise-parameter measurements are not accompanied by an uncertainty analysis. As low-noise amplifiers become quieter and more widespread, small differences in the noise parameters become increasingly important, and the lack of reliable uncertainties becomes increasingly problematic. The problem of uncertainty propagation in measurements of amplifier noise parameters does not admit a simple analytical solution. The four noise parameters are nonlinear functions of the underlying measured quantities, and in a typical measurement the noise parameters are determined from a least-squares fit to an overdetermined system of equations.

A convenient way to evaluate uncertainties in such cases is by Monte Carlo simulation. Simulations have been used to investigate the optimal pattern of source reflection coefficients in noise-parameter measurements [1–3], but those papers did not look into the issue of uncertainty propagation in detail. In this paper we use a Monte Carlo simulation to estimate the uncertainties in 'typical' measurements of noise parameters of a low-noise amplifier (LNA) and to investigate how the uncertainties in the noise parameters depend on the uncertainties in the underlying quantities, such as the noise temperature of the non-ambient noise source, the ambient temperature, the reflection coefficients of the terminations, the scattering parameters of the amplifier, the power measurements, and variations in the connections. An abbreviated account of this work can be found in [4].

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J. Randa is with the Radio Frequency Technology Division, National Institute of Standards and Technology, Boulder, CO 80305, USA

W. Wiatr is with the Warsaw University of Technology, Institute of Electronic Systems, Warsaw, Poland

2 Model and procedures

We begin by reviewing the measurement process that we wish to simulate. The standard method for measuring amplifier noise parameters employs redundant measurements [5] and measurement of noise temperature or power [6, 7], rather than noise figure, for several different input impedances. A number of different terminations (i) of known reflection coefficient $\Gamma_{G,i}$ and noise temperature $T_{G,i}$ are connected to the input of the amplifier, and the output power $P_{out,i}$ is measured for each. There is an equation that relates the output power to the amplifier noise parameters, the amplifier scattering parameters S_{jk}, and the noise temperature and reflection coefficient of the termination. Consequently each measurement yields an equation relating the noise parameters (which we wish to determine) to known or measured quantities. By measuring a number (N_{meas}) of different terminations (usually between 10 and 20), one obtains an overdetermined set of nonlinear equations, which is then solved for the noise parameters by a least-squares fit. The amplifier gain is usually included with the noise parameters in the set of unknowns to be determined, and we do so in this paper.

There are several different parameterisations for the noise characteristics of amplifiers, and the exact form of the equation for P_{out} depends on the particular set of noise parameters one chooses to use. The form that we used for the simulation is

$$P_{out} = (k_B B)|S_{21}|^2 \left\{ \frac{(1-|\Gamma_G|^2)}{|1-\Gamma_G S_{11}|^2} T_G + \left| \frac{\Gamma_G}{1-\Gamma_G S_{11}} \right|^2 X_1 + X_2 + 2\,\mathrm{Re}\left[\frac{\Gamma_G}{1-\Gamma_G S_{11}} X_{12} \right] \right\} \quad (1)$$

where $k_B B$ is Boltzmann's constant times the bandwidth, P_{out} is the output power measured by a reflectionless power meter, Γ_G and T_G are the reflection coefficient and noise temperature of the input termination, S_{11} and S_{21} are scattering parameters of the amplifier, and X_1, X_2, and X_{12} are the noise parameters to be determined. The Xs are simply related to the usual noise-wave matrix elements [8] by

$$k_B X_1 = \left\langle |\hat{b}_1|^2 \right\rangle, \quad k_B X_2 = \left\langle \left|\frac{\hat{b}_2}{S_{21}}\right|^2 \right\rangle, \qquad (2)$$
$$k_B X_{12} = \left\langle \hat{b}_1 \left(\frac{\hat{b}_2}{S_{21}}\right)^* \right\rangle$$

where the brackets denote time average, and $\hat{b}_1$ and $\hat{b}_2$ are the amplitudes of the noise waves that would emerge from input (1) and output (2) ports of the amplifier if both terminations were reflectionless and noiseless.

Although all our computations are performed in terms of the Xs, the analysis program computes the IEEE set of parameters [9], and all results are presented in terms of them. This corresponds to parameterising the effective input noise temperature T_e by

$$T_e = T_{e,min} + t\frac{|\Gamma_{opt} - \Gamma_G|^2}{|1 + \Gamma_{opt}|^2(1 - |\Gamma_G|^2)} \qquad (3)$$

The two sets of noise parameters are related by

$$t = X_1 + |1 + S_{11}|^2 X_2 - 2\mathrm{Re}[(1 + S_{11})^* X_{12}]$$
$$T_{e,min} = \frac{X_2 - |\Gamma_{opt}|^2[X_1 + |S_{11}|^2 X_2 - 2\mathrm{Re}(S_{11}^* X_{12})]}{(1 + |\Gamma_{opt}|^2)} \qquad (4)$$
$$\Gamma_{opt} = \frac{\eta}{2}\left(1 - \sqrt{1 - \frac{4}{|\eta|^2}}\right)$$

with

$$\eta = \frac{X_2(1 + |S_{11}|^2) + X_1 - 2\,\mathrm{Re}(S_{11}^* X_{12})}{(X_2 S_{11} - X_{12})}. \qquad (5)$$

A good description of the use of Monte Carlo simulation for uncertainty analysis is given in [10]. For the simulation, we first chose 'true' values for the underlying quantities. These comprise the noise and scattering parameters of the amplifier and the noise temperature and reflection coefficient of each termination. We then chose uncertainties for the S_{jk}, $T_{G,i}$, $\Gamma_{G,i}$, and $P_{out,i}$. All measurement distributions were taken to be gaussian. We also chose a value for the connector variability. This accounts for the variation from one connection to the next or from one connector to another. Because of this connector variability the true values of the reflection coefficients and S-parameters vary from connection to connection. The 'true' values chosen for the S-parameters and reflection coefficients are thus average true values.

We generated simulated measured values for the S_{jk}, $T_{G,i}$, and $\Gamma_{G,i}$ in the standard manner, randomly choosing a value from a gaussian distribution centred at the true value. For the complex quantities, real and imaginary parts were generated independently. To generate the simulated power measurement we first calculated the true output power from the equation for output power, using the true values for the noise parameters and the termination noise temperatures, and using the true values of the S-parameters and the reflection coefficient for that connection. The true values for that connection were generated from the average true values using the connector variability as the standard deviation of the gaussian distribution. Once the true output power for the given connection was calculated, the measured value was generated using the uncertainty in the power measurement as the standard deviation. A complete stimulated measurement set then consisted of the measured values for S_{jk} and the measured $T_{G,i}$, $\Gamma_{G,i}$, and $P_{out,i}$ for each of the N_{meas} terminations.

The complete simulated measurement set can be analysed and the noise parameters and gain can be determined in the same way as for real data. The analysis program we used was the eight-term linear model of [11, 12]. None of the results of the present paper should be sensitive to the particular analysis program used. As a check, we compared results for a few test cases to the results obtained using a completely different analysis program and found excellent agreement. To assess the uncertainties in the noise parameters, we generated a large number N_{sim} of simulated measurement sets with the given uncertainties in the underlying quantities. Each simulated measurement set was analysed to produce a set of 'measured' noise parameters, yielding N_{sim} measured values for each parameter. The average and standard deviation of the measured values were computed, and the standard deviation was identified as the uncertainty for a single measurement of that quantity. (Statistics for Γ_{opt} were computed on real and imaginary parts, not on magnitude and phase.)

We tried several different values of N_{sim}, ranging up to 1000. The results for 100 simulations were essentially the same as for 1000, and so we used $N_{sim} = 100$ in all the subsequent simulations. For the complete set of measurements we used 13 different terminations. One was a hot source with $T_{G,1} = 9920\,\mathrm{K}$ and $\Gamma_{G,1} = 0.028070 + 0.022718j$. The others were all at ambient temperature ($T_{G,i} = 296\,\mathrm{K}$), and their reflection coefficients were distributed around the complex plane as shown in Fig. 1. In addition to the hot source, there was one matched load, and the other terminations were reflective or partially reflective loads with various phases. We did not investigate in any detail the effect of changing the number of different terminations used. We did test the effect of eliminating one termination, but the main focus of the paper is the dependence of the noise-parameter uncertainties on the underlying measurement uncertainties. For later reference, we have also shown the true value used for Γ_{opt} in the simulations.

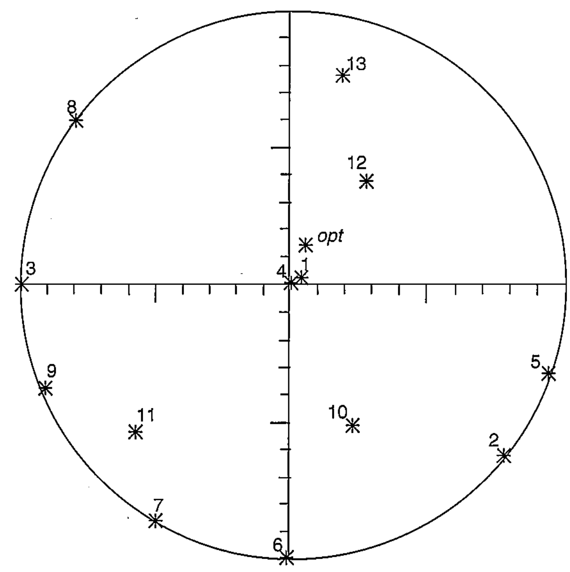

Fig. 1 *Distribution of reflection coefficients of terminations in and on unit circle*

3 Simulation results

The true values for the amplifier were chosen to be $|S_{21}|^2 \equiv G_0 = 2399$ (33.80 dB), $T_{e,min} = 109.6\,\mathrm{K}$ ($F_{min} =$ 1.392 dB), $\Gamma_{opt} = 0.050 + 0.142j$, and $t = 176.3\,\mathrm{K}$. The amplifier's scattering parameters were $S_{11}=0.0181-0.1215j$, $S_{12}=0.0018+0.0007j$, $S_{21}=-39.9609+28.3203j$, $S_{22}=0.1372-0.0300j$. These choices correspond to the values measured for a particular LNA at 11 GHz. We denote the standard deviations (or the standard uncertainties [13, 14]) by σ_Γ for the real or imaginary part of the reflection coefficients and for any S-parameter except S_{21}, σ_{S21} for the real or imaginary part of S_{21}, and σ_{con} for the connector variability. For the ambient noise temperature, the hot noise temperature, and the power measurement, we use fractional standard deviations $\sigma_{a,frac}$, $\sigma_{h,frac}$, and $\sigma_{p,frac}$, respectively. The uncertainties estimated in this paper are type-B uncertainties only [13, 14]. In an actual application there will also be type-A uncertainties that are evaluated statistically in the course of the fitting procedure.

A set of values for the underlying uncertainties was chosen to serve as a baseline, to which other results could be compared. For this purpose we used ($\sigma_{S21}=0.01$, $\sigma_{con}=0.001$, $\sigma_{a,frac}=0.005$, $\sigma_{h,frac}=0.005$, and $\sigma_{p,frac}=0.001$. For the reflection coefficients and the S-parameters other than S_{21}, we used $\sigma_\Gamma=0.002$ if the reflection coefficient was 0.5 or less, and $\sigma_\Gamma=0.003$ if the reflection coefficient was greater than 0.5. These choices are not meant to have any particular significance; they just provide a point of comparison. For the most part they are quite good, but achievable, uncertainties.

With these underlying uncertainties, the 100 simulated measurement sets yielded the following results:

$$\begin{aligned} G_0 &= 2,400 \pm 13 (33.80 \pm 0.02\,\text{dB}) \\ T_{e,min} &= 109.5 \pm 2.5\,\text{K} \\ F_{min} &= 1.391 \pm 0.027\,\text{dB} \\ t &= 176.2 \pm 1.9\,\text{K} \\ \Gamma_{opt} &= (0.050 + 0.140j) \pm (0.013 + 0.010j) \end{aligned}$$

The average value for each parameter agrees well with the input value, as it should. The uncertainty is the sample standard deviation of the distribution. For the reduced gain G_0 and minimum noise figure F_{min} the statistics were computed for the linear quantity, and the results were then converted to decibels.

The effect of increasing the uncertainties in the underlying quantities was investigated by increasing one uncertainty while holding the others fixed at their base values. The results for the power, reflection coefficient, and hot noise temperature uncertainties are given in Tables 1–3. In each Table, the first row of results is for the baseline set of uncertainties, and the following rows demonstrate the effect of relaxing the pertinent uncertainty. The two entries for σ_Γ in the first row of Table 2 reflect the fact that different uncertainties were used for small $|\Gamma|$ and for large $|\Gamma|$, as discussed. For the other rows of Table 2, the same uncertainty was used for all values of $|\Gamma|$. In Table 3 the final two rows correspond to uncertainties of 0.1 and 0.15 dB in the hot noise temperature.

Table 1: Effect of fractional power uncertainty

$\sigma_{p,frac}$	u_G, dB	u_{Tmin}, K	u_{Fmin}, dB	u_t, K	$u_{Re\Gamma}$	$u_{Im\Gamma}$
0.001	0.024	2.5	0.027	1.9	0.013	0.011
0.005	0.033	3.5	0.038	2.5	0.020	0.013
0.010	0.051	5.6	0.061	3.6	0.032	0.020

Table 2: Effect of uncertainty in reflection coefficients

σ_Γ	u_G, dB	u_{Tmin}, K	u_{Fmin}, dB	u_t, K	$u_{Re\Gamma}$	$u_{Im\Gamma}$
0.002, 0.003	0.024	2.5	0.027	1.9	0.013	0.011
0.005	0.024	2.7	0.030	2.7	0.020	0.016
0.010	0.027	3.5	0.038	5.0	0.037	0.032

Table 3: Effect uncertainty in noise temperature of hot source

$\sigma_{h,frac}$	u_G, dB	u_{Tmin}, K	u_{Fmin}, dB	u_t, K	$u_{Re\Gamma}$	$u_{Im\Gamma}$
0.005	0.024	2.5	0.027	1.9	0.013	0.011
0.010	0.048	4.6	0.050	2.6	0.013	0.011
0.020	0.096	8.9	0.098	4.2	0.013	0.011
0.0223	0.112	10.4	0.114	4.8	0.013	0.011
0.0351	0.170	15.6	0.173	7.0	0.013	0.011

The general features of the results in Tables 1–3 are consistent with intuition: the uncertainty in the power measurement affects everything, though not as strongly as one might expect; σ_Γ has a strong effect on the uncertainty in Γ_{opt} and weaker effects on G_0, T_{min}, and t; and the uncertainty in the hot noise temperature has a strong effect on all parameters except Γ_{opt}, on which it has no effect. The present simulation does not include any correlations between successive measurements. Therefore, over the course of the 13 different measurements in each measurement set, the errors tend to cancel or average out. A few readings may be high, but a few others will be low. If correlations are present, e.g. if the power readings or termination temperatures are systematically high, or if all reflection coefficients are wrong in the same direction, the uncertainty could be significantly larger. The effect of correlations on the uncertainties will be studied in subsequent work.

For guidance in estimating achievable uncertainties we have also computed the uncertainties in noise parameters resulting from a few sets of underlying uncertainties that we consider typical or representative of common situations. The four cases are labelled average (meant to represent average industrial laboratory measurements), good (representing measurements by a very good industrial laboratory or a good standards laboratory), and two different very good cases (meant to represent national standards laboratories). Two different very good cases were included to test the difference between using two different types of nonambient noise sources. VG-h uses a hot diode source with $T=9200$ K and with a noise-temperature uncertainty typical of a good national laboratory calibration $u_T=0.5\%$. VG-c uses a cryogenic noise source with $T=78$ K and with an uncertainty equal to that of NIST's cryogenic primary standard $u_T=0.8\%$ [15, 16]. The underlying uncertainties for these four cases are given in Table 4. The Good entries are the same as the baseline case already defined. Average has a much larger uncertainty in the hot noise source, as

Table 4: Underlying uncertainties used in representative cases

Case	$u_{a,frac}$	$u_{h/c,frac}$	$u_{p,frac}$	u_Γ	u_{con}	u_{S21}
Average	0.005	0.020	0.002	0.005	0.001	0.010
Good	0.005	0.005	0.001	0.002, 0.003	0.001	0.010
VG-h	0.001	0.005	0.001	0.002	0.001	0.010
VG-c	0.001	0.008	0.001	0.002	0.001	0.010

might be the case if it were not calibrated by a good standards laboratory, and also has larger uncertainties in the power and reflection coefficient measurements. VG-h is quite similar to good, but it improves the control of the ambient temperature, and it also has a smaller uncertainty for the large reflection coefficients, i.e. the reflective terminations. VG-c is the same as VG-h except for the nonambient noise temperature and its uncertainty.

The uncertainties in the noise parameters for these representative cases are tabulated in Table 5. The results generally require no explanation, but two features warrant comment. The seemingly innocuous change in u_Γ between good and VG-h has a significant effect on the uncertainty in Γ_{opt}. Also, VG-c has significantly smaller uncertainties for G and T_{min} (and therefore also for F_{min}) than VG-h, despite having a larger fractional uncertainty in the noise temperature of the source. This is not so surprising when one considers that the cryogenic standard with $u_T = 0.8\%$ could be used to calibrate a hot source with an uncertainty of $u_T = 0.5\%$ [16]. (This is due to the fact that the important temperature uncertainty is that in the scale $T_{amb} - T_{cry}$, and this is smaller due to the small uncertainty in T_{amb}.)

Table 5: Noise parameter uncertainties for representative cases

Case	u_G, dB	u_{Tmin}, K	u_{Fmin}, dB	u_t, K	$u_{Re\Gamma}$	$u_{Im\Gamma}$
Average	0.101	9.0	0.099	4.6	0.020	0.016
Good	0.024	2.5	0.027	1.9	0.013	0.011
VG-h	0.024	2.4	0.026	1.5	0.008	0.007
VG-c	0.019	1.5	0.016	1.4	0.008	0.007

The final test we performed was to measure the effect of omitting one of the 13 terminations in the measurement set. Using the baseline set of underlying uncertainties, we computed the uncertainties in the noise parameters for the set of 12 measurements resulting from omitting one termination, each in turn. We did not consider omitting the hot noise source since that is needed to determine G_0 and T_{min}. The results are presented in Table 6, with each termination denoted by its number in Fig. 1. The salient feature of the results is that there is little effect in most cases. The uncertainty in the gain is not significantly increased by omission of any one termination. By far the greatest effect on other uncertainties was caused by omitting termination 10. That increased all uncertainties significantly (except u_G), by as much as a factor of three for Im Γ_{opt}. Omitting termination 4 increased the uncertainty in T_{min} from 2.5 to 2.9 K, with a corresponding increase in F_{min}; and omitting termination 11 led to a significant increase in the uncertainty for Re Γ_{opt}. Omitting terminations 5, 8, or 12 resulted in a slight increase in one or two uncertainties, and omission of any one of the other terminations (numbers 2, 3, 6, 7, 9, or 13) had no significant effect on any of the noise-parameter or gain uncertainties. Referring to Fig. 1, it is not clear why termination 10 would have such a large effect on the uncertainties. A cautionary note is that the fact that a particular termination can be safely omitted in this case does *not* imply that it serves no purpose. That termination could be very important for other values of noise parameters.

Table 6: Effect of omitting one termination from measurement set

Omitted	u_G, dB	u_{Tmin}, K	u_{Fmin}, dB	u_t, K	$u_{Re\Gamma}$	$u_{Im\Gamma}$
None	0.024	2.5	0.027	1.9	0.013	0.011
2	0.024	2.5	0.028	1.9	0.013	0.010
3	0.024	2.5	0.027	1.9	0.014	0.011
4	0.024	2.9	0.032	1.9	0.015	0.011
5	0.024	2.5	0.027	2.2	0.014	0.011
6	0.024	2.5	0.027	2.0	0.014	0.012
7	0.024	2.6	0.028	1.9	0.013	0.011
8	0.024	2.5	0.027	2.2	0.013	0.014
9	0.024	2.5	0.027	1.9	0.013	0.010
10	0.024	3.0	0.032	3.0	0.028	0.034
11	0.025	2.7	0.029	1.9	0.019	0.011
12	0.024	2.6	0.026	2.0	0.016	0.013
13	0.024	2.5	0.027	2.0	0.014	0.010

4 Discussion

We performed a Monte Carlo study of the dependence of the uncertainties in measured noise parameters on the uncertainties in the underlying quantities, including hot noise temperature, reflection coefficients of terminations, and power measurements. This was done for a common method of noise-parameter measurement and for a single set of values of noise parameters. We also presented results for the uncertainties for several special cases meant to represent common measurement environments. The present treatment suffers from some significant limitations, which will be addressed in future work. These comprise the effect of nonzero reflection coefficient of the power measuring system, the inclusion of correlations among the underlying measurement uncertainties, and allowing nongaussian distributions in generating the simulated measurements. A software package that would include these extensions is planned. It could then be used to evaluate the uncertainties in general measurements of noise parameters.

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