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## LETTER TO THE EDITOR

# Magnetic field dependence of quantized Hall effect breakdown voltages

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**Abstract.** When large currents are passed through a high-quality quantized Hall resistance device, the voltage drop along the device is observed to assume discrete, quantized states when plotted against the magnetic field. These quantized voltage states are interpreted as occurring when electrons are excited to higher Landau levels and then return to the original Landau level. The quantization is found to be a function of magnetic field, and consequently can be more difficult to verify and determine than previously suspected.

The quantum Hall effect [1] occurs when current is passed through a two-dimensional electron gas (2DEG) formed in a semiconductor device which is cooled to very low temperatures in the presence of a large magnetic field. In high-quality devices the current flow within the 2DEG is nearly dissipationless for currents around 25 $\mu$A. At high currents, however, energy dissipation suddenly appears in these devices [2, 3]. This is called breakdown of the quantum Hall effect.

The dissipative breakdown voltage, $V_x$, can be detected by measuring voltage differences between potential probes placed on either side of the device in the direction of current flow. We found [3] that there is a distinct set of dissipative voltage states, with transient switching among these states observed on microsecond time-scales. Bliek *et al* [4] proposed the existence of a new quantum effect to explain the structures in their curves of $V_x$ versus magnetic field at currents near breakdown for samples with narrow constrictions. Cage *et al* [5] then found that, in wide samples, the distinct states are quantized in voltage. Other laboratories have observed dissipative voltages at breakdown of the quantum Hall effect (private communication), but none have yet confirmed that these voltage states are quantized. We show here that the voltage is indeed quantized, but that the quantization is more complicated than previously suspected because it is a function of the magnetic field.

Our sample is a GaAs/$Al_xGa_{1-x}As$ heterostructure grown by molecular beam epitaxy with $x = 0.29$. It is designated as GaAs(7). It has a zero magnetic field mobility of 100 000 $cm^2\,V^{-1}\,s^{-1}$ at 4.2 K, exhibits excellent integral quantum Hall effect properties, and is used as the new United States resistance standard. The inset of figure 1 shows the geometry of this sample. It is 4.6 mm long and 0.4 mm wide. The two outer Hall potential probe pairs are displaced from the central pair by $\pm 1$ mm. The magnetic field is perpendicular to the sample; its direction is such that probes 2, 4 and 6 are near the source potential S, which is grounded. Probes 1, 3 and 5 are near the potential of the drain D. The dissipative voltages $V_x$ were measured between potential probes 2 and 4, hereafter denoted as $V_x(2,4) \equiv V_x(2) - V_x(4)$.

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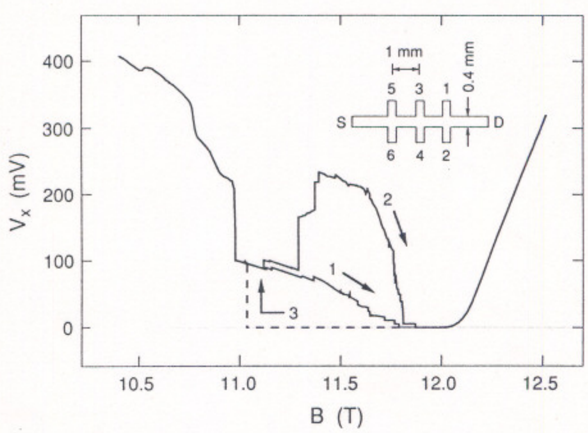


**Figure 1.** Two sweeps of $V_x(2,4)$ versus $B$ for the $i = 2$ plateau at $+210$ $\mu$A and 1.3 K. These sweeps, which follow separate paths 1 and 2, are in the increasing $B$ direction. The broken line, path 3, shows hysteresis for a sweep in the decreasing $B$ direction. The inset shows the sample geometry.

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Figure 1 shows sweeps of $V_x(2,4)$ versus the magnetic field $B$ for the $i = 2$ (12 906.4 Ω) quantized Hall resistance plateau at a temperature of 1.3 K and a current, $I$, of $+210\ \mu$A, where positive current corresponds to electrons entering the source and leaving the drain. This current is approaching the 230 $\mu$A critical current value at which $V_x$ never reaches zero for this particular probe pair. One of two distinct paths always occurred for positive current when sweeps were made in the direction of increasing $B$. Those paths are labelled 1 and 2 in the figure. This path 'bifurcation' is quite unusual. It occurred only for the $V_x(2,4)$ probe pair at positive current, and only for the $i = 2$ plateau. Hysteresis was observed when the magnetic field was decreased; this path is indicated by the broken line, labelled 3.

Figure 2 shows seventeen consecutive sweeps of $V_x(2,4)$ versus $B$ over a magnified region on the low-magnetic-field side of the $V_x$ minimum for both + and $-210\ \mu$A. The four sweeps along path 1 at $+210\ \mu$A formed one set of curves. Another set was generated by the four sweeps over path 2 at $+210\ \mu$A. No bifurcation was observed for the eight sweeps with increasing $B$ at $-210\ \mu$A; that set of curves is labelled path 4 in the figure. Finally, the broken line, labelled 3, was the same for all sweeps with decreasing $B$ for both + and $-210\ \mu$A. $V_x$ is always zero in the broken-line sweep. This indicates that a dissipationless state exists over this region of $V_x$ minimum.

The data of figure 2 clearly show discrete, well defined voltage states, with switching between the states. We next demonstrate that these discrete voltage states are quantized, and that this quantization is a function of magnetic field. A family of 19 shaded curves has been drawn through the data in figure 2. This family was generated by forcing all 19 shaded curves, and the broken line, to have equal voltage separations at each value of magnetic field. The voltage separations are, however, allowed to vary as $B$ is varied. The voltage separation (quantization) varies between 5.22 mV and 7.85 mV over the magnetic field range of this figure, which contains a $V_x = 0.0$ mV ground state and 19 excited states.

The higher-lying excited states are difficult to see in the multiple sweeps of figure 2 because of switching between states. Figure 3, therefore, shows just one of those sweeps along path 4. The higher-lying states are just as well quantized (i.e. well-fitted by the shaded curves) as the lower-lying states. We had found in [5] that the $V_x$ versus $B$ curves, themselves, can be time-averages of several voltage states, but histograms [5] of high-speed measurements for the present data indicated that there are no subdivisions of the displayed states.

The breakdown activity shown in figure 2 occurs in the region between, but not including, the Hall probe pairs 1,2 and 3,4 because the Hall resistances of both probe pairs are well quantized. The Hall voltage curves of both pairs would be horizontal, straight lines at 2710 mV if they had been included, with the same resolution, in this figure.

We next interpret the dissipative voltage quantization, treating the region between the Hall probe pairs 1,2 and 3,4 as a black box. Quantization already exists in the quantum Hall effect: the quantized Hall resistance occurs when the conducting electrons in the 2DEG occupy all the allowed states of the lowest Landau levels. We assume that the dissipation arises from transitions in which electrons from the originally full Landau levels are excited to states in higher Landau levels and then return to the lowest Landau levels. The electrical energy loss per carrier for $M$ Landau level transitions is $M\hbar\omega_c$, where $\omega_c = eB/m^*$ is the cyclotron angular frequency and $m^*$ is

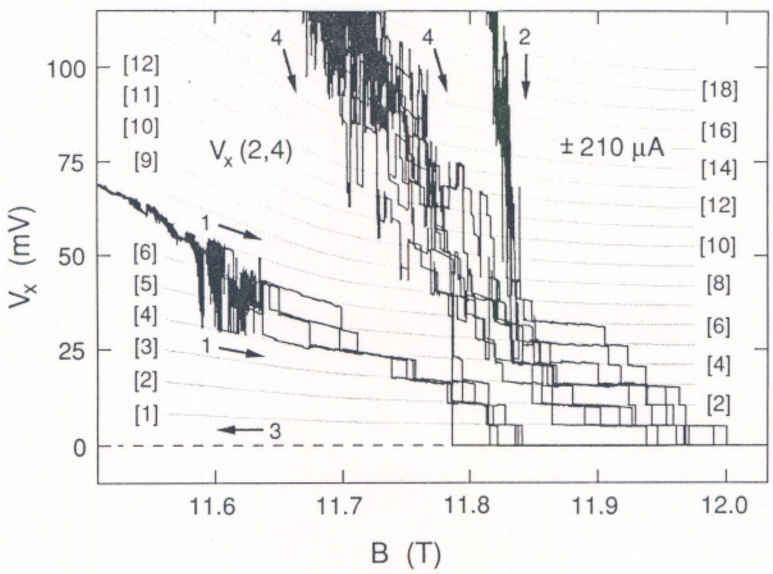


**Figure 2.** Seventeen sweeps of $V_x(2,4)$ versus $B$ for $\pm 210\ \mu$A. A family of 19 shaded curves are fitted to these data. The text explains how the shaded curves were generated. The Landau level transition numbers $M$ are shown in brackets.

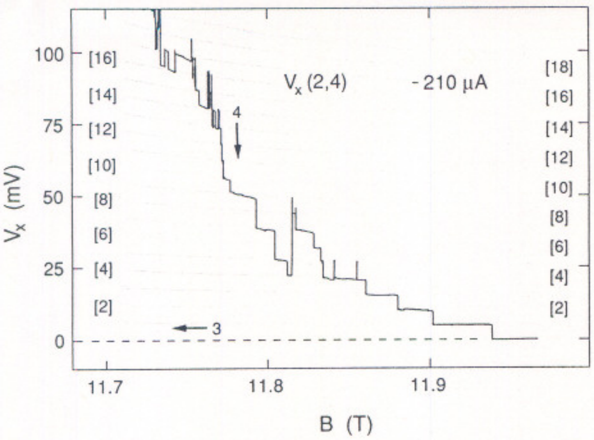


**Figure 3.** One of the $V_x(2,4)$ versus $B$ curves shown in figure 2 for path 4 at $-210\ \mu$A.

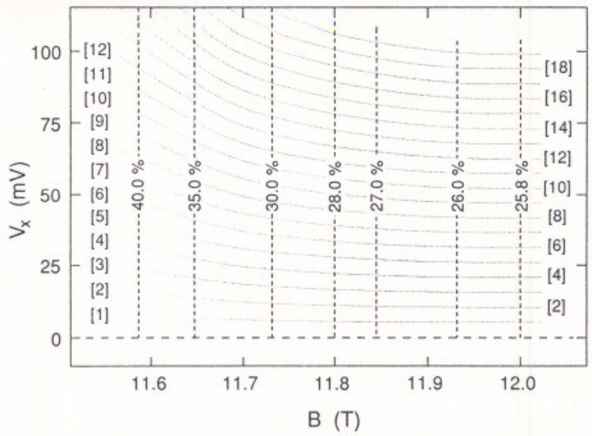


**Figure 4.** The fraction $f$ (expressed as a percentage) of electrons making the Landau level transitions for the 19 shaded curves shown in figure 2. See equation (1) for the definition of $f$. The shaded curves were generated with an accuracy of $\sim 1\%$, and a resolution of $\sim 0.1\%$.

the reduced mass of the electron (0.068 times the free-electron mass in GaAs). The power loss is $IV_x$. If (i) the ground state involves several filled Landau levels, (ii) only electrons in the highest-filled Landau level undergo transitions, and (iii) electrons of both spin sublevels of a Landau level undergo the transitions, then $IV_x = r(2/i)M\hbar\omega_c$, where $r$ is the total transition rate and $i$ is the Hall plateau number. Thus

$$fM = \left(\frac{re}{I}\right) M = \left(\frac{i}{2}\right)\left(\frac{m^*}{\hbar}\right)\left(\frac{V_x}{B}\right) \qquad (1)$$

where $f$ is the ratio of the transition rate $r$ within the black box to the rate $I/e$ that electrons transit the device; $f$ can also be interpreted as the fraction of conducting electrons that undergo transitions. We associate values of $M$ with the shaded curves in figures 2 and 3, where they are shown in brackets. $I$, $V_x$ and $B$ are measured quantities, and $i$, $m^*$ and $\hbar$ are constants. Therefore, $f$ and $r$ can be determined from the $V_x$ versus $B$ plots and equation (1).

If $f$ and $r$ were constant, then $V_x \propto B$ in equation (1), but it is clear from figure 2 that this cannot be the case for these particular data because the slope of $V_x$ versus $B$ has the opposite sign. Therefore, $f$ and $r$ must vary with magnetic field. The fraction $f$ (expressed as a percentage) of electrons that make the transitions is shown in figure 4, where $f$ varies between 25.8% and 40%, corresponding to transition rates between $3.4 \times 10^{14}\ \mathrm{s}^{-1}$ and $5.2 \times 10^{14}\ \mathrm{s}^{-1}$. The fact that $f$ varies seems to contradict our observations in [5] that $f$ was a constant 26.5% for the $i = 2$ plateau with this probe set, but most of the histograms used in [5] were obtained for path 2 curves in the magnetic field region between 11.75 and 12.0 T where $f$ is nearly constant and is about 27%.

The existence of quantized voltage states seems to imply that those electrons in the highest-filled ground state Landau level which pass through the breakdown region have an excitation probability of 100%. The fact that $f$ is not 100% suggests that some of the current bypasses the breakdown region. Voltage quantization also indicates that both $f$ and $r$ are constant for fixed values of $I$ and $B$.

We conclude that the fraction $f$ conducting electrons that make the Landau level transitions can be quite large, but is not necessarily 100%. Also, in general, $f$ is a function of $B$. These facts can greatly complicate the identification of voltage quantization for most breakdown data because the voltage separations will not be constant if $f$ and $r$ are not constant, so the voltages appear not to be quantized even when they actually are.

One can always obtain the product $fM$ from the data by using equation (1), but the value of $f$ can only be determined if $M$ can be unambiguously deduced. Most breakdown data require very careful measurements to deduce the quantization, and in many cases $M$ may be impossible to definitively determine if sweeps of $V_x$ versus $B$ do not display switching between states and/or enough deviations between sweeps like those shown in figure 2.

We have treated the breakdown region as a black box, but suggest that the mechanism responsible for the quantized dissipation is made up of two parts: (i) Landau level excitations involving the emission of acoustic phonons to conserve energy and momentum, as employed by Heinonen *et al* [6] and later used in the QUILLS model of Eaves and Sheard [7], with refinements and extensions by Cage *et al* [8]; and (ii) the return to the ground state via emission of either photons or optical phonons. Whatever the actual transition mechanism, it is surprising just how well quantized the dissipative voltage states are, up to at least the nineteenth state.

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