MINIMIZING TIME DILATION IN PENNING TRAP ION CLOCKS*

JOSEPH N. TAN, J.J. BOLLINGER, WAYNE M. ITANO, and D.J. WINELAND
National Institute of Standards and Technology
Time and Frequency Division, 325 Broadway
Boulder, Colorado 80303, USA

ABSTRACT

The relativistic time-dilation shift is expected to give one of the largest systematic uncertainties in a stored-ion clock. We show that in a Penning trap, it can be minimized if laser-cooled ions are prepared in a particular spheroid. Recent observation of Bragg scattering from crystals formed by the stored ions may be useful in stabilizing the time-dilation shift.

One of the goals at NIST-Boulder is to realize an ion clock with absolute accuracy surpassing 1 part in 10^{15}. Experiments at NIST with $^9Be^+$ ions stored in a Penning trap have shown that an RF oscillator can be steered by a nuclear spin-flip "clock" transition (~ 303 MHz) with $\sigma_y \approx 3\times10^{-12}/\sqrt{\tau}$ fractional frequency stability and 5×10^{-15} systematic uncertainty due to relativistic time dilation (second-order Doppler shift).[1,2] If collisional shifts[2] and other environmental perturbations are assumed to be negligible, then the expected performance of a stored ion clock is limited by the "projection-noise"[3] frequency stability $\sigma_y = 1/\omega_o\sqrt{NT_R\tau}$ and the uncertainty in the time-dilation shift $\Delta\nu/\nu_o = -\langle v^2\rangle/2c^2$. The frequency stability σ_y depends on the clock transition frequency $\omega_o/2\pi \equiv \nu_o$, the number of ions N, the Ramsey interrogation period T_R, and the averaging time $\tau > T_R$. The possibility of using "squeezed" atomic states to obtain higher frequency stability than the projection-noise limit is being explored.[4] The time-dilation shift depends on the mean squared velocity $\langle v^2\rangle$ of the ions. We present here a detailed treatment[5] of the time-dilation effects in Penning traps to bring out the important factors for optimized performance.

The simplest stored-ion clock consists of one trapped ion, with charge q and mass M. In a Penning trap, the motions of the confined ion [6] consist of a harmonic oscillation (frequency $\omega_z/2\pi$) along the trap's symmetry axis (z-axis), a fast cyclotron orbit ($\omega_c/2\pi$) about the magnetic field, and a slow magnetron orbit ($\omega_m/2\pi$) about the trap center, which generates a $q\mathbf{v}\times\mathbf{B}$ force for radial confinement. Time-dilation shifts of internal transitions can be made negligible by cooling these motions to their ground states. Typically, a hyperfine transition which is field-independent (to first order) at a particular field B_o is selected for clock operations. The potentially high accuracy in a single ion, however, exacts a cost in low signal-to-noise ratio and long averaging times.

The signal-to-noise ratio and frequency stability can be improved dramatically by using large numbers of ions. Unfortunately, to remain confined indefinitely, an ion cloud or plasma[7] must rotate (spin) about the trap axis with a frequency ω_r in the range $\omega_m <$

$\omega_r < \omega_c - \omega_m$. Hence, the time-dilation shift also increases with the number of ions because $v = \omega_r r$ and, in general, the ions in a larger "cloud" are distributed farther from the trap axis. Laser cooling greatly reduces the thermal motions of the ions superimposed on this rigid rotation; we can assume the effects of thermal motions ($\ll 1$ K typically) are negligible. As ions are laser-cooled, they first condense into a liquid drop, bounded by a spheroid $(x^2 + y^2)/r_s + z^2/z_s = 1$ with an aspect ratio $\alpha \equiv z_s/r_s$ which varies with the rotation frequency ω_r.[7] When the $q\mathbf{v} \times \mathbf{B}$ radial restoring force is weak ($\omega_r \longrightarrow \omega_m$) or when the centrifugal force is strong ($\omega_r \longrightarrow \omega_c - \omega_m$), the spheroid is stretched into a thin circular disk ($\alpha \longrightarrow 0$). Hence, near these lower and upper bounds of ω_r, the time-dilation shift for the thin disk of ions is very large. Our study shows that the time-dilation shift goes to a minimum at a particular (low) value of ω_r which brings the ions closer to the trap axis.

We first consider a single ion species. In the regime ($\omega_z/\omega_c \ll 1$) of experimental interest, the minimum shift is attained if the ions are prepared in a spheroid with aspect ratio $\alpha = z_s/r_s \simeq 0.46$, rotating at $\omega_r \simeq 1.8\omega_m$. The optimal shape of the spheroid is independent of system parameters, to first order in ω_z/ω_c, and the fractional frequency shift near the minimum is[5]

$$\frac{\Delta\nu}{\nu_o} \simeq -\Lambda\left(\frac{\omega_z}{\omega_c}\right)^2\left(3N\frac{\omega_z\tilde{r}}{c}\right)^{2/3}\left[0.186 + 0.15\left(\frac{\Delta r_s}{r_{so}}\right)^2\right], \tag{1}$$

where $\tilde{r} \equiv q^2/Mc^2$ and Δr_s is the deviation from the optimal spheroid radius r_{so}. For a single species, $\Lambda = 1$. (An approximate extension to sympathetically cooled systems – two ion species – has also been obtained.[5])

TABLE I: Expected Performance for some systems with $N = 10^6$, $T_R = 100$ s, $\alpha = 0.46$, and $r_{so} = 4.2$ mm.

Ion	ν_o (GHz)	B_o (T)	$\sigma_y(1s)$	minimum $\Delta\nu/\nu_o$
$^{9}Be^{+}$	0.303016	0.8194	53×10^{-15}	-241×10^{-15}
$^{25}Mg^{+}$	0.291996	1.2398	55×10^{-15}	-105×10^{-15}
$^{67}Zn^{+}$	$\simeq 1$	$\simeq 8.0$	16×10^{-15}	-2.5×10^{-15}
$^{199}Hg^{+}$	20.9	43.9	0.76×10^{-15}	-0.084×10^{-15}
$^{201}Hg^{+}$	7.73	3.91	2.1×10^{-15}	-11×10^{-15}

Neglecting order (ω_z/ω_c) corrections, Table I gives the expected performance for some systems with $N = 10^6$ ions, $T_R = 100$ s, and fixed spheroid dimensions ($\alpha = 0.46$, $r_{so} = 4.2$ mm). The minimum shift is independent of the ion mass. Hence, clock transitions at high magnetic fields are desirable since the minimum is inversely proportional to the square of the magnetic field. By stabilizing the radius of the ion spheriod near r_{so}, the optimal value, it should be possible to stabilize the time-dilation shift. Even a modest stabilization of $|\Delta r_s/r_{so}| \leq 0.1$ will reduce the fluctuations in the time-dilation shift to $< 1\%$.

Significantly improved performance, hence, requires good control and characterization of the ion distribution. Recent experiments show that long-range order emerges when sufficiently large ion clouds are laser-cooled.[8] For the first time, Bragg scattering from clouds with $N \gtrsim 2.7 \times 10^5$ ions reveals diffraction patterns which are consistent with a bcc lattice, predicted for a unbounded system (bulk behavior). Because of the rigid rotation of the ion cloud, the dots in a Laue pattern are rotated into circular Bragg peaks. Experiments to remove this angular averaging by using stroboscopic methods are underway. One method is to mask the diffraction pattern except for a small hole located on one of the circular Bragg peaks. If the diffraction pattern consists of dots, then the autocorrelation of photons passing through the hole must be periodic. We have recently observed such periodic autocorrelations.[9] (By using the photon arrival to gate an unmasked camera, a dot pattern has also been observed.) The autocorrelation provides the fastest and most accurate method that we have for measuring the ion cloud rotation frequency ω_r. It potentially can be used to stabilize ω_r at the optimum value for minimum time-dilation shift. The design and realization of an ion frequency standard, of course, must take into account other important systematic offsets.[10] For instance, preliminary studies showed that the $^9Be^+$ hyperfine clock transition (303 MHz) has an unexpectedly large pressure shift $\{(-1.7 \pm 0.4) \times 10^{-5}/$ Pa for $CH_4\}$.[11] For some ions, a cryogenic environment may be required to eliminate this effect.

References

1. J.J. Bollinger, *et al., Phys. Rev. Lett.* **54** (1985) 1000.

2. J.J. Bollinger, *et al., IEEE Trans. Instrum. Meas.* **40**, No. 2 (1991) 126.

3. W.M. Itano, *et al., Phys. Rev.* **A47** (1993) 3554.

4. D.J. Wineland, *et al., Phys. Rev.* **A50** (1994) 67.

5. J.N. Tan, J.J. Bollinger, and D.J. Wineland, *IEEE Instrum. Meas.* **44**, (1995) 144.

6. See review by L.S. Brown, and G. Gabrielse,*Rev. Mod. Phys.* **58**, No. 1 (1986) 233.

7. L.R. Brewer, *et al., Phys. Rev.* **A38** (1988) 859.

8. J.N. Tan, J.J. Bollinger, B. Jelenkovic, and D.J. Wineland, to appear in *Phys. Rev. Lett.*

9. Preliminary results to appear in *Proc. Int. Conf. Physics of Strongly Coupled Plasmas*, Binz (Germany), Sept. 11-15, 1995 (World Scientific).

10. D.J. Wineland, *et al., IEEE Trans. Ult. Fer. Freq. Control* **37**, No. 6 (1990) 515.

11. D.J. Wineland, *et al.*, in *Laser Manipulation of Atoms and Ions*, ed. E. Arimondo, W.D. Phillips, and F. Strumia (North-Holland, Amsterdam, 1992) pp. 553-567.