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THE RELATIVISTIC RED SHIFT WITH 2x10^{-17} UNCERTAINTY AT NIST, BOULDER, COLORADO, U.S.A.*

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Abstract

We have estimated the relativistic red shift correction due to gravity, necessary to reference to the geoid the measurements of the new cesium fountain frequency standard at the National Institute of Standards and Technology (NIST) in Boulder, Colorado, USA. The frequency correction due to the red shift is given by $\Delta f/f = (W_P - W_0)/c^2$, where c is the speed of light, W_P the gravity potential at the location of the cesium fountain and W_0 the gravity potential on the geoid. Note we are using the geodetic convention in which gravity potentials are positive. We have computed the geopotential number $C = W_0 - W_P$ in three ways: (1) Based on the global gravitational model EGM96. (2) Based on the regional, high-resolution geoid model G96SSS. (3) Based on the value provided in the National Geodetic Survey's data sheet for the NIST reference marker. We have estimated the offsets between the reference surfaces associated with each of the above three values of C. After referencing the three C values to a geoid surface defined with respect to the current best estimate of an "ideal" mean-Earth ellipsoid, the three computations of C gave the following $\Delta f/f$ results: (1) -1797.83×10^{-16}, (2) -1798.94×10^{-16} and (3) -1798.91×10^{-16}. The minus sign implies that the cesium fountain runs faster in the laboratory in Boulder than a standard clock located on the geoid. The values from (2) and (3) are expected to be the most accurate and are also *independent*. We currently estimate $\Delta f/f$ on the cesium fountain's location at NIST to be -1798.93×10^{-16}, with an estimated error of $\pm0.2\times10^{-16}$.

Keywords: geodetic leveling, geoid, geopotential, mean-Earth ellipsoid, relativistic red shift, vertical datum

Introduction and Theoretical Background

With the advent of new primary frequency standards whose uncertainties approach 1 part in 10^{15}, there is a need for improved estimates of the relativistic red shift. This is an effect predicted by relativity theory as the sum of a special and a general relativistic effect. In general relativity, a clock at a higher gravitational potential runs faster relative to a clock at a lower potential. In relativity, *"higher"* potential means less negative, since the convention used is such that potential has (in general) negative value, approaching zero as a particle moves towards infinity away from an attracting body. Thus the effect of the geopotential on a clock would cause it to run faster as it moves away from the Earth, or in our case, higher above the geoid. Note that geodesy uses the *opposite* sign convention for geopotentials than that used in relativity theory. In geodesy, all potentials are positive, so that a higher potential would generally be closer to the Earth. In this paper we will use the geodetic convention, in which all geopotentials are positive. A second effect in relativity enters, the so-called second-order Doppler shift of special relativity, in which a standard clock runs slower as it moves faster, relative to a clock at rest. The rotation of the Earth, therefore, gives rise to a centripetal potential that also changes the clock's frequency. We differentiate between the potential due to *gravitation* and that due to *gravity*: the former arises from the presence of attracting masses *only*, the latter contains in addition the centripetal potential due to the Earth's rotation [6, section 2-1]. It is the gravity potential that we need to consider here, therefore the term "gravitational red shift" is somewhat misleading and has been avoided herein.

A primary frequency standard which contributes to International Atomic Time (TAI) must be corrected to run at the rate clocks would run on the Earth's geoid. It is therefore necessary to determine the difference in gravity potential $(W_0 - W_P)$, between the geoid (0) and the location of a primary frequency standard (P), in order to correct for this frequency offset, according to [1]

$$(f_0 - f_P)/f = \Delta f/f = (W_P - W_0)/c^2, \qquad (1)$$

where $f = (f_0 + f_p)/2$, and c denotes the speed of light. Note that if the point P is above the geoid, we generally have $W_P < W_0$, using the convention in which potentials are positive. Hence, Δf is negative in this case, since this clock correction would make the clock in Boulder run slower, to match the rate of a standard clock on the geoid.

The *geopotential number* $C = W_0 - W_P$ [6, page 56] is given by:

$$C = W_0 - W_P = \int_{H=0}^{H=H_P} g dH \ , \qquad (2)$$

where g is the magnitude of the gravity acceleration vector, and dH is the length increment along the plumb line (positive upward). The (path-independent) line integral in equation (2) starts from a reference equipotential surface whose gravity potential is W_0 (on which every point has *orthometric* height equal to zero) and ends at the station location where $W = W_P$ and $H = H_P$. Although the reference equipotential surface can be defined unambiguously through a prescribed value of W_0, such a definition has limited practical value for the physical realization of this surface, since absolute potentials cannot be measured. In theory, any equipotential surface of the gravity field is a suitable reference surface for orthometric heights *worldwide*. However, the human conception of "heights" and historic practices, make it convenient for such a reference surface (*vertical datum*) to be "close" to the Mean Sea Surface (MSS). Historically, the vertical datum of a country (or a set of countries) has been realized by prescribing a certain value to the orthometric height (or the geopotential number) of some tide gauge station(s). The geopotential numbers and orthometric heights of other points could then be determined using spirit leveling and gravity observations, through the evaluation of a discrete counterpart (summation) of equation (2) [6, chapter 4].

The presence of a quasi-stationary (i.e., non-vanishing through averaging over long time periods) component within the Dynamic Ocean Topography (DOT) results in departures of MSS from an equipotential surface ranging (geographically) between -2.1 m and +1.3 m approximately. Due to these departures (and in some cases due to additional considerations related to mapping applications), different vertical datums refer to different equipotential surfaces. Therefore, given a datum-dependent C value, the determination of $\Delta f/f$ with respect to a unique equipotential surface requires the estimation of that datum's offset from that unique equipotential surface. A unique equipotential surface – *the geoid* – that closely approximates (in some prescribed fashion) the MSS, has to be defined and realized through the operational development of models [5,7]. There exist global geoid models, developed through the combination of satellite tracking data, surface gravimetry, and satellite altimetry. A state-of-the-art such model, complete to degree and order 360 (corresponding to half-wavelength resolution of ~55 km at the equator), is EGM96 [8]. The resolution of such global models is limited (primarily) by the available surface gravimetric data used in their development. Detailed (i.e., higher-resolution) local or regional geoid models are developed by incorporating the information contained within dense gravity and topography data into a global geoid model. This adds high (spatial) frequency details to the broader geoid features represented within a global model. G96SSS [16] is such a regional geoid model for the United States. Global and regional geoid models can also be used to estimate the geopotential number C, given the geocentric coordinates of the point P. We distinguish therefore two general approaches for the computation of C (and hence $\Delta f/f$): one based on spirit leveling and gravity observations, and another based on the use of geoid models (either global or regional/local). Each approach has its own advantages and disadvantages, and its own error characteristics. It is important however to recognize that each computational method (and/or model used) may yield a result that refers to a different equipotential surface. Since the various reference surfaces may be offset by several decimeters, estimation of their relative offsets becomes important if one desires to compare the various results at the level of a decimeter or less.

It is useful to recall the correspondence between the approximate magnitude changes of H, C, and $\Delta f/f$. Near the Earth's surface $g \approx 9.8ms^{-2}$ and since $c = 299792458\,ms^{-1}$, a change in H by one meter implies roughly a $9.8m^2s^{-2}$ change in C, and therefore a change in $\Delta f/f$ of -1.1×10^{-16}. Our present requirement is that $\Delta f/f$ be computed with an error not exceeding $\pm1\times10^{-16}$. Therefore, the total error in an absolute determination of the geopotential number C, consisting of the error in W_0 (absolute) and the error in $W_0 - W_P$ (relative), should not exceed $\sim\pm9.8m^2s^{-2}$ (equivalently, the absolute orthometric height H_P of our station should be determined to better than $\pm1m$).

Computational Aspects

In the following paragraphs we discuss the specific computations involved in the estimation of $\Delta f/f$, according to three methods. The first two methods are based on geoid model information (global and regional respectively), while the third method is based on spirit leveling and gravity observations. The first two methods share some (long-wavelength) errors, but the third method is *independent* of the other two.

Method 1

The concept of a *mean-Earth ellipsoid* [6, section 2-21] is of central importance in gravimetric geodesy (and in our specific application). This purely mathematical construct is a rotating ellipsoid of revolution (i.e., bi-axial), whose surface is also an equipotential surface of its gravity field. The gravity potential on its surface is pre-supposed to equal the gravity potential on the geoid. Four parameters are necessary and sufficient to define uniquely its size, shape, rotation, and gravity field. One may pre-suppose that these parameters are numerically equal to the corresponding parameters of the real Earth. Then, the departures of the geoid from such an "ideal" ellipsoid (called *geoid undulations* and denoted by *N*) have vanishing zero-degree term (i.e., their average over the whole Earth equals zero). Therefore, by suppressing the zero-degree term in the spherical harmonic expansion of *N*, one obtains "automatically" geoid undulations that refer to this "ideal" mean-Earth ellipsoid, *without* the need to know the specific scale (semi-major axis) of this ellipsoid. Specification of the scale and the gravity field of this "ideal" ellipsoid require numerical specification of its defining parameters. These values can be determined only from analyses of various geodetic observations and therefore contain random (and possibly systematic) errors. Here we will define this "ideal" mean-Earth ellipsoid, in a *tide-free* system [15], by adopting the current best estimates for the values of the following parameters [2]:

Equatorial radius: $a = 6378136.46\ m$
Flattening: $f = 1/298.25765$
Geocentric gravitational constant: (3)
$GM = 3.986004418\times10^{14}\ m^3s^{-2}$
Mean rotational speed:
$\omega = 7292115\times10^{-11}\ rad\ s^{-1}$

We should emphasize here that the mean-Earth ellipsoid defined by the above four values is only as "ideal" as the current accuracy of these values allows. These values are constantly being refined through improved geodetic determinations. GM is currently determined most accurately from analyses of laser ranging data acquired on high altitude geodynamic satellites, and so is f (more precisely the second-degree zonal gravitational coefficient, J_2, from which f can be derived). ω is deduced most accurately from Very Long

Baseline Interferometry. The equatorial radius "a" is currently determined best from analyses of satellite radar altimeter data over the oceans. Jekeli discusses the fundamental concepts involved in the determination of the best estimate of the equatorial radius [7]. The adopted defining values of equation (3) imply a value of the gravity potential on the geoid equal to:

$$W_0 = 62636856.88\ m^2 s^{-2}\ , \qquad (4)$$

with an estimated error of $\pm 1.0\ m^2 s^{-2}$ [2].

The gravity potential W_P is given by:

$$W_P = V(r_P, \theta_P, \lambda_P) + \Phi(r_P, \theta_P) \qquad (5)$$

where $V(r_P, \theta_P, \lambda_P)$ is the gravitational potential, $\Phi(r_P, \theta_P)$ the centrifugal potential, and $(r_P, \theta_P, \lambda_P)$ are geocentric radius, geocentric co-latitude (90° minus latitude), and longitude, respectively. One has [6, chapter 2]:

$$V(r_P, \theta_P, \lambda_P) = \qquad (6)$$

$$\frac{GM}{r_P}\left[1 + \sum_{n=2}^{\infty}\left(\frac{a}{r_P}\right)^n \sum_{m=-n}^{n} \bar{C}_{nm} \cdot \bar{Y}_{nm}(\theta_P, \lambda_P)\right]$$

$$\Phi(r_P, \theta_P) = \frac{1}{2}\omega^2 r_P^2 \sin^2\theta_P \qquad (7)$$

with

$$\bar{Y}_{nm}(\theta_P, \lambda_P) = \bar{P}_{n|m|}(\cos\theta_P) \cdot \begin{cases} \cos m\lambda_P & if\, m \geq 0 \\ \sin|m|\lambda_P & if\, m < 0 \end{cases} . \qquad (8)$$

$\bar{P}_{nm}(\cos\theta_P)$ is the fully-normalized Associated Legendre Function of the first kind [6, sections 1-11, 1-14], of degree n and order m, and $\bar{C}_{nm}$ are the (unitless) fully-normalized potential coefficients. The numerical values of $\bar{P}_{nm}(\cos\theta_P)$ were evaluated here using (a modification of) routine LEGFDN, originally written by O.L. Colombo [3, page 131]. EGM96 [8] provides currently the most accurate estimate of a set of $\bar{C}_{nm}$, complete to degree and order 360.

The geocentric Cartesian coordinates of our reference point (P) at the NGS marker for NIST Boulder in ITRF94 are:

$$X_P = -1288380.79m$$
$$Y_P = -4721667.99m$$
$$Z_P = 4078642.02m$$

(9)

No accuracy estimate for these coordinates was available, but their errors are expected to be well below one meter. A new survey of reference markers at NIST is expected to take place in the summer of 2000, to improve the accuracy to 1 cm or better

We converted these coordinates to $(r_P, \theta_P, \lambda_P)$ and evaluated equation (6) (truncated to maximum degree and order 360), and equations (7) and (5). We obtained:

$$W_P = 62620698.79\ m^2 s^{-2}\ , \qquad (10)$$

which, implies due to equations (1) and (4):

$$\Delta f / f = -1797.83 \times 10^{-16}\ . \qquad (11)$$

There are two types of error associated with the use of EGM96: (a) error of commission due to the fact that the coefficients $\bar{C}_{nm}$ are imperfectly known, and, (b) error of omission due to the truncation to degree 360, of the infinite series in (6). The commission error of EGM96 has two components. The first one (long wavelength) can be computed rigorously from the error covariance matrix that accompanies the part of the model up to degree and order 70. The second component, corresponding to degrees 71 to 360, is only available in terms of a global Root Mean Square (RMS) estimate that does not account for the specific geographic location of our station. This estimate can be computed from the standard deviations of the EGM96 coefficients above degree 70. The omission error of EGM96 can also be estimated based on some theoretical model describing the decay of the gravitational spectrum of the Earth *globally*. Such computation yields also a global RMS value, without geographic specificity. Details on the EGM96 geoid error assessment can be found in [8, sections 7.3.3.1 and 10.3.2]. Based on that assessment, we estimate the total (commission plus omission) geoid undulation error of EGM96 to be approximately $\pm 0.6m$, in an RMS sense over the conterminous USA. To obtain this estimate we proceeded as follows. Over the conterminous USA the RMS geoid error obtained from rigorous error covariance propagation up to degree 70 is $\pm 0.26m$, while the quadratic summation of the EGM96 coefficient standard deviations (i.e., neglecting coefficient error correlation) gives $\pm 0.19m$. We used the ratio of these two quantities (~1.4) to scale the higher-degree (71 to 360) commission error, as well as the omission error beyond degree 360, thereby "converting" these two global RMS estimates to corresponding values that are more representative of our specific area. We fully recognize here that this approach is only approximate. Considering the mountainous terrain of the region around our station (which is expected to increase primarily the omission, but also the commission error of the model), it is not unreasonable to estimate the EGM96 undulation error at our station (P) to be between $\pm 0.6m$ and $\pm 1.0m$. Pavlis et al. reported an evaluation of EGM96 using independent data [11]. Over 5168 benchmarks distributed over the conterminous USA, the standard deviation of the differences between the EGM96 geoid undulation estimates and *independent* estimates obtained from GPS positioning and spirit leveling is approximately $\pm 0.40m$, which verifies that our present error assessment is not unreasonable (it may actually be slightly pessimistic). This EGM96 geoid error estimate for our site implies that the error of the $\Delta f / f$ value given in (11) is not expected to exceed $\pm 1 \times 10^{-16}$.

Method 2

A significant reduction of the omission error encountered with EGM96 can be effected through the use of a detailed regional geoid model. We have used the 2'x2' gravimetric geoid G96SSS [16] as follows:

1. We first computed the geoid undulations implied by EGM96 (to degree 360) at the 2'x2' grid nodes of G96SSS, using [13]:

$$N_{EGM96}(\theta_P, \lambda_P) = \tag{12}$$
$$\frac{GM}{r_P \gamma_P} \sum_{n=2}^{\infty} \left(\frac{a}{r_P} \right)^n \sum_{m=-n}^{n} \bar{C}^*_{nm} \cdot \bar{Y}_{nm}(\theta_P, \lambda_P) + \frac{\Delta g_B}{\bar{\gamma}} H$$

In equation (12), γ_P is normal gravity at P, $\bar{\gamma}$ is an average value of normal gravity between the projections of P on the ellipsoid and telluroid, Δg_B is the Bouguer gravity anomaly, and $\bar{C}^*_{nm}$ denote the potential coefficient remainders after the even zonal reference coefficients of the ellipsoidal (normal) field are subtracted from $\bar{C}_{nm}$. Detailed discussion of the underlying theory and the details of the numerical implementation of equation (12) can be found in [13]. Notice that the geoid undulations from (12) refer to our current best estimate of the ideal mean-Earth ellipsoid.

2. We subtracted the G96SSS geoid undulations, N_{G96SSS}, *exactly* as these are given on the distributed CD, from the undulations computed from equation (12). The average value of these differences over the domain of G96SSS provides an estimate of the shift that is required in order to reference the N_{G96SSS} values to an ideal mean-Earth ellipsoid. We found:

$$N_{G96SSS}(ideal) = N_{G96SSS} + 0.40m \quad . \tag{13}$$

3. From the geocentric Cartesian coordinates of P, we computed its geodetic coordinates with respect to an ellipsoid of the (a, f) given in equation (3). This yields:

$$\begin{aligned} \varphi_P &= 39°59'43.331'' \\ \lambda_P &= 254°44'15.016'' \quad , \\ h_P &= 1634.62m \end{aligned} \tag{14}$$

therefore the orthometric height H_P, as implied by the geocentric positioning data (h_P) and the $N_{G96SSS}(ideal)$ value is:

$$H_P(ideal) = h_P - N_{G96SSS}(ideal) \quad . \tag{15}$$

The evaluation of equation (15) requires N_{G96SSS} at the point P. This value was obtained using bicubic spline interpolation of the 2'x2' grid on which the G96SSS values are given.

4. From $H_P(ideal)$ we computed the geopotential number of P using Helmert's equation [6, equation 4-26]:

$$C = H_P(ideal)\cdot[g_P + 0.0424 H_P(ideal)] \quad (16)$$

where C is in $g.p.u.$ ($1 g.p.u. = 10 m^2 s^{-2}$), $H_P(ideal)$ in km, and g_P (the value of gravity acceleration at P) is in Gal ($1 Gal = 10^{-2} ms^{-2}$). $g_P = 9.796022 ms^{-2}$, a value obtained from the National Geodetic Survey's (NGS) data sheet for the NIST reference marker. The approximations involved in equation (16) introduce errors of only a few centimeters [6, page 169]. More accurate formulations in place of equation (16) (which take also into account the terrain correction) are possible [6, page 169]; however such formulations were not implemented here.

5. From equation (16) we obtained:

$$C = 16168.04 m^2 s^{-2}\ , \quad (17)$$

which finally yields from equation (1):

$$\Delta f / f = -1798.94\times 10^{-16}\ . \quad (18)$$

Unlike EGM96, the G96SSS regional geoid model is not accompanied by propagated error estimates. Its accuracy has been assessed only through comparisons with *independent* geoid undulation estimates obtained from GPS positioning and leveling observations [16]. Based on this accuracy assessment, we estimate the error in N_{G96SSS} to be between $\pm 0.15 m$ and $\pm 0.20 m$. Considering also a $\pm 0.15 m$ error in the ellipsoidal height h_P, this implies an error for the $\Delta f / f$ value given in (18) of about $\pm 0.20\times 10^{-16}$ to $\pm 0.25\times 10^{-16}$.

Method 3

The $\Delta f / f$ values given in equations (11) and (18) were computed based on a global and a regional geoid model, respectively. We turn now to the $\Delta f / f$ computation from spirit leveling and gravity measurements, as shown in equation (2). We performed this computation as follows:

1. From the NGS data sheet for our reference marker we obtained its *dynamic height* [6, page 163] value $H_P^{dyn} = 1649.034 m$. This value is related to the geopotential number of P by [6, equation 4-9]:

$$H_P^{dyn} = C / \gamma_0\ , \quad (19)$$

where $\gamma_0 = 9.806199 ms^{-2}$ is the value of normal gravity on the GRS80 reference ellipsoid, at $\varphi = 45°$ (this value was taken from the NGS data sheet, exact to the digits given there). From equation (19) we therefore computed:

$$C_{NAVD88} = 16170.76 m^2 s^{-2}\ , \quad (20)$$

where the subscript "*NAVD88*" emphasizes the fact that this value refers to the equipotential surface that passes through the origin point (Father Point/Rimouski located in Quebec) of the North American Vertical Datum 1988 [17, 1992].

2. To estimate the offset between the NAVD88 reference equipotential surface and the "ideal" geoid surface, we proceeded (in principle) as described in [12]. From NGS [*Milbert,* private communication, 1998] we have available a set of 5168 GPS/leveling points distributed (not evenly) over the conterminous USA. The geodetic coordinates and the NAVD88 orthometric heights of these points are given. The Cartesian coordinates were obtained from GPS positioning and are given with respect to the ITRF94(1996.0) reference frame. From these Cartesian coordinates, the geodetic coordinates (φ, λ, h) have been computed with respect to the GRS80 reference ellipsoid and were provided to us. For our computation however, we need h to be defined with respect to the "ideal" values of (a, f) given in equation (3). We performed this conversion using [14, equation 64]:

$$\left.\begin{aligned} h(ideal) &= h(GRS80) - w\cdot\Delta a + \frac{a(1-f)}{w}\sin^2\varphi\cdot\Delta f \\ w &= (1 - e^2\sin^2\varphi)^{1/2} \end{aligned}\right\} \quad (21)$$

where:

$$\left.\begin{aligned} \Delta a &= 6378136.46 - 6378137.m \\ \Delta f &= 1/298.25765 - 1/298.257222101 \end{aligned}\right\}\ . \quad (22)$$

Using equation (12) we computed the EGM96-implied geoid undulations, N_{EGM96}, at the locations of these 5168 points, with respect to an ideal mean-Earth ellipsoid, in the tide-free system. We then formed the differences:

$$d = h(ideal) - H_{NAVD88} - N_{EGM96} \quad . \tag{23}$$

3. The mean value of d over the 5168 points provides an estimate of the offset between the NAVD88 reference surface and the "ideal" geoid surface. For reasons explained in [13], we computed this mean value in three ways: Using all 5168 points

$$\Rightarrow \bar{d}_1 = -0.448m$$

Using 2067 points whose distance is not less than 25 *km* (thinned set)

$$\Rightarrow \bar{d}_2 = -0.477m$$

Using 438 points of the thinned set where H<100m

$$\Rightarrow \bar{d}_3 = -0.235m$$

Rapp [13, page 286] discusses the reason for the value $\bar{d}_3$ to be considered more reliable than the other two. On the other hand, $\bar{d}_3$ is computed on the basis of only a subset of the total number of available points. Estimating the "best" value of $\bar{d}$ depends on the "relative weights" that one is willing to assign to these three estimates. We adopted the value $\bar{d} = -0.300m$ as our current "best" estimate of the offset between the NAVD88 reference equipotential surface and the geoid surface. The minus sign implies that the equipotential surface passing through the origin of NAVD88 is *below* the geoid surface that is realized through the EGM96 model, when the latter is referenced to our current best estimate of a mean-Earth ellipsoid. Smith and Milbert report a value of $\bar{d} = -0.314m$ with an uncertainty of $\pm 0.156m$ [16]. Their value is based on comparisons over 2951 GPS/leveling points over the conterminous USA and agrees very well with our estimate. We should also mention that in the above analysis the permanent tide effect was consistently accounted for. All three quantities in equation (23) were expressed in the tide-free system.

4. The offset $\bar{d}$ can now be input to equation (16), in the place of $H_P(ideal)$, to estimate the correction dC necessary to convert C_{NAVD88} to $C(ideal)$. We find:

$$dC = -2.94\ m^2s^{-2} \Rightarrow C(ideal) = 16167.82\ m^2s^{-2} \tag{24}$$

which implies:

$$\Delta f / f = -1798.91 \times 10^{-16} \quad . \tag{25}$$

Errors in the estimate of $\Delta f / f$ given in (25) arise from errors in the NAVD88 dynamic height value provided in the NGS data sheet for our reference marker, and errors in our estimation of the NAVD88 datum offset. The NGS data sheet for our reference marker contained no error estimates, other than the designation that "First order, Class II" leveling was performed to determine our station's height. Zilkoski et al. discuss a comparison of NAVD88 heights with corresponding *independent* estimates from Canadian leveling observations over the USA-Canada border [17]. Over 14 points the maximum difference found was 0.11 m. This value does not necessarily apply to our station; nevertheless (and in lieu of more precise information) a reasonable estimate of our station's dynamic height error may be about $\pm 0.15m$. Considering an error of the same magnitude in our estimate of the NAVD88 datum's offset, we conclude that the $\Delta f / f$ value given in (25) is probably accurate to $\pm 0.23 \times 10^{-16}$.

It is noteworthy that the results from the two *independent* Methods 2 and 3 agree to 0.03×10^{-16}, which is better than our specific requirement by almost two orders of magnitude. In addition, the difference between the EGM96-implied value (Method 1) and the results from the other two Methods is well within the expected errors of the corresponding values. Although our specifications could probably have been met by the use of EGM96 alone, this exercise permitted additional cross validation of our results and, we hope, improved the overall reliability of our answer.

Based on the analysis described here and the three methods used to evaluate $\Delta f/f$, we estimate its value for our reference marker to be -1798.93×10^{-16}, with an error of $\pm0.2\times10^{-16}$. We should mention however that we have not accounted here for luni-solar tidal effects. At this level of accuracy the effects of (at least) the semi-diurnal lunar tide M_2 (and possibly of other constituents) must be considered. One should therefore interpret our $\Delta f/f$ result as an average value over multiples of the main tidal constituents' periods. Finally we should note that the geocentric Cartesian coordinates of our reference point (P) given in equation (9) were formally considered here to refer to a well-defined reference frame (ITRF94). There is still some ambiguity regarding the specific reference frame of these coordinates, and therefore a re-occupation of our site with a GPS receiver is highly desirable.

Summary and Future Prospects

Based on our work, it appears that the existing measurements and models of the Earth's gravity field may not support estimates of the relativistic red shift correction to better than the 10^{-17} level for frequency standards on the Earth. Since this number contributes to the error budget of a primary frequency standard in an RMS sense, this implies that a primary frequency standard in an Earth-bound laboratory will have difficulty contributing to TAI at better than the 10^{-16} level. In the next decade it seems reasonable to expect frequency standards to reach accuracies challenging our current accuracy in the determination of the red shift correction. Currently two geopotential mapping missions are in preparation, which are expected to support a significant advance in the present application: NASA's Gravity Recovery And Climate Experiment (GRACE), and ESA's Gravity Field and Steady-State Ocean Circulation (GOCE) missions. The primary objective of the GRACE mission is stated to be [9]:

> "The primary objective of the GRACE mission is to provide gravity models with accuracies that better existing global and high spatial resolution models of the Earth's gravity field by at least an order of magnitude, on a monthly basis, for a period of up to 5 years. The temporal sequence of gravity field estimates provide the mean (or static) gravity field, as well as a time history of its temporal variability."

GRACE is scheduled for launch in 2001, and promises to deliver centimeter-level geoid undulation accuracy with half-wavelength resolution of 200 to 300 km (depending on the altitude where GRACE will be deployed finally). GOCE (proposed launch in 2004) is expected to improve even further the resolution, allowing cm-level geoid undulation accuracy down to ~80 km resolution [4]. The global geopotential models expected from these missions, in combination with locally available detailed surface gravity and topography data may permit point geoid undulation determination approaching cm-level accuracy. In addition, radar altimeter data from satellites such as TOPEX/Poseidon and its follow-on Jason-1 in combination with the global geopotential models from GRACE and GOCE, should permit improvements in the determination of the equatorial radius of the mean-Earth ellipsoid, which directly affects the accuracy of W_0. These advances may permit determination of $\Delta f/f$ accurate to few parts in 10^{18}.

On the opposite side, development of frequency standards accurate to 10^{-17} or better may provide one possibility for the verification and error calibration of geopotential differences estimated from data acquired (in part) from the GRACE and GOCE missions. This could be attempted following ideas such as those proposed originally by Bjerhammar [1]. In addition, frequency standards of such high accuracy, located on different continents, provide an alternative technique well recognized among geodesists for connecting different vertical datums. While there is promise for standards of such accuracies, methods for transferring such time and frequency measurements appear to be lacking. The current best time transfer methods appear to be at the level of 200 ps stability, or about 2×10^{-15} frequency transfer at 1 day [10]. In conclusion, it appears that technology advances in the development of frequency standards and advances in gravity field determination over the upcoming years are expected to benefit both disciplines in complementary ways.

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