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1. INTRODUCTION TO OPTICAL RADIOMETRY

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1.1 Background

Radiometry is the science of measuring electromagnetic radiation in terms of its power, polarization, spectral content, and other parameters relevant to a particular source or detector configuration. An instrument which measures optical radiation is called a radiometer. While in many parts of the world, the term radiometer is exclusively applied to devices which monitor radiance, we will use the word in a more general sense to mean a device which measures one of several optical-power-dependent quantities. Radiance, the optical power from or through an area within some solid angle, is one of several optical terms that will be defined and discussed in this chapter. It has been the authors' experience that the meaning of terms like radiance and other radiometric expressions is one of the more vexing problems faced by scientist new to the field. It is our hope that this issue, among others, will be clarified by this chapter.

A radiometer will have as its essential component a detector or sensor of the optical radiation and, with it, associated optical and electronic elements to generate a signal that is representative of the quantity being monitored. A major technical challenge in radiometry involves the characterization of a source of radiation which in turn requires the characterization of a detector system that will measure the source optical power and from which the characteristics of the source can be determined. For example, it is necessary to characterize light sources used for illumination of buildings so as to most efficiently use electricity, and at the same time provide adequate illumination for the inhabitants of a building. This in turn requires that specialized optical detectors be used that allow the illumination engineer to measure the light in a manner that is relevant to human vision by properly accounting for the visual response of the human eye. There are many uses of radiometry in industrial application to monitor manufacturing processes and in scientific and technical activities that utilize the sensing of optical radiation to deduce information about a wide range of physical, chemical, and biological processes.

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The earth remote-sensing community relies upon complex radiometric systems to explore the earth's radiation budget, to monitor land and ocean environmental health, and to explore global climate change issues [1]. The science of radiometry encompasses all these varied needs for sensing and measuring light, and as a result it is a multifaceted discipline with many different techniques to meet varied technical needs. In cases where a radiometer is developed to sense some particular physical phenomena or process, the radiometer and its associated optical system is often called a sensor or optical sensor, or even given a specific name that denotes a purpose, such as pyrometer, which is an optical sensor for measuring temperature. The exact meanings of the terms detector, sensor, or radiometer in general needs to be decided from their context. This book is intended to be an introduction for the reader to the recent innovations in radiometry that have been developed to take advantage of the technical advances of the past several decades.

Early efforts in radiometry were associated with the desire to understand visual sensations. Many of the early scientific writings from the time of the Greeks and Romans through the middle ages involved attempts to understand the eye and its relation to visual phenomena. Scientists as distinguished as Newton, Kepler, and Descartes spent considerable effort in attempts to understand vision, and in addition they contributed significantly to understanding the function of the eye's lens and formulated theories to explain color perception [2, 3]. The history of the study of vision is a central theme in the evolution of scientific thought, and the reader is encouraged to pursue this fascinating topic in some of the references cited here [2].

Most of the early works on radiometry were in an area we now call photometry. Photometry is the science of measuring light taking into account the wavelength response or sensitivity of the human eye. Hence, photometry is one of a number of radiometric techniques that use a wavelength selective detector system to measure a quantity of interest. Instruments that are designed to measure light as the human eye does are called photometers. The reader should be aware that in various parts of the world and in other scientific disciplines, the term photometer can refer to some other sort of instrument for measuring light that is not directly related to human vision. Photometry will be covered extensively in Chapter 7.

The first efforts in quantitative radiometry are attributed to Bouguer and Lambert, who developed photometers and attempted to quantify the measurement of the visual effects of visible radiation in the period, 1725–1760 [3]. Lambert laid out the theoretical foundations of photometry, the principles of which remain in modern practice. He established mathematical relationships that include the law of addition of illuminations, the inverse square law, the cosine law of illumination, the cosine law of emission, etc. The concepts laid out by Lambert for photometry have been extended and generalized to measurements involving the infrared and ultraviolet parts of the electromagnetic spectrum and form an integral part of modern radiometric practice.

This chapter will review the history of radiometry and define the terminology and basic methodology that underpin modern radiometric practices. The remaining chapters will develop in greater detail the important critical elements of modern radiometry.

1.2 Basics of Radiometry and Important Milestones

The initial impetus for radiometry in the 18th and 19th centuries had been the development of quantitative measurements in the physical sciences and, in particular, the efforts by astronomers to quantify the varying intensity of the observed stars. The astronomer Sir William Herschel (1800) discovered infrared radiation by comparing the temperature rise of liquid in glass thermometers placed in different spectral parts of the dispersed solar radiation. He showed that heating occurred in the red portion of the spectrum and also in the invisible portion at longer wavelengths as part of his efforts to develop means to quantify stellar intensity measurements [3]. Similarly, in 1802, ultraviolet light was discovered by Johann Ritter, who used chemical activity of light in analyzing the spectrum of white light and noted that the activity extended to shorter wavelengths than the visible blue light [4]. There were many contributions to the understanding of electricity and magnetism in the 19th century that culminated in the developing of a comprehensive theory of electromagnetism by Maxwell in 1864 [5]. Maxwell's equations predicted the existence of electromagnetic waves traveling at the speed of light. The existence of these waves was confirmed by Hertz in 1887 [6]. By the end of the 19th century, there were measurements of the speed of light by a variety of laboratory, terrestrial, and astronomical techniques that gave an experimental value for the speed of light and the waves that Hertz discovered. The presently accepted value of the speed of light is 299,792,458 m/s and is an exact number by international convention [7]. Maxwell's work provided an underpinning for the explanation of all forms of electromagnetic radiation including that of light from the ultraviolet through the infrared. Table 1.1 shows the internationally recognized designations for the various wavelength regions [8] commonly used in radiometric measurements discussed in this book.

Toward the end of the 19th century, there was intense interest in the scientific community to correctly explain the observations being made concerning the spectrum of radiation from high-temperature sources. This in-

TABLE 1.1. Commonly Named Wavelength Regions

Region	Wavelength interval
UV-C	100–280 nm
UV-B	280–315 nm
UV-A	315–400 nm
Visible	380–780 nm
IR-A	780–1400 nm
IR-B	1.4–3 μm
IR-C	3–1.0 mm

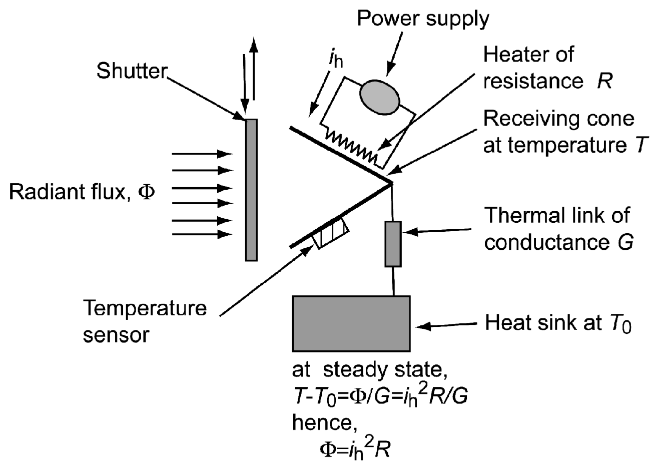

FIG. 1.1. Schematic diagram of an electrical substitution radiometer.

terest helped precipitate the development of new kinds of optical radiation detectors that were designed to make direct measurements of the amount of radiation in portions of dispersed spectra. One of the first radiometers developed was a device that compared optical power to electrical power and hence became known as an electrical substitution radiometer. A schematic representation of such a device is shown in Figure 1.1. The device has a shutter that can be opened to allow light to fall upon the receiving cone which is usually coated with an absorbing material. This will cause the temperature of the cone to rise to some equilibrium value T that depends on the conductance of the thermal link and the heat sink that is maintained at T_0 and other parameters of the system. When the shutter is closed, an electrical current i_h is passed through the heater that then maintains the temperature T of the cone. Neglecting correction due to various losses, this equivalence of temperature implies that the optical power Φ is equal to the electrical power $i_{\mathrm{h}}^2 R$. Devices using this principle today are of fundamental importance in radiometry and their modern application is discussed in Chapter 2.

Kurlbaum and Ångstrom, working separately in the 1890s, are credited with developing the first electrical substitution radiometers for measuring a physical process [9]. They performed measurements of the spectral distribution of the radiation from blackbody sources using these radiometers. The attempt to understand blackbody radiation was an important scientific topic at the end of the 19th century. The spectrum from a blackbody was shown to rise in radiance from low levels at short wavelengths, reach a maximum, and then decrease in radiance at longer wavelengths. As we mentioned earlier in this chapter, radiance is the amount of optical power from a surface area that is emitted into a solid angle. The wavelength position of the maximum radiance shifted toward shorter wavelengths at higher temperatures. A perfect blackbody source is a radiator that would absorb and emit with unit efficiency, while any realistic source will have an efficiency that is wavelength-dependent and is less than unity. In the early 20th century, Coblentz at the Bureau of Standards in the US developed a thermopile detector system that relied upon electrical substitution to measure the Stefan–Boltzmann constant and the constants in Planck's radiation law [10–12]. Coblentz's radiometer was conceptually similar to that shown in Figure 1.1, except the receiving surface was an absorbing thermocouple array which directly gave a signal. Similar work, too extensive to review here, was carried on in a number of laboratories worldwide. A later section of this book will deal with the details of blackbody sources, and an excellent description of the development of the various early electrical substitution radiometers can be found in Hengstberger's book [9]. Electrical substitution radiometers are sometimes called absolute radiometers because they measure the optical power directly using fundamental physical relationships and do not rely upon some other type of optical device for their calibration.

Planck, after attempts by Rayleigh, Boltzmann, and others, developed a theory that correctly accounted for the spectral distribution of blackbody radiation. Planck's theory necessitated the hypothesis that the radiators emitting energy in the blackbody source emit energy only in multiples of a quantity that is proportional to the frequency of the radiation [13]. This insight by Planck is credited with the development of modern ideas on the quantum nature of physical phenomena. The proportionality constant between the energy $h\nu$ the quantized unit of light, later named the photon, and its frequency ν is called Planck's constant h.

One outgrowth of Planck's formulation of the photon nature of light is the seeming variance with the wave nature of light as predicted by Maxwell's equation. Most of the issues in radiometry can be understood using geometrical optics or wave notions, but in some parts of this book it will be appropriate to introduce descriptions based upon the quantum theory of radiation in which the photon or particle nature of light is necessary for understanding [14]. For example, the photoelectric effect was first properly described by Einstein by invoking the quantum nature of light [14, 15]. Classical wave theory of light suggested that the energy of the photoelectrons emitted from surfaces when light is incident should increase with the intensity of the light. Instead it was found that the energy distribution of the electrons only depended upon the frequency of the light which led Einstein to suggest the explanation that the energy of the light was proportional to its frequency in the same manner that Planck had hypothesized to explain blackbody radiation. These two experiments, blackbody radiation and the photoelectric effect, and their explanations, are a major underpinning of modern quantum theory.

An important milestone in improving the accuracy in radiometry that demands special note was the development of a cryogenic electrical substitution radiometer by Quinn and Martin at the National Physical Laboratory in the UK. This instrument enabled the measurement of the Stefan–Boltzmann constant to an uncertainty of 100 parts in a million [16]. The advantage of cryogenic operation, usually at liquid helium temperature (4 K), is that various sources of error that affect the establishment of equivalence between the electrical and optical heating of the cavity are eliminated or greatly reduced. These include errors caused by radiative loss, conductive loss in electrical leads, convection losses, and others. Cryogenic radiometers are now the standards by which most national metrology institutes maintain their radiometric quantities and they have had a profound impact on lowering the measurement uncertainties associated with radiometric measurements. Chapter 2 is devoted to the important topic of cryogenic radiometers and their use.

The next section describes the nomenclature associated with the practice of radiometry and photometry and discusses the essential geometrical aspects of radiometry that are essential to its understanding and use. In Section 1.4, the basic problem of radiometry is introduced in terms of the measurement equation. The measurement equation is a method of analyzing radiometric measurement arrangements and, for example, allows for the output of a radiometer to be expressed in terms of the source quantities and the geometrical, electrical, optical, and other relevant properties of the optical sensor system. This is the key to understanding the sources of uncertainty and deducing the expected quality of a measurement. In general, the measurement equation allows one to extract physically meaningful data from a measurement of optical radiation with some detector system. The use of measurement equations is an integral part of most of this book, and their use in deducing uncertainty estimates is explored as appropriate. An example of calculating the uncertainty in a radiometric measurement using the measurement equation is presented in Section 1.5.2, and Chapter 6 deals with uncertainty according to internationally accepted practice [17].

1.3 Radiometric Terminology

The radiometric terminology in this book conforms to the definitions accepted by the International Standards Organizations (ISO) and the International Commission on Illumination (CIE) [8, 18]. Table 1.2 summarizes few of the commonly used radiometric quantities and their corresponding photometric quantities. The symbols representing spectral radiometric quantities, for example, spectral irradiance, are formed by adding a subscript appropriate to the spectral quantity, for example, wavelength λ, to symbolize the spectral irradiance E_λ. The denominators of spectral units have an additional unit of length and, in the case of spectral irradiance, the dimension is W/m^3. More commonly the wavelength is measured in nanometers (nm), and the dimension becomes $W/(m^2\,nm)$.

The radiometric quantities listed in Table 1.2 can be visualized with reference to Figure 1.2a, in which an emitting surface designated by dA_1 acts as a source of radiation that impinges upon a receiving surface designated by dA_2. For the purposes of this discussion, dA_1 emits uniformly in all directions and the two surfaces are both centered on and perpendicular to the centerline. A source of radiation emitting equally in all directions is called a

TABLE 1.2. Radiometric and Photometric Quantities and their Units

Radiometric quantity	Symbol	Units	Units	Symbol	Photometric quantity
Radiant energy	Q	J	lm s	Q_v	Luminous energy
Radiant flux (power)	P, Φ	W	lm	Φ_v	Luminous flux
Irradiance	E	W/m^2	$(lm/m^2) = lx$	E_v	Illuminance
Radiance	L	$W/(m^2\,sr)$	$lm/(m^2\,sr)$	L_v	Luminance
Radiant intensity	I	W/sr	(lm/sr) = cd	I_v	Luminous intensity
Radiant exitance	M	W/m^2	lm/m^2	M_v	Luminous exitance
Radiant exposure	H	$W\,s/m^2$	lx s	H_v	Luminous exposure
Radiance temperature	T	K	K	T_c	Color temperature

Note: J = joule, W = watt, lm = lumen, lx = lux, m = meter, sr = steradian, s = second, cd = candela, K = kelvin.

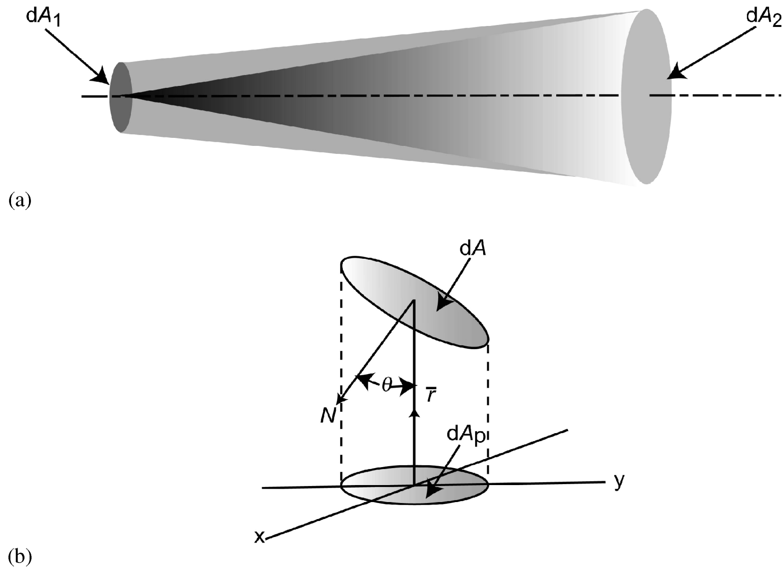

FIG. 1.2. (a) Schematic of a source of radiation at dA_1 which illuminates a second surface dA_2. (b) Schematic which shows the projected area dA_p of an area dA.

Lambertian emitter. The radiant intensity is conceptualized as the power from a point on the surface emitted into the solid angle shown by the cone of light starting at the origin of dA_1 and intersecting dA_2, and hence is the power per steradian. More practically, the radiant intensity is the amount of power per steradian passing through a surface subtending a given solid angle which can be realized in a situation when the observer is far from a small source that can be considered a point source. The term intensity in optics is often used in differing ways and can cause great confusion [19, 20]. The *radiance* of the source area dA_1 is defined by the amount of optical power from dA_1 within the angular space defined by the truncated cone of radiation emitted from the entire surface element dA_1 and which is incident upon the area dA_2. Radiance is the optical power per area of the source per steradian of solid angle defined in some direction of the propagation and, in this case, the solid angle is defined by the distance between the two surfaces and the size of the areas. Assuming the total optical power passing through the surface dA_2 is evenly distributed over the area dA_2, the *irradiance* is defined by dividing the radiant flux by the area dA_2. Irradiance is then the power per unit area in some region of space and is a very useful quantity for describing the energy obtainable from an optical source at a given position. For example, the energy from the Sun is usually given in irradiance, W/m^2, for use in estimating the amount of solar energy available in some configuration. These radiometric quantities are developed more fully in the following sections.

The quantity, $\mathrm{d}A\cos\theta$, often in radiometry and is called the projected area $\mathrm{d}A_\mathrm{p}$. This concept can be seen from the geometry shown in Figure 1.2b where an area $\mathrm{d}A$, whose normal vector N is oriented at an angle θ with respect to a plane defined by coordinates x,y and which has a normal direction shown by $\bar{\mathbf{r}}$. The inclination produces a projected area $\mathrm{d}A_\mathrm{p} = \mathrm{d}A\cos\theta$ in the xy plane and represents the area of $\mathrm{d}A$ as viewed from the xy plane. This concept is useful in describing the amount of flux passing through a plane due to some external source such as might be represented by an emitting surface element $\mathrm{d}A$.

Radiometric quantities can be functions of wavelength λ, frequency, v, or wavenumber, σ. The quantities λ, v, and σ are related by

$$\lambda = \frac{c}{nv} = \frac{1}{\sigma} \tag{1.1}$$

In Eq. (1.1), c is the velocity of light, and n the index of refraction of the medium in which the light is propagating. If radiometric quantities are functions of λ, v, and σ, they are designated by the same term preceded by the adjective *spectral* and by the same symbol followed by λ, v, or σ, in parentheses to indicate the functional dependence; for example, spectral emissivity $\varepsilon(\lambda)$. Some of the quantities, for example, radiance, can be functions of wavelength (or frequency or wavenumber) and it is then called the *spectral radiance* and is represented by the symbol for the quantity with the subscript λ (v or σ), depending on the quantity chosen for the independent variable. Using this notation, the spectral radiance would be represented symbolically by L_λ with the functional dependence on λ implicit, as indicated here, or included the explicitly by formally including the variable in parentheses, i.e., $L_\lambda(\lambda)$. The subscript indicates here, as in calculus notation, that the quantity L_λ is differential with respect to λ and, hence, in this case, the spectral radiance is the radiance per wavelength interval. The equations governing the spectral radiometric quantities can be converted into one or the other of the possible independent variables by using the ordinary rules of algebra and calculus for substitution of variables in equations.

The photometric quantities on the right side of Table 1.2 are obtained from the corresponding spectral radiometric quantities by integrating the spectral radiometric quantity weighted with the function called the *spectral luminous efficiency*, $V(\lambda)$, over the visible wavelength region [21]. The $V(\lambda)$ function is used for characterizing the human visual response under good lighting conditions, and there are other functions defined for the human visual response under lighting conditions that are less than optimal.

1.3.1 Radiance

The primary quantity measured by radiometers is optical power Φ incident upon the detector. While in some cases the power is the quantity of interest, in most radiometric measurements, one is trying to deduce some other quantity such as the radiance of a source or the irradiance incident upon some surface. As mentioned in Section 1.2, we develop the detailed concepts of radiance first and then show how the irradiance is related to the radiance and geometrical factors. In the previous section, these quantities were defined in general terms as shown in Figure 1.2 and in this section, we develop the ideas of radiance and the other quantities in the detail necessary to fully define radiometric measurement arrangements and provide the framework necessary to estimate appropriate uncertainties.

The relationship between quantities such as power, radiance, and irradiance can be demonstrated by considering a general type of radiometric measurement situation that is shown in Figure 1.3. In Figure 1.3, x_1 and y_1 describe a coordinate system centered on a source that emits radiation from a differential element of area $\mathrm{d}A_1$ positioned on the larger area shown as A_1. This source is characterized by its radiance, which is the amount of optical power per unit area of the source emitted per unit of solid angle. In order to visualize these quantities, it is useful to describe a bundle radiation from $\mathrm{d}A_1$

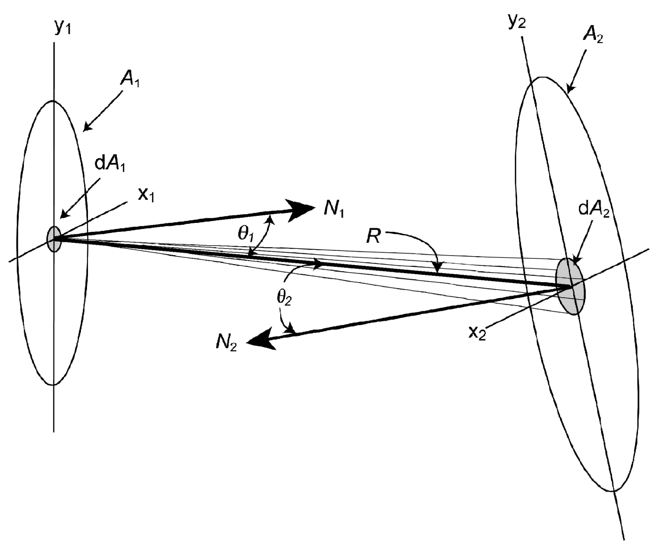

FIG. 1.3. Generalized configuration of optical radiation passing from one surface to another. The surface A_1 is considered a source region that originates rays of optical radiation passing onto surface A_2.

that is incident upon a second surface in a differential element dA_2 of a larger area A_2 centered on coordinate system x_2 and y_2. The lines connecting surface elements dA_1 and dA_2 in Figure 1.3 indicate some, but certainly not all, of the possible path rays of light traverse between the surfaces.

The radiation incident upon A_2 can be characterized in terms of the radiance, but often the irradiance, the amount of optical power per unit area, is a more useful quantity for characterizing radiation incident upon a surface. A discussion of Figure 1.3 shows the relationship between these fundamental quantities. A_1 and A_2 are shown in Figure 1.3 as being circular for schematic reasons, but they can be of any shape that describes a source of radiation and a surface of interest through which it propagates. For a typical example in a radiometric measurement, A_2 is the entrance aperture of a detector system and A_1 describes the aperture of the source providing the radiation.

R is a line whose length is the distance between the origins of the two surface area elements, and N_1 and N_2 are the normal vectors to the surfaces at angles θ_1 and θ_2 with respect to R. The coordinate systems are centered on the apertures for convenience. It is useful to discuss this general type of radiometric configuration in order to define the radiometric quantities involved and to become aware of the consequences of approximations made when simplifying for actual measurement arrangements.

If L_1 is the radiance of the source at dA_1, the amount of flux $\Delta\Phi_1$ in the beam that leaves the element of area dA_1 and that passes through element of area dA_2 is

$$\Delta\Phi_1 = \frac{L_1 dA_1 \cos\theta_1 dA_2 \cos\theta_2}{R^2} \tag{1.2}$$

This equation defines radiance and underscores its fundamental properties in describing the propagation of fluxes of optical radiation. Equation (1.2) relates the optical power passing through a region of space to a property of the source, the radiance L, and purely geometric considerations. In general, the radiance is a function of the coordinates defining dA_1 as well as the angles that define the direction of propagation of the light leaving surface dA_1, and thus evaluating Eq. (1.2), in real circumstances, can be difficult. In many cases, simplifying assumptions must be made to obtain a result. It is important, however, to start with this complex definition of radiance in order to understand the implications of the approximations that are made to evaluate the flux in configurations used in practical radiometric measurements. Terms on the left portion of the right-hand side of Eq. (1.2) can be grouped and the expression written in a different manner. Using the following definition

$$d\omega_2 = \frac{dA_2 \cos\theta_2}{R^2} \tag{1.3}$$

where $d\omega_2$ is the solid angle subtended at dA_1 by area dA_2, we can rearrange Eq. (1.2) in the following form to explicitly define radiance

$$L_1 = \frac{\Delta\Phi_1}{d\omega_2\, dA_1 \cos\theta_1} \tag{1.4}$$

This equation defines the radiance in terms of the optical flux and geometry of the source and some area through which the flux passes and is useful, along with Figure 1.3, in conceptualizing the meaning of the quantity radiance. We see in Table 1.2 that radiance has units of $W/(m^2\, sr)$ and from Eq. (1.4) that radiance is the optical power of a source emitted into a solid angle defined by the region of space in which the power is directed. It is important to keep track of the directions and angles included between source and observer in determining radiance due to the angular component, cos θ_1, in Eq. (1.4).

In the absence of any dissipative mechanisms in the space between A_1 and A_2, the flux in the beam leaving dA_1 within the solid angle shown in the construction of our example, is equal to that which passes through dA_2. One can create an equation much like that of Eq. (1.2) that describes the relationship between the radiance and flux at dA_2 and show that the radiance is a conserved quantity in the beam [22–24]. This can easily be seen if we introduce a radiance L_2 that represents the radiance of the beam at surface element dA_2 and realize that Eq. (1.2) can express the flux $\Delta\Phi_2$ at that surface by permuting the variable subscripts, $1 \leftrightarrow 2$, and generate the same equation as Eq. (1.2) due to the symmetry in the variables. This argument which assumes that the radiance is not varying over the elements of area dA_1 and dA_2 leads to Eq. (1.5a) which states the conservation of radiance in a beam which is propagating in a non-dissipative medium whose index of refraction is unity. This fundamental relationship underscores the importance of the concept of radiance in radiometry. This relationship undergoes a slight modification if the index of refraction of the medium in which the optical radiation is traveling differs from unity as assumed so far in our discussion. Application of Snell's law to the angles defining the angular quantities in the definitions of radiance leads to a factor of n^2 normalizing the radiance, where n is the index of refraction at the defining surfaces for the determination of the radiance, dA_1 and dA_2 in this example. Using n_1 and n_2 to represent the index of refraction of the medium at the surfaces dA_1 and dA_2 respectively, we can write Eq. (1.5b), which defines the conserved quantity in circumstances when the index of refraction is different from unity [25]. The quantity L/n^2 is sometimes referred to as the *reduced radiance* and becomes the generalized conserved quantity in the presence of media with varying indices of refraction. This relationship depends upon the conservation of flux in a beam and hence factors due to any scattering because of index variation or other factors are not accounted for in this formulation and would have to be dealt with separately.

$$L_1 = L_2 = \text{constant} \tag{1.5a}$$

$$\frac{L_1}{n_1^2} = \frac{L_2}{n_2^2} = \text{constant} \tag{1.5b}$$

1.3.2 Radiance in Terms of Projected Area and Projected Solid Angle

Dropping subscripts on the solid angle and letting the elemental areas and solid angles become differentials by taking limits, and realizing that the flux depends upon two quantities, Eq. (1.4) can be rewritten in terms of the projected area as

$$L = \frac{\mathrm{d}^2\Phi}{\mathrm{d}\omega\,\mathrm{d}A_\mathrm{p}} \tag{1.6}$$

Written in this way, the radiance is seen as the amount of flux per unit projected area of the source per unit solid angle subtended by the area to which the flux is headed. The reader is referred to the literature for details of the mathematical limiting procedures to arrive at the mathematical correct differential form of radiance [23, 25, 26].

An alternative and sometimes useful variant of the development of the projected area is the introduction of the concept of projected differential solid angle $\mathrm{d}\Omega = \cos\theta\,\mathrm{d}\omega$. The projected solid angle Ω is then given by

$$\Omega = \int_\omega \mathrm{d}\Omega = \int_\omega \cos\theta\,\mathrm{d}\omega \tag{1.7}$$

In this notation the expression for the radiance in Eq. (1.6) is written as

$$L = \frac{\mathrm{d}^2\Phi}{\mathrm{d}\Omega\,\mathrm{d}A} \tag{1.8}$$

1.3.3 Radiant Flux and Irradiance

Inspection of Eq. (1.2) indicates that the total radiant flux Φ_1 passing through the surface A_2 which originates from surface A_1 can be expressed with an integral relation over the surface variables of the two surfaces of interest, i.e., A_1 and A_2 as shown in Figure 1.3. Therefore,

$$\Phi_1 = \int_{A_1}\int_{A_2} \frac{L_1 \mathrm{d}A_1 \cos\theta_1 \, \mathrm{d}A_2 \cos\theta_2}{R^2} \tag{1.9}$$

An additional implicit consideration in Eq. (1.9) is the fact that L_1, in addition to being a function of the spatial coordinates of the emitting surface, is also in general a function of the wavelength. In this situation, the relationship would describe the *spectral flux* leaving the surface that is a result of the *spectral radiance* of the surface.

In general, the integral in Eq. (1.9) is difficult to perform, particularly if L_1 is a function of the spatial and angular coordinates. Additionally if the apertures are large, the angles θ_1 and θ_2 and the distance R all have complicated relationships across the areas that make the solution of Eq. (1.9) very difficult. The propagation of optical flux represented by Eq. (1.9) is related to problems in other areas of physics such as heat transfer and some of the techniques developed for those problems can be utilized for calculations of radiometric flux transfer [27, 28].

Although Eq. (1.9) is in general difficult to solve exactly, in many situations in radiometry it is fortunate that circumstances exist or assumptions can be made which allow for simplification and solution of the complex integral relationship. Often sources of radiometric interest can be considered Lambertian; they radiate a constant radiance in all directions into a hemisphere, and furthermore they emit the same at every point in the source plane A_1. If this is the case, we can rewrite Eq. (1.9) in the following manner by removing the radiance from the functional considerations,

$$\Phi_1 = L_1 \int_{A_1}\int_{A_2} \frac{\mathrm{d}A_1 \cos\theta_1 \mathrm{d}A_2 \cos\theta_2}{R^2} = L_1 A_1 \pi F_{12} = L_1 T_{12} \tag{1.10}$$

where F_{12} only depends on the geometry and is called the configuration factor, which is obtained by evaluation of the double integral. Other terms for the configuration factor include view, shape, or exchange factor. For many cases in ordinary radiometry, the configuration factor can be looked up in literature [27, 28] or evaluated using numerical techniques and computers. The quantity $T_{12} = A_1 \pi F_{12}$ occurs often in radiometry and is called the *throughput* of the particular optical arrangement. In the absence of dissipative effects, when the flux is conserved in an optical system, the invariance of the radiance implies the invariance of the throughput of the system. Using the same arguments as above, one can readily show the reciprocity relations for radiometric systems, $A_1 F_{12} = A_2 F_{21}$ or $T_{12} = T_{21}$, which indicates the reversibility of the propagation of the optical beam.

Additional insight and quantities can be seen by considering a bundle of rays leaving surface A_1 in Figure 1.3 and applying Eq. (1.8) to calculate the flux in a bundle of rays leaving the surface with some simplifications. In this approximation, we assume that the projected solid angle of the beam is independent of the position on A_1. This means, in practice, that the source size and aperture size of A_2 are small compared to the distance R. Reinserting subscripts into Eq. (1.8) and integrating since we can separate the integrands by assumption, we have

$$\Phi_1 = L \int_{A_1} \mathrm{d}A_1 \int_{\omega_2} \cos\theta_1 \, \mathrm{d}\omega_2 = L A_1 \Omega_{12} \tag{1.11}$$

In this approximation, often useful in radiometry, the throughput $T_1 = A_1\Omega_{12}$, can often be simply approximated and the flux and radiance directly and simply related. In more complicated situations where the dimensions are large, the more exact expressions utilizing the configuration factor must be used. For example, in a simple case where the source is a uniform small circular aperture A_1 separated by a distance R that is large ($R > 20$ radius of A_1) from a receiving aperture of A_2 of similar small size and the two apertures are perpendicular to the line connecting their centers, we have the flux incident upon A_2, Φ_2, equal by construction to the flux leaving A_1, Φ_1,

$$\Phi_2 = \Phi_1 \cong L_1 \frac{A_1 A_2}{R^2}, \qquad E_2 = \frac{\Phi_2}{A_2} = \frac{L_1 A_1}{R^2} \tag{1.12}$$

In this case, the irradiance E_2 at the surface A_2 is simply related to the radiance of the beam and geometric factors. The irradiance and its spectral analog, the *spectral irradiance*, are important radiometric quantities because they specify the optical power in a cross-sectional area of an optical beam. The irradiance is an important quantity because most detectors measure optical power, and by knowing the aperture area defining what the detector intercepts, the irradiance can be determined and related to radiance by equations like Eq. (1.12).

The more general case of two circular apertures whose centers are on a common centerline and which are oriented such that their aperture planes are normal to the centerline is an important arrangement in radiometry and will be discussed here as a further example. Letting r_1 be the radius of A_1 and r_2 the radius of A_2, the configuration factor determined by the integral in Eq. (1.10) becomes [27, 29]

$$F_{12} = \frac{1}{2}\left[\left(\frac{r_1^2 + R^2 + r_2^2}{r_1^2}\right) - \left[\left(\frac{r_1^2 + R^2 + r_2^2}{r_1^2}\right)^2 - 4\frac{r_2^2}{r_1^2}\right]^{1/2}\right] \tag{1.13}$$

From Eq. (1.10) and writing $A_1 = \pi r_1^2$ and assuming the radiance is a constant over the surface, we can write an exact expression for the optical power at the second aperture due to the radiance from A_1 as

$$\Phi = L\frac{\pi^2}{2}[(r_1^2 + r_2^2 + R^2) - [(r_1^2 + r_2^2 + R^2)^2 - 4r_1^2 r_2^2]^{1/2}] \quad (1.14)$$

In many cases R is sufficiently larger than either of the radii of the apertures and the term can be factored out and the expression expanded and simplified. We have then,

$$\begin{aligned}\Phi &= \frac{L\pi r_1^2 \pi r_2^2}{(r_1^2 + r_2^2 + R^2)}\left(1 + \frac{r_1^2 r_2^2}{(r_1^2 + r_2^2 + R^2)^2} + \text{higher terms}\right) \\ &= \frac{LA_1 A_2}{D^2}[1 + \delta +] \end{aligned} \quad (1.15)$$

where we have $D^2 = r_1^2 + r_2^2 + R^2$ and $\delta = r_1^2 r_2^2 / D^4$. QA :1

This result gives a correction to the approximations used in Eq. (1.12) and is useful for many radiometric applications and for uncertainty analysis, although with the ready availability of modern computers, the exact expression can often be easily derived. As long as the source is Lambertian, this equation is useful for calculating the radiance from a measured flux and for estimating the contributions to the uncertainty by evaluating the significance of higher-order terms in the above expansion. In this example, if an optical detector were behind the aperture A_2, its measurement of the optical power Φ would allow the determination of the radiance of the source at A_1. It can be seen by comparison of Eq. (1.12) to the results in Eq. (1.15) that the more exact calculation using the configuration factor results in an equation of the same general form but with corrections to the geometrical factors. The irradiance at A_2 in the approximations implied by Eq. (1.15) is simply related to the radiance of the source and a geometric factor and is found by dividing both sides of Eq. (1.15) by A_2.

The reader is referred to the extensive literature on specific optical systems to find approximations used for the configuration factor to calculate the throughput for various specific optical arrangements, including those with lenses and other beams forming and steering devices [22–24, 30, 31]. In any radiometric system where these equations and approximations are used, it is necessary to keep in mind that the degree of approximation used also introduces uncertainties into the measurement process that must be accounted for in the uncertainty budget for the measurement. The simplifications shown above, such as by Eq. (1.15), rely upon the uniformity of the optical beam and the Lambertian nature of the source as well as the lack of dissipation of the beam by scattering or absorption of optical radiation. If a source is not Lambertian and uniform, extensive measurements and characterizations are necessary to understand the relationship between the radiance of the source and any measured optical power.

1.3.4 Radiant Intensity

Another quantity that is useful in radiometry for certain applications is called the *radiant intensity* and is usually denoted by the symbol I. This quantity usually is associated with point sources or those of negligible dimensions and is the amount of flux per solid angle. In terms of the variables used above, the radiant intensity is

$$I = \frac{d\Phi}{d\omega} \tag{1.16}$$

This quantity corresponds to the flux in the bundle of radiation shown in Figure 1.3 if the source element of area dA_1 is collapsed to a point. The corresponding photometric quantity is the *luminous intensity*, which is measured in the SI base unit of the candela.

1.4 Radiometric Measurements

The main measurement problems posed in radiometry are the characterization of a source of optical radiation for its radiance, spectral radiance, or photometric quantities, and the development, characterization, and calibration of a detector system to make such measurements. For example, by measuring the spectral radiance of a blackbody source, its temperature can be inferred. In other applications, optical sensor systems that operate in narrow wavelength regions are used to deduce properties of celestial bodies or to monitor earth resources from orbiting spacecraft. These specific applications, as well as others, are discussed in the following chapters of this book. QA :2

The problems of correctly characterizing sources and detectors have been the traditional driving forces for improvements in radiometry. These issues continue to attract attention in order to meet current demands for increased accuracy of measurement for remote sensing, industrial applications, and scientific studies. Separate chapters of this volume will deal with the calibration and characterization of modern photodetector systems, and others will deal with the characterization of optical radiation sources that can be used as calibration sources. These once poorly connected efforts are merging as the technology for building and characterizing stable and accurate photodetector systems, which allows for very accurate determination of source characteristics, and hence the traditional separate technologies of characterizing sources and detectors is merging into one of characterizing detector systems [32, 33]. While sophisticated optical detector systems often form the basis of fundamental standards maintenance for both sources and detectors, well understood and characterized optical radiation sources are often very useful for the calibration of the spectral characteristics of optical sensor systems.

1.4.1 Detector Responsivity

A detector of optical radiation generates an output, usually electrical, which can be related to the amount of optical flux incident upon the detector. For example, in solid-state detectors, the absorption of a photon results in the promotion of a charge carrier to the conduction band and a resultant current in an electrical circuit. At a particular wavelength, this current is found to be proportional to the optical flux over a large dynamic range [25, 34]. Various types of solid-state devices are used to detect optical radiation from the X-ray to the infrared wavelength region and are the backbone of many complicated radiometric sensor instruments [34, 35].

Figure 1.4 shows a typical configuration of how a solid state or other type of detector might be used to make a measurement of the properties of a source of radiation. A source of optical radiation that has a defining aperture of area A_S and radius r_S is a distance d from an aperture A_D with a radius r_D. A_D defines the flux boundary of the radiation that passes through a filter F and is incident upon a detector D. For solid-state detectors, the output signal is usually a current, and hence a signal response r which is proportional to the current produced by the detector is generated by the signal amplifier. In electrical substitution radiometers the signal would be proportional to the electrical power needed to generate a response equivalent to the optical power. This relationship between optical power and the output signal response r is called R, the responsivity of the detector; hence we have

$$r(\lambda) = R(\lambda)\Phi(\lambda) \tag{1.17}$$

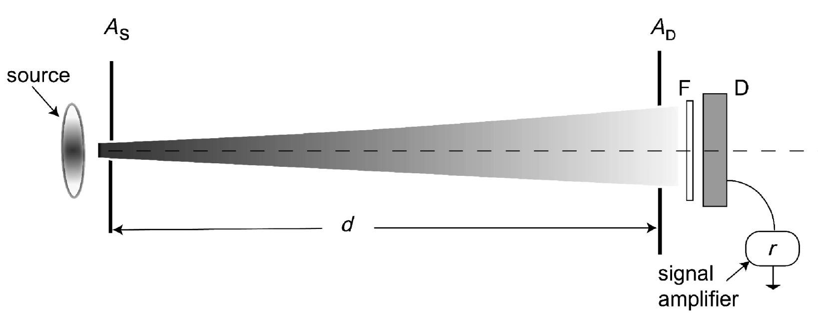

FIG. 1.4. A Lambertian source behind aperture A_S illuminates a defining aperture A_D. The radiation passes through an optical filter F and is measured by a detector D that generates a signal response r.

As indicated, the signal, responsivity, and flux in general depend upon the wavelength of the radiation. This is indicated in Eq. (1.17) by explicitly indicating the functional dependence upon the wavelength λ. In general, the responsivity R of a detector can depend upon a host of parameters including temperature and other environmental quantities, polarization of the light, spatial effects on its own receiving surface, angle of incidence, and others. The complete characterization of the various factors in the response of a detector is essential to the accurate operation of the system in which it is employed and to properly generate an error budget for the system. A system composed of a detector, electronics, and optics including wavelength selection, is often referred to as a *sensor*. The terms detector and sensor as well as radiometer are sometime used interchangeably and, as mentioned earlier in this chapter, the meaning must be inferred from the context of the discussion. Subsequent chapters of this book describe the characterization and calibration of various detectors and their use in measurement systems.

1.4.2 The Measurement Equation

Henry Kostkowski and Fred Nicodemus of the National Bureau of Standards (now NIST) introduced the concept of a "measurement equation" in radiometry [23, 26]. The measurement equation describes receiver output due to the optical radiation received from a specific source configuration. It is a *system equation*; i.e., it models the system performance in terms of the subsystem and component specifications and provides not only the measurement quantities required, but also serves as the basis for estimating the uncertainties of the measurement.

1.4.2.1 Measurement equation for a filter radiometer

The situation depicted in Figure 1.4 is a simple, yet common example encountered in radiometry. If we assume the source is a Lambertian emitter with a spectral radiance $L_\lambda(\lambda)$, the source and detector apertures, A_S and A_D with radii r_S and r_D, respectively, are circular and perpendicular to the line connecting their centers and are a distance d apart, and the detector has a uniform spatial response, then we can use the results of the previous section to generate the measurement equation. The transmittance of the filter is represented by $\tau(\lambda)$ and the spectral responsivity of the detector to optical power will be designated $R(\lambda)$. Using the results of Eqs. (1.15) and (1.17), we can write the signal response r in terms of an integral over wavelength

$$r = \frac{A_S A_D (1 + \text{correction})}{(r_S^2 + r_D^2 + d^2)} \int_\lambda L_\lambda(\lambda)\tau(\lambda)R(\lambda)\,\mathrm{d}\lambda \qquad (1.18)$$

This result is obtained by using the expression for the optical flux as shown in Eq. (1.15), modifying it by the transmittance of a filter and integrating over the wavelength region where the filter has significant transmittance. To determine the spectral radiance of the source from this equation, further assumptions need to be made in order to evaluate the integral. If the functional form of the spectral radiance is known, such as if it is a blackbody, as the transmittance and detector responsivity are known, the equation can often be iteratively solved for the spectral radiance and hence the temperature of the source. In other situations, the bandpass of the filter is narrow compared to the variations in the incident *radiance* distribution. This allows the values in the integral to be calculated by taking appropriate average values of the spectral quantities over the bandpass $\Delta\lambda$ of the instrument. Defining the quantities in front of the integral sign in Eq. (1.18) as C, we can rewrite the equation in a more useful form

$$\begin{aligned} r &= CL_{\lambda_0}(\lambda_0)\tau(\lambda_0)R(\lambda_0)\Delta\lambda \\ &= R_{\mathrm{L}}L_{\lambda_0} \end{aligned} \tag{1.19}$$

where R_{L} is called the *total radiance responsivity* of the radiometer. In this approximation, it is assumed that the spectral radiance is a constant in the wavelength interval determined by $\Delta\lambda$, and $\Delta\lambda$ is chosen so that the product of the transmittance, responsivity and $\Delta\lambda$ gives the value that the integral over these quantities would yield. In other words, the integral over these quantities is replaced by a width and average value of the integrands. The value of λ_0 in the equation is chosen such that the value of the functions evaluated at this point and the value for the bandpass give the best estimate of the integral approximation. This equation is often used in radiometry and it is important to remember the assumptions that have been made and how they may contribute to the uncertainty of the measurement. For example, one must evaluate how well this approximation for the integral relation actually reproduces the more exact expression and add a component in the error budget for the added uncertainties due to the approximations. In Section 1.5, we discuss how the radiometer could be alternatively calibrated with a known spectral radiance source.

1.4.2.2 Measurement equation for a spectral radiometer

A spectral radiometer is a device which performs wavelength selectivity in its measurement of optical radiation. While in one sense the device shown in Figure 1.4 has wavelength selectivity, it is not normally referred to as a spectral radiometer since its wavelength selectivity is fixed. Figure 1.5 shows schematically the principal components of a spectral radiometer that is designed to measure spectral radiance. Substitution of different collection optics could easily make this device a spectral irradiance or spectral flux

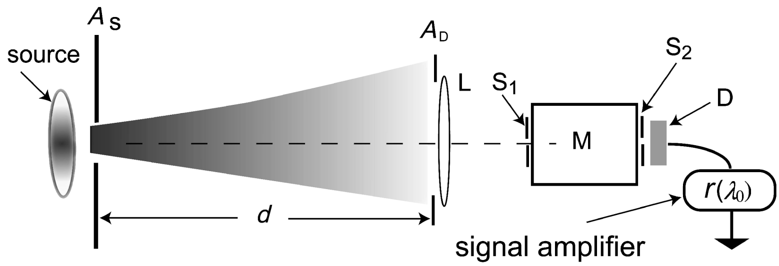

FIG. 1.5. A source illuminates an aperture A_S that transmits radiation to a defining aperture A_D. The radiation is focused by a lens L onto a monochromator system M that disperses the radiation onto a detector D. The output from D is conditioned by a signal amplifier that generates a wavelength-dependent signal $r(\lambda_0)$ at each wavelength setting λ_0.

radiometer with minimal fundamental changes in the formulation we present here.

In Figure 1.5, a source illuminates an aperture A_S which transmits light that fills the detector aperture A_D that is a distance d from the source aperture. As in the previous discussion of the filter radiometer, the two apertures are perpendicular to the axis joining their centers. The optical radiation passing through A_D is imaged by a lens L onto a monochromator with an entrance slit S_1 and an exit slit S_2. The dispersed radiation exiting the monochromator is incident upon a detector D which has its output connected to a signal amplifier that conditions the detector output and produces a response $r(\lambda)$ (that is dependent upon the monochromator's wavelength setting specified by λ_0. The overall spectral responsivity $R(\lambda_0, \lambda)$ of the imaging system, the monochromator, and the detector is a function of wavelength where the monochromator is set, λ_0, and the wavelength range λ over which the instrument has sensitivity. Additionally, the responsivity is a function of the many variables that characterize the various elements of the system including the transmittance and reflectance of the various optical elements, the responsivity of the detection element, and dissipative effects such as scattering and diffraction.

It is often convenient to factor the overall system spectral responsivity $R(\lambda_0, \lambda)$ into a term that represents the wavelength selectivity called the slit scattering function $\rho(\lambda_0, \lambda)$ which has a finite amplitude only in the region around the set wavelength λ_0 and a term $R^f(\lambda)$ that represents the overall responsivity as a function of the wavelength [23, 26].

$$R(\lambda_0, \lambda) = \rho(\lambda_0, \lambda) R^f(\lambda) \tag{1.20}$$

In a case like we have described, the function $R^f(\lambda)$ can be further factored into terms representing transmittance, reflection, detector responsivity, and other terms that characterize the optical system. Each situation that a particular instrument poses will likely be amenable to various ways of accounting for the total responsivity of the instrument and, hence, we will keep to the general case for our discussion and leave the specific cases to other chapters in this volume as well as the literature [23]. The responsivity of a spectral instrument normally has a nonzero amplitude in a region $\Delta\lambda$ around the set wavelength of λ_0 as shown in Figure 1.6 by the slit scattering function $\rho(\lambda_0, \lambda)$. For our example, we will assume the shape $\rho(\lambda_0, \lambda)$ will not vary with the wavelength setting of the monochromator and that the wavelength variation of the magnitude of the responsivity can be accounted for by responsivity factor $R^{\mathrm{f}}(\lambda)$ which in this context is the amplitude variation of the system response as the wavelength is varied.

The slit scattering function represents the wavelength selectivity of the monochromator and is due to the imaging of the entrance aperture on the exit aperture. It also contains information about scattering and diffraction in the monochromator optics such as its grating and mirrors, and accounts for transmittance of the monochromator away from the central wavelength λ_0. In an idealized case of equal entrance and exit apertures, the slit scattering function is a triangular shape with a width representing the resolution of the instrument. This function must be determined experimentally by using appropriate narrow wavelength light sources such as lasers. Details of these techniques can be found in the literature [23, 26].

Inserting these expressions for the responsivity in Eq. (1.20) into Eqs. (1.18) and (1.19), and ignoring the small corrections, we can generate the

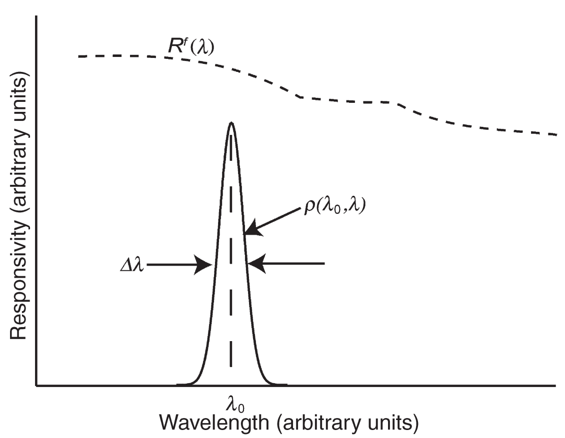

FIG. 1.6. Schematic of a responsivity relationship with an instrument with a slit scattering function $\rho(\lambda_0, \lambda)$ and a responsivity factor $R^{\mathrm{f}}(\lambda)$.

measurement equation for the spectral radiometer.

$$r(\lambda_0) = \frac{A_S A_D}{(r_S^2 + r_D^2 + d^2)} \int_\lambda L_\lambda(\lambda)\rho(\lambda_0, \lambda)R^f(\lambda)\,d\lambda$$
$$= C \int_\lambda L_\lambda(\lambda)\rho(\lambda_0, \lambda)R^f(\lambda)\,d\lambda \quad (1.21)$$

We have incorporated the geometric terms in the constant C. This constant will be different for different configurations caused by changes in apertures or the distances involved in the measurements. If rectangular slits were used instead of circular apertures, the configuration factor appropriate for slits could be obtained from the literature and the appropriate constant factor determined. The slit scattering function is usually very narrow compared to structure and variation in the spectral radiance and, as in the previous example, the term $L_\lambda(\lambda)$ can be removed from the integral and replaced by its suitably averaged value $L_\lambda(\lambda_0)$ to arrive at Eq. (1.22).

$$r(\lambda_0) = CL_\lambda(\lambda_0) \int_\lambda \rho(\lambda_0, \lambda)R^f(\lambda)\,d\lambda \quad (1.22)$$

1.5 Radiometric Calibration and Uncertainties

The measurement equations, such as Eqs. (1.19) and (1.22), form the basis for the uncertainty analysis in determining the radiometric quantities. In general, radiometric calibration of the sensor is performed by using the measurement equation to deduce the unknown radiometric quantity by *in situ* comparison with that of a standard under an identical geometrical set-up. In that case, the associated geometrical factors cancel, leaving the solution for the unknown radiometric quantity in terms of just the two measured output signals (the unknown and the standard) and the known value for the standard. Alternatively, the standard could be used to evaluate the responsivity of the sensor first, and then the calibrated responsivity is used in the solution of the measurement equation to measure the unknown quantity from signals measured under the same or known geometrical conditions. In either case, the solutions are expressed as equations that are often referred to as calibration equations.

For example, in the case of a spectral radiometer, the measurement equation, Eq. (1.22), can be used to determine an unknown spectral radiance if the slit scattering function and the responsivity factor can be determined using appropriate approximations and accounting for uncertainties introduced. This is most often accomplished by employing a known spectral source, in this case a spectral radiance source, to determine the value of the integral at each wavelength of interest. Using the superscript c to denote the values of the signal response and other components of Eq. (1.22) when a calibration source is used, we can write the relationship shown in Eq. (1.23). The signal output at each wavelength can be measured, and since the spectral radiance of the calibration source is known, the value of the integral in Eq. (1.22) can be determined using

$$\frac{r^c(\lambda_0)}{C^c L^c_\lambda(\lambda_0)} = \int_\lambda \rho(\lambda_0, \lambda) R^{\mathrm{f}}(\lambda)\, \mathrm{d}\lambda \tag{1.23}$$

By inverting Eq. (1.22) to determine the unknown spectral radiance using Eq. (1.23), we get the equation referred to as the calibration equation as shown in Eq. (1.24). It shows the unknown spectral radiance in terms of the signal response and the calibration quantities.

$$L_\lambda(\lambda_0) = \frac{C^c}{C} \frac{L^c_\lambda(\lambda_0)}{r^c(\lambda_0)}\, r(\lambda_0) \tag{1.24}$$

If the configuration factors are the same in both the calibration and the use of the instrument to perform measurements, the ratio of the configuration factors cancel, and if not, the geometrical terms involved in C can sometimes indicate their ratio. Care must be exercised in a system with imaging optics to ensure that the entrance slit on the monochromators is illuminated in the same way in the calibration as in the use of the instrument. Another potential error is introduced by the fact that the slit scattering function does not go to zero outside the region of $\Delta\lambda$ but in fact has some finite value. If the calibration source and the source to be measured have different functional forms, then further uncertainties can occur due to the differences in accounting for the contributions in the wings of the slit scattering function's transmittance. Koskowski treats corrections to the measurement equation due to proper accounting of the transmittance in the wings away from the set wavelength λ_0 of the slit scattering function [26].

If the spectral radiometer is designed to be a spectral irradiance sensor system, the arguments follow similar paths, except one uses a calibration source of known spectral irradiance. Irradiance sensor systems often employ an integrating sphere with a known aperture as a collection device that is placed in front of the monochromator. Another technique to calibrate spectral instruments involves using tunable laser systems that calibrate spectral instruments by scanning a narrow wavelength laser line across the portion of the spectrum covered by the spectral instrument [36]. These techniques are further illustrated in Chapters 3 and 4. Another example is the measurement of the spectral radiance of an unknown source using the filter radiometer shown in Figure 1.4. It requires that the geometric factors, the filter transmittance, and detector responsivity be known in order to complete a measurement. Equivalently, if a calibration source of known spectral radiance is available, one can determine the total radiance responsivity R_L and measure the unknown radiance as a ratio of the known radiance. In using this technique, it is important to note that the unknown spectral radiance source should have similar relative spectral radiance distributions as the calibration source, or extra uncertainties in the measurement will result from the difference. This comes about because of the averaging process used to reduce Eq. (1.18) to that of Eq. (1.19), which relies on being able to average the values in the integrand of Eq. (1.19). If a calibration source and an unknown source both do not have either roughly constant values over the bandpass or the same general functional form, then errors in applying Eq. (1.19) will result. A detailed examination of the uncertainty analysis for filter radiometers can be found in the literature [37].

In the measurement equation examples so far discussed, we have assumed that the detector is uniform and that there are no spatial, polarization, or temporal dependencies. In the more general case, this is not always realized, and a more detailed consideration of the measurement equation is necessary. For the calibration and uncertainty analysis of radiometers or complex electro-optical sensors for all the recognized dependencies, the goal is first to design calibration experiments using a standard source if necessary, and independently characterize the radiometer or sensor's overall system responsivity R^T in the spectral, spatial, temporal, and polarization domains according to

$$R^{T}(\lambda_0, \lambda, \theta, \phi, t, P) = R(\lambda_0, \lambda)R(\theta, \phi)R(t)R(P) \tag{1.25}$$

where $R(\lambda_0, \lambda)$ is the overall system spectral responsivity, $R(\theta, \phi)$ is the *spatial responsivity*, also called the field of view responsivity, $R(t)$ is the *temporal responsivity* and $R(P)$ is the *polarization responsivity*. The measurement equations such as Eqs. (1.19) and (1.22) are generally derived for the major domain, that is, the spectral part, with certain assumptions made regarding the spatial and other domains. Therefore, the quantity that is most important to measure independently is the overall system spectral responsivity, $R(\lambda_0, \lambda)$ of the radiometer or sensor system. For spatial and other domains, deviations from the assumptions are assessed and applied as corrections to the measurement equations. Solutions to the modified measurement equations are obtained from results of the calibration experiment at the system level and are compared with predictions from component level specifications and measurements. This procedure allows for accurate calibration of the radiometer or the sensor and determination of the overall uncertainty budget. The example discussed in Appendix A is an illustration of the prediction from component level specifications and measurements for a filter radiometer used as a transfer standard for NASA remote-sensing applications. QA :3

We see that the spectral characterization is the major part of the calibration experiment. For broad-band filter radiometers, the shape of the spectral bandpass function for the radiometer system must be measured and compared to the calculated one to assess the uncertainties as the product of optical transmittance (reflectance) of apertures, filters, mirrors, etc. The calculated transmittance may not reflect the reality because multiple reflections and diffraction effects may alter the spectral characteristics of the throughput of the system. The spectral characterization is usually accomplished by scanning with a calibrated monochromator output with sufficient resolution and recording the system response across the wavelength band. This method of using monochromator output often suffers from the problems of not having sufficient spectral resolution or sufficient intensity in the output. However, modern developments in using tunable laser sources such as the SIRCUS facility at NIST solved these problems not only for the broadband filter radiometers but also for the complex spectroradiometers discussed in Chapter 4 [36, 38].

Also, in the case of filter radiometers, there is the problem of out-of-band leakage. In some applications, adequate long wavelength blocking is a major problem because of lack of availability of suitable materials and the difficulty of designing interference filters to serve the purpose. Short wavelengths can create fluorescence in the optical components and contribute to the out-of-band leakage. These factors, as well as contributions due to scattering, must be carefully assessed at the component level and at the system level to assess the overall uncertainty of the spectral responsivity of the sensor system. One method commonly used is to vary the temperature of a blackbody source to systematically change the peak of the Planckian radiation and measure the responsivity of the system. Long wavelength leakage can be detected and corrected by this method especially for mid or long wave infrared filter radiometers [22]. Again, the tunable laser facility such as SIRCUS is found to be very useful to assess this leakage in the case of the spectroradiometers used by NASA for sea surface temperature measurements and thus enables a correction to be applied to the overall system spectral responsivity. As mentioned earlier, comprehensive discussions of various methods to determine $R(\lambda_0, \lambda)$ can be found in the literature [22, 23]. It should be noted that the term *relative spectral responsivity* is introduced by some authors and refers to quantities such as we have defined but perhaps normalized to an integral or peak value. It is important to ascertain the precise definitions of terms in a particular discussion from the context.

The spatial characterization of a detector or a sensor is very important, because these devices often exhibit spatial non-uniformity. In the case of a sensor based on array detector, the spatial non-uniformity is evaluated by a calibration experiment where the array is flooded by a spatially uniform source at one irradiance level. Such sources are discussed in Chapter 5. In the case of a radiometer based on a single detector element, it is often evaluated by scanning the surface area of the detector with a small spot of radiation from a stable laser or a stable incoherent monochromatic source. The uncertainty evaluation would be different based on the application, whether the radiance, irradiance, or radiant power is measured by the system. These issues are discussed in Chapter 3.

The angular field-of-view characterization is also very important to assess the system performance for the desired linear field of view. It is necessary to know the spatial field of view responsivity $R(\theta, \phi)$ for the system, because errors can be made if a non-uniform source is measured with a non-uniform spatial field of view responsivity. As discussed by Wyatt the errors are due to on-axis performance and/or the measure of off-axis (out-of-field) rejection [22]. The on-axis performance is assessed by the response to a point source at angles close to the optical axis in comparison with the ideal designed field of view of the radiometer system. The off-axis performance is assessed by the response to a point source at angles far from the optical axis and evaluated to many orders of magnitude below the on-axis performance depending on the requirements of the system performance.

The *modulation transfer function* (MTF) is a parameter that describes the optical system response to spatial frequencies and is especially important for imaging systems with array detectors. Imaging radiometers are not dealt with extensively in this volume and the reader is referred to the references for discussion of this topic [26, 39].

It is important to characterize the temporal responsivity, $R(t)$, of a radiometer system, because the flux from the source being observed may change with time, or because intentional chopping of the radiation is employed to discriminate against background. All detectors have a characteristic response time before a signal is detected and an integration time for the signal to reach a stable value for measurement. Therefore, it is important to characterize the frequency response of the system. The frequency response is measured by observing the output response to a modulated light source [22]. This topic is discussed in Chapters 3 and 4.

It is also important to characterize the polarization responsivity, $R(P)$, of the radiometer sensor because mirrors and other possible materials in the optical beam path may introduce polarization or have polarization-dependent properties. For example, scattered optical radiation is frequently polarized, and hence the polarization sensitivity of a sensor measuring it must be characterized by employing polarizer and retarder combinations in various ways [22, 23]. This topic is not pursued in depth in this book.

Further more, *noise and drift* are important, because they affect the repeatability and reproducibility of the data. In general, noise and drift are characterized by employing a stable source of radiation, such as a blackbody, and collecting data at intervals throughout the dynamic range of the system for inclusion as a part of the calibration. Most optical sensor systems exhibit some degree of nonlinearity. The evaluation of the non-linearity of radiometers, associated corrections and uncertainties is very important and is discussed in Chapters 3 and 4.

The calibration of a radiometer system requires an experiment which includes characterization and determination of the corrections and sources of uncertainties listed above. This effort will yield the radiometer response as a function of the radiant, spectral, spatial, temporal, and polarization properties of an appropriate transfer standard. The transfer standard could be a well characterized source or another radiometer which has been calibrated and is traceable to international standards. The resulting equations, such as Eq. (1.24), are often called calibration equations and are derived from approximations in order to perform an inversion of the measurement equation. These approximations introduce elements of uncertainty which must be accounted for.

1.5.1 Uncertainty Nomenclature According to the ISO Guide

No measurement is complete unless it is associated with an uncertainty statement. The acceptance of the ISO guide to the Expression of Uncertainty in Measurement by the National Metrology Laboratories of various countries around the world paved the way for a uniform approach for the expression of uncertainty [17]. The basic concepts and nomenclature are discussed in Chapter 6.

The uncertainty in the result of a measurement generally consists of several components of uncertainty based on the measurement equation for the measurand Y. If we denote the estimated value for Y as y and the best estimates for various other component variables in the measurement equation as $x_1, x_2, x_3, \ldots, x_M$, the measurement equation can be written as

$$y = f(x_1, x_2, x_3, \ldots, x_M) \tag{1.26}$$

The total uncertainty, called the *combined standard uncertainty* $u_c(y)$, for M statistically independent mean values of components is calculated using Eq. (1.26) and the law of propagation of uncertainties

$$u_c(y) = \left[\sum_{j=1}^{M} \left[\frac{\partial f}{\partial x_j} \right]^2 u^2(x_j) \right]^{1/2} \tag{1.27}$$

However, if correlations are present between the components, then the covariances of the variables must be estimated and Eq. (1.27) must be modified with additional terms reflective of the covariance between the variables. This subject is further treated extensively in Chapter 6. The *relative combined standard uncertainty* is defined as $u_{c,r}(y) = u_c(y)/y_a$ and is frequently used because it expresses an uncertainty as a fraction of the measurand, which is a useful way to compare results.

The uncertainty determined by statistical techniques on the basis of direct measurements is referred to in the *ISO guide* as Type A while those which are evaluated by other means (e.g., on the basis of scientific judgment) as Type B. Finally, the *expanded standard uncertainty* is denoted in the *ISO guide* as U and is obtained for an approximate level of confidence (the interval that will cover the true value of the estimated parameter with a given confidence) using the *coverage* factor k. Thus we have $U = ku_c(y)$ and the measurand $Y = y_a \pm U$, where y_a is the measurement result. For example, approximately 95% of the measurements will fall within $\pm 2u_c(y)$, of the mean which corresponds to the case $k \approx 2$ if the distribution represented by y_a and $u_c(y)$ is approximately normal and the sample is large. A 99% level of confidence corresponds to $k \approx 3$. The *ISO guide* also recommends the use of a *coverage factor* of 2 as a default multiplier in which case the level of confidence is approximately 95%.

However, the earth remote sensing community has been dealing with measurement of small changes in signals over extended time periods and has introduced the concepts of *accuracy* and *stability* in a quantitative fashion for time-series analysis of data. Accuracy is defined by the ISO guide as the "closeness of the agreement between the result of the measurement and the true value of the measurand" [17]. So, the term accuracy is measured by the bias or systematic error of the data, that is, the difference between the short-term average of the measured value of a variable and the truth. The short-term average value is the average of a sufficient number of successive measurements of the variable under identical conditions such that the random error is negligible relative to the systematic error. The term stability may be thought of as the extent to which the accuracy remains constant with time. Stability is measured by the maximum excursion of the short-term average measured value of a variable under essentially identical conditions over a decade. The smaller the maximum excursion, the greater the stability of the data set. Chapter 10 uses this terminology and further discussion on this topic can be found in Reference [40].

1.5.2 Application to Radiometric Uncertainty Analysis

As a simple example to illustrate the evaluation of combined standard uncertainty, $u_c(y)$, let us consider Eq. (1.12) for measuring the flux Φ reach-

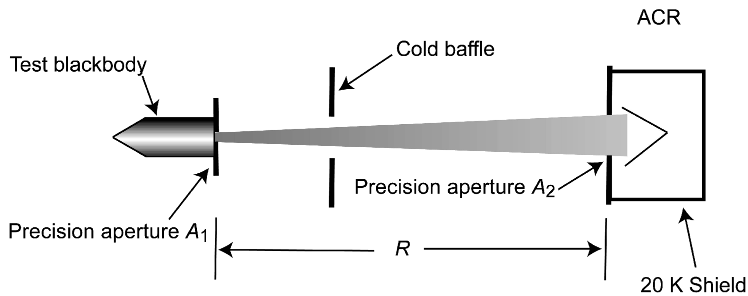

FIG. 1.7. Schematic of a blackbody calibration using an ACR.

ing the aperture of area A_2 of an absolute cryogenic radiometer (ACR) in Figure 1.7. The acronym ACR used here should not be confused with the same acronym commonly used in the literature for Active Cavity Radiometers, which are of non-cryogenic type [17]. This equation is useful as a close approximation for the calibration of the radiance temperature of a point source cryogenic blackbody that is equipped with pinhole apertures. Such blackbodies are used to calibrate infrared sensors in chambers that simulate the cold space background like what sensors in space operate in.

The setup shown in Figure 1.7 applies to the calibration of such blackbodies. The blackbody illuminates a precision aperture A_1 through which radiation passes to the limiting aperture A_2 on the ACR input. The ACR in Figure 1.7 is an electrical substitution radiometer that has the same response for all wavelengths of light and is used to measure the total optical power in absolute units of watts. These radiometers are discussed in detail in Chapters 2 and 3 and function like the radiometer shown in Figure 1.1. In order to calibrate the radiance temperature of the blackbody for its various temperature settings as read by the contact thermometers on the blackbody core, the Stefan–Boltzmann law for the *radiant exitance* M is used. The *radiant exitance* is the flux per unit area of the source emitted into a hemisphere around the source. For a blackbody as a Lambertian source, integration of Eq. (1.4) for the whole hemisphere yields, $M = \pi L$. Therefore, Stefan–Boltzmann law for the set up shown in Figure 1.7 gives the flux Φ in terms of temperature T and geometric factors as

$$\Phi \cong \frac{M}{\pi}\frac{A_1 A_2}{R^2}, \qquad M = \sigma T^4 \tag{1.28}$$

where σ is the Stefan–Boltzmann constant whose value is $5.6704 \times 10^{-8}\,(\mathrm{W\,m^{-2}\,K^{-4}})$. If r_1 and r_2 are the radii of circular precision apertures A_1 and A_2, R is distance between those apertures in Figure 1.7, the flux measured by the ACR is

$$\Phi \cong \frac{A_1 A_2}{R^2} \frac{\sigma}{\pi} T^4 \tag{1.29}$$

Equation (1.29) is the measurement equation to deduce the *radiance temperature* T of the blackbody as

$$T = \left[\frac{\Phi R^2}{\sigma r_1^2 \pi r_2^2} \right]^{1/4} \tag{1.30}$$

Using the procedures defined in the ISO guide to the Expression of Uncertainty in Measurement, the *combined relative standard uncertainty* $u_{c,r}(T)$ will have components of uncertainty consisting of four components arising from r_1, r_2, R, and Φ as

$$u_{c,r}(T) = \frac{1}{4}[u_r^2(\Phi) + 4u_r^2(r_1) + 4u_r^2(r_2) + 4u_r^2(R)]^{1/2} \tag{1.31}$$

The components $u_r(r_1)$, $u_r(r_2)$, and $u_r(R)$ are the relative uncertainties of the measurements of the geometrical terms and contains Type A uncertainties due to measurement statistics as well as uncertainties associated with temperature effects and other corrections to the values used in the calculation. The relative uncertainty in the flux measurement $u_r(\Phi)$ has both Type A and Type B contributions. The uncertainties in the flux measurement have contributions from the ACR itself and diffraction corrections, which are usually wavelength dependent. These various factors need to be evaluated and combined using the square root of the sum of the squares of the individual contributions provided the parameters are uncorrelated. Should correlation among the parameters be important, more detailed treatment like that discussed in Chapter 6 may be necessary. Any correction made to the flux due to diffraction will have an uncertainty associated with it and hence will contribute to the overall uncertainty in the flux in Eq. (1.31). The corrections for diffraction in radiometry are discussed in Chapter 9.

The treatment given here is a simplified analysis for a single setting of the blackbody temperature. A comprehensive analysis that includes data at different temperatures of the blackbody, a regression analysis to establish a calibration equation to predict radiance temperature in terms of the contact temperature reading of the blackbody, and a detailed analysis of the uncertainties involved in all the contributing components are presented in Appendix A.

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